Bevezetés a fizika filozófiájába

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Hogyan is kell érteni a relativitás elvét a klasszikus és a relativisztikus fizikában?

1 . It is a widely accepted view that special relativity, beyond its claim about space and time, is a theory providing a powerful method for the physics of objects moving at constant velocities. The basic idea is the following: Consider a physical object at rest in an arbitrary inertial frame K. Assume we know the relevant physical equations and know the solution of the equations describing the physical properties of the object in question when it is at rest. All these things are expressed in the terms of the space and time coordinates x_1, x_2, x_3, t and some other quantities defined in K on the basis of x_1, x_2, x_3, t. We now inquire as to the same physical properties of the same object when it is, as a whole, moving at a given constant velocity relative to K. In other words, the issue is how these physical properties are modified when the object is in motion. The standard method for solving this problem is based on the *relativity principle/Lorentz covariance*. It follows from the covariance of the laws of nature relative to Lorentz transformations that the same equations hold for the primed variables $x'_1, x'_2, x'_3, t', \ldots$ defined in the co-moving inertial frame K'. Moreover, since the moving object is at rest in the co-moving reference frame K', it follows from the relativity principle that the same rest-solution holds for the primed variables. Finally, we obtain the solution describing the system moving as a whole at constant velocity by expressing the primed variables through the original $x_1, x_2, x_3, t, \ldots$ of K, applying the Lorentz transformation.

This is the way we usually solve problems such as the electromagnetic field of a moving point charge, the Lorentz deformation of a rigid body, the loss of phase suffered by a moving clock, the dilatation of the mean life of a cosmic ray μ-meson, etc.

I would like to show that this method, in general, is not correct; the system described by the solution so obtained is not necessarily identical with the original system set in collective motion. The reason is, as will be shown, that Lorentz covariance in itself does not guarantee that the physical laws in question satisfy the relativity principle in general. The principle of relativity actually only holds for the equilibrium quantities characterising the equilibrium state of dissipative systems.

The relativity principle

2 . The first formulation of the relativity principle appeared in the following passage of Galilei's *Dialogue*:

> ... the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also. (Galilei 1953, p. 187)

In Einstein's formulation it is the following:

> If, relative to K, K' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to K' according to exactly the same general laws as with respect to K. (Einstein 1920, p. 16)

Finally, in a typical text book formulation, relativity principle is the following assertion:

> All the laws of physics take the same form in any inertial frame.

Let us try to unpack what this principle actually asserts. First of all it must be clear that the *same law* of physics must take the same form in all inertial frames. What are the same laws of physics in different inertial frames? Of course, the laws of physics can be identified by means of the physical phenomena they describe. If so, then one can think that the same physical phenomenon must be described by the same solution of the same equations in all frames. This is however obviously not the case. For example, the motion of the plasma of the same solar flare is described differently by two observers in two different inertial frames. Thus, the opposite must be true: *different* physical phenomena are described by the same solutions of the same equations in different inertial frames. So, our first task will be to clarify what are those different physical phenomena the description of which must have the same form in all inertial frame.

3 . The second problem is how the phrase "same form" should be understood. For, in terms of different variables, one and the same physical law in one and the same inertial frame of reference can be expressed in different forms. Therefore we have to add to the principle that the physical laws must be expressed in terms of the same physical quantities. This immediately raises the next question of how the physical quantities defined in different inertial frames are identified. Obviously, we identify those physical quantities that have identical empirical definitions. It is however far from obvious how these identical empirical definitions are actually understood.

The empirical/operational definitions require *etalon* measuring equipments. But how do the observers in different reference frames share these *etalon* measuring equipments? Do they all base their definitions on the same *etalon* measuring equipments? They must do something like that, otherwise any comparison between their observations would be meaningless. But, is principle of relativity really understood in this way? Is it true that the laws of physics in K and K', which ought to take the same form, are expressed in terms of physical quantities defined/measured with one and the same standard measuring equipments? Not exactly! "The cause of all these correspondences of effects is the fact that *the ship's motion is common to all the things* [italics mine] contained in it"—Galilei writes in the above quoted passage. Or, consider how Einstein applies the principle:

> Let there be given a stationary rigid rod; and let its length be l as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod. We now inquire as to the length of the moving rod, and imagine its length to be ascertained by the following two operations:
>
> (a) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way *as if all three were at rest* [italics mine].
>
> (b) ...
>
> In accordance with the principle of relativity the length to be discovered by the operation (a)–we will call it "the length of the rod in the moving system"–must be equal to the length l of the stationary rod. (Einstein 1905)

That is to say, if the standard measuring equipment defining a physical quantity X^K is, for example, at rest in K and, therefore, moving in K', then the observer in K' does not define the corresponding $X^{K'}$ as the physical quantity obtainable by means of the original standard equipment—being at rest in K and moving in K'—but rather as the one obtainable by means of the same standard equipment *in another state of motion*, namely when it is at rest in K' and moving in K.

4 . Let us return to the first problem posed at the end of Point **2**. Now we can specify those different physical phenomena the description of which must have the same form in all inertial frame. For, what we told about the measuring equipments, also holds for the physical systems to be measured. That is to say, the principle says that the description of the behaviour of a system when it is co-moving with inertial frame K takes the same form as the description of the same system when it is co-moving with inertial frame K'.

5 . Putting all these details together, now we are ready to give a more accurate formulation of the relativity principle:

Relativity Principle *The laws of physics describing the behaviour of a system co-moving as a whole with inertial frame K, expressed in terms of the results of measurements obtainable by means of measuring-rods, clocks, etc., co-moving with K takes the same form as the laws of physics describing the similar behaviour of the same system when it is co-moving with inertial frame K', expressed in terms of the measurements with the same equipments when they are co-moving with K'.*

Whether or not the relativity principle holds is, it must be clear, a contingent fact of nature. If the laws of physics known *in any one inertial frame of reference*, say K, account for all physical phenomena then these laws unambiguously predetermine whether the principle holds or not. The reason is that these laws also describe the behaviour of moving (relative to K) physical systems including both the measuring equipments co-moving with another inertial frame K' and the system to be measured co-moving with K'.

Nevertheless, there are still vague points here. But before entering in the discussion of these further problems, let us recall how the relativity principle implies Galilean/Lorentz covariance.

Galilean and Lorentz covariance

6 . Consider two inertial frames of reference K and K'. Assume that K' is moving at constant velocity v relative to K along the axis of x. Assume that laws of physics are known and empirically confirmed in inertial frame K, including the laws describing the behaviour of physical objects in motion relative to K. Denote $x(A), y(A), z(A), t(A)$ the space and time tags of an event A, obtainable by means of measuring-rods and clocks at rest relative to K, and denote $x'(A), y'(A), z'(A), t'(A)$ the similar data of the same event, obtainable by means of measuring-rods and clocks co-moving with K'. In the approximation of classical physics ($v \ll c$), the relationship between $x'(A), y'(A), z'(A), t'(A)$ and $x(A), y(A), z(A), t(A)$ can be described by the Galilean transformation:

$$
\begin{aligned}
t'(A) &= t(A) && (1)\\
x'(A) &= x(A) - v\,t(A) && (2)\\
y'(A) &= y(A) && (3)\\
z'(A) &= z(A) && (4)
\end{aligned}
$$

Due to the relativistic deformations of measuring-rods and clocks, the exact relationship is described by the Lorentz transformation:

$$
\begin{aligned}
t'(A) &= \frac{t(A) - \frac{v\,x(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} && (5)\\
x'(A) &= \frac{x(A) - v\,t(A)}{\sqrt{1-\frac{v^2}{c^2}}} && (6)\\
y'(A) &= y(A) && (7)\\
z'(A) &= z(A) && (8)
\end{aligned}
$$

Since physical quantities are defined by the same operational procedure in all inertial frames, the transformation rules of the space and time coordinates (usually) predetermine the transformations rules of the other physical variables. So, depending on the context, we will mean by Galilean/Lorentz transformation not only the transformation of the space and time tags, but also the corresponding transformation of the other variables in question.

Following Einstein's 1905 paper, the Lorentz transformation rules are usually derived from the relativity principle—the general validity of which we are going to challenge in this essay. As we will see, this derivation is not in contradiction with our final conclusions. Nevertheless, it is worth while to mention that Lorentz transformation can also be derived independently of the principle of relativity, directly from the facts that a clock slows down by factor $\sqrt{1-v^2/c^2}$ when it is gently accelerated from K to K' and a measuring-rod suffers a contraction by factor $\sqrt{1-v^2/c^2}$ when it is gently accelerated from K to K' (see Point **37**).

7 . In classical physics, the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames can be connected through the Galilean transformation. According to special relativity, the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames are connected through the Lorentz transformation. Consequently, the laws of physics must preserve their forms with respect of the Galilean/Lorentz transformation. Thus, it must be emphasised, the Galilean/Lorentz covariance is a consequence not only of the fact that the laws of physics satisfy the relativity principle but also of the other physical fact that the space and time tags in different inertial frames are connected through the Galilean/Lorentz transformation.

8 . Let us now try to unpack the verbal formulations of the relativity principle in a more mathematical way. Let $\mathcal{E}$ be a set of differential equations describing the behaviour of the system in question. Let us denote by ψ a typical set of (usually initial) conditions determining a unique solution of $\mathcal{E}$. Let us denote this solution by $[\psi]$. Denote $\mathcal{E}'$ and ψ' the equations and conditions obtained from $\mathcal{E}$ and ψ by substituting every x_i with x_i', and t with t', etc. Denote $G_v(\mathcal{E}), G_v(\psi)$ and $\Lambda_v(\mathcal{E}), \Lambda_v(\psi)$ the set of equations and conditions expressed in the primed variables applying the Galilean and the Lorentz transformations, respectively (including, of course, the Galilean/Lorentz transformations of all other variables different from the space and time coordinates). Finally, in order to give a strict mathematical formulation of relativity principle, we have to fix two further concepts, the meaning of which are vague: Let a solution $[\psi_0]$ is stipulated to describe the behaviour of the system when it is, as a whole, at rest relative to K. Denote ψ_v the set of conditions and $[\psi_v]$ the corresponding solution of $\mathcal{E}$ that are stipulated to describe the similar behaviour of the system as $[\psi_0]$ but, in addition, when the system was previously set, as a whole, into a collective translation at velocity v.

Now, what relativity principle states is *equivalent* to the following:

$$G_v(\mathcal{E}) = \mathcal{E}' \tag{9}$$

$$G_v(\psi_v) = \psi_0' \tag{10}$$

in the case of classical mechanics, and

$$\Lambda_v(\mathcal{E}) = \mathcal{E}' \tag{11}$$

$$\Lambda_v(\psi_v) = \psi_0' \tag{12}$$

in the case of special relativity.

9 . Although, in conjunction with the Galilean/Lorentz transformation rules, relativity principle implies Galilean/Lorentz covariance, the relativity principle, as we can see, *is not equivalent* to the Galilean covariance (9) in itself or the Lorentz covariance (11) in itself. It is equivalent to the satisfaction of (9) in conjunction with condition (10) in classical physics, or (11) in conjunction with (12) in relativistic physics.

10 . Note, that $\mathcal{E}$, ψ_0, and ψ_v as well as the transformations G_v and Λ_v are given by contingent facts of nature. It is therefore a contingent fact of nature whether a certain law of physics is Galilean or Lorentz covariant, and, *independently*, whether it satisfies the principle of relativity. The relativity principle and its consequence the principle of Lorentz covariance are certainly normative principles in contemporary physics, providing a heuristic tool for constructing new theories. We must emphasise however that these normative principles, as any other fundamental law of physics, are based on empirical facts; they are based on the observation that the behaviour of any moving physical object satisfies the principle of relativity. I will show, however, that the laws of relativistic physics, in general, do not satisfy this condition.

11 . Before we begin analysing our examples, it must be noted that the major source of confusion is that there still exists some vagueness in the relativity principle (Point **5**). Namely, the vagueness of the concepts like "a system co-moving as a whole with an inertial frame" and "the similar behaviour of the same system when it is co-moving with a given inertial frame". In other words, the vagueness of the definitions of conditions ψ_0 and ψ_v. In principle any $[\psi_0]$ can be considered as a "solution describing the system's behaviour when it is, as a whole, at rest relative to K". Given any one fixed ψ_0, it is far from obvious, however, what is the corresponding ψ_v. When can we say that $[\psi_v]$ describes the similar behaviour of the same system when it was previously set into a collectives motion at velocity v? As we will see, there is an unambiguous answer to this question in the Galileo covariant classical physics. But ψ_v is vaguely defined in relativity theory. Note that Einstein himself uses this concept in a vague way, for example in the passage quoted in Point **3**. (What exactly does "a uniform motion of parallel translation with velocity v ... imparted to the rod" mean?)

The following examples will illustrate that the vague nature of this concept complicates matters. In all examples we will consider a set of interacting particles. We assume that the relevant equations describing the system are Galilean/Lorentz covariant, that is (9) and (11) are satisfied respectively. As it follows from the covariance of the corresponding equations, $G_v^{-1}(\psi_0')$ and, respectively, $\Lambda_v^{-1}(\psi_0')$ are conditions determining new solutions of $\mathcal{E}$. The question is whether these new solutions $\left[G_v^{-1}(\psi_0')\right]$ and $\left[\Lambda_v^{-1}(\psi_0')\right]$ are identical with $[\psi_v]$—the one determined by ψ_v. If so then the relativity principle is satisfied.

The relativity principle in classical mechanics

12 . Let us start with an example illustrating how the relativity principle works in classical mechanics. Consider a system consisting of two point masses connected with a spring (Fig. 1). The equations of motion in K,

$$m\frac{d^2x_1(t)}{dt^2} = k\left(x_2(t) - x_1(t) - L\right) \tag{13}$$

$$m\frac{d^2x_2(t)}{dt^2} = -k\left(x_2(t) - x_1(t) - L\right) \tag{14}$$

are indeed covariant with respect to the Galilean transformation, that is, expressing (13)–(14) in terms of variables x', t' they have exactly the same form as before:

$$m\frac{d^2x_1'(t')}{dt'^2} = k\left(x_2'(t') - x_1'(t') - L\right) \tag{15}$$

$$m\frac{d^2x_2'(t')}{dt'^2} = -k\left(x_2'(t') - x_1'(t') - L\right) \tag{16}$$

Consider the solution of the (13)–(14) belonging to an arbitrary initial condition ψ_0:

$$\begin{aligned} x_1(t=0) &= x_{10} \\ x_2(t=0) &= x_{20} \\ \left.\tfrac{dx_1}{dt}\right|_{t=0} &= v_{10} \\ \left.\tfrac{dx_2}{dt}\right|_{t=0} &= v_{20} \end{aligned} \tag{17}$$

The corresponding "primed" initial condition ψ_0' is

$$\begin{aligned} x_1'(t'=0) &= x_{10} \\ x_2'(t'=0) &= x_{20} \\ \left.\tfrac{dx_1'}{dt'}\right|_{t'=0} &= v_{10} \\ \left.\tfrac{dx_2'}{dt'}\right|_{t'=0} &= v_{20} \end{aligned} \tag{18}$$

Applying the inverse Galilean transformation we obtain a set of conditions $G_v^{-1}(\psi_0')$ determining a new solution of the original equations:

$$\begin{aligned} x_1(t=0) &= x_{10} \\ x_2(t=0) &= x_{20} \\ \left.\tfrac{dx_1}{dt}\right|_{t=0} &= v_{10} + v \\ \left.\tfrac{dx_2}{dt}\right|_{t=0} &= v_{20} + v \end{aligned} \tag{19}$$

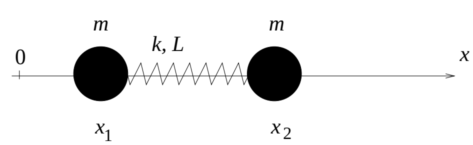


Figure 1. *Two point masses are connected with a spring of equilibrium length L and of spring constant k*

One can recognise that this is nothing but ψ_v. It is the set of the original initial conditions in superposition with a uniform translation at velocity v. That is to say, the corresponding solution describes the behaviour of the same system when it was (at $t = 0$) set into a collective translation at velocity v, in superposition with the original initial conditions.

13 . In classical mechanics, as we have seen from this example, the equations of motion not only satisfy the Galilean covariance, but also satisfy the condition (10). The principle of relativity holds for *all details of the dynamics* of the system. There is no exception to this rule. In other words, if the world were governed by classical mechanics, relativity principle would be a universally valid principle. With respect to later questions, it is worth noting that the Galilean principle of relativity therefore also holds for the equilibrium characteristics of the system, if the system has dissipations. Imagine for example that the spring has dissipations during its distortion. Then the system has a stable equilibrium state in which the equilibrium distance between the particles is L. When we initiate the system in collective motion corresponding to (19), the system relaxes to another equilibrium state in which the distance between the particles is the same L.

Violation of relativity principle in relativistic physics

14 . Let us turn now to the relativistic examples. It is widely held that the new solution determined by $\Lambda_v^{-1}(\psi_0')$, in analogy to the solution determined by $G_v^{-1}(\psi_0')$ in classical mechanics, describes a system identical with the original one, but co-moving with the frame K', and that the behaviour of the moving system, expressed in terms of the results of measurements obtainable by means of measuring-rods and clocks co-moving with K' is, due to Lorentz covariance, the same as the behaviour of the original system, expressed in terms of the measurements with the equipments at rest in K—in accordance with the principle of relativity. However, the situation is in fact far more complex, as I will now show.

15 . Imagine a system consisting of interacting particles (for example, relativistic particles coupled to electromagnetic field). Consider the solution of the Lorentz covariant equations in question that belongs to the following general initial conditions:

$$\mathbf{r}_i(t=0) = \mathbf{R}_i \tag{20}$$

$$\left.\frac{d\mathbf{r}_i(t)}{dt}\right|_{t=0} = \mathbf{w}_i \tag{21}$$

(Sometimes the initial conditions for the particles unambiguously determine the initial conditions for the whole interacting system. Anyhow, we are omitting the initial conditions for other variables which are not interesting now.) It follows from the Lorentz covariance that there exists a solution of the "primed" equations, which satisfies the same conditions,

$$\mathbf{r}_i'(t'=0) = \mathbf{R}_i \tag{22}$$

$$\left.\frac{d\mathbf{r}_i'(t')}{dt'}\right|_{t'=0} = \mathbf{w}_i \tag{23}$$

Eliminating the primes by means of the Lorentz transformation we obtain

$$t_i^\star = \frac{\frac{v}{c^2}R_{xi}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{24}$$

$$\mathbf{r}_i^{new}\left(t=t_i^\star\right) = \begin{pmatrix} \frac{R_{xi}}{\sqrt{1-\frac{v^2}{c^2}}} \\ R_{yi} \\ R_{zi} \end{pmatrix} \tag{25}$$

and

$$\left.\frac{d\mathbf{r}_i^{new}(t)}{dt}\right|_{t_i^\star} = \begin{pmatrix} \frac{w_{xi}+v}{1+\frac{w_{xi}v}{c^2}} \\ w_{yi} \\ w_{zi} \end{pmatrix} \tag{26}$$

It is difficult to tell what the solution deriving from such a nondescript “initial” condition is like, but it is not likely to describe the original system in collective motion at velocity v. The reason for this is not difficult to understand. Let me explain it by means of a well known old example (Dewan and Beran 1959, Evett and Wangsness 1960, Dewan 1963, Evett 1972, Bell 1987, Nikolic 1999, Field 2004).

16 . In stead of two rockets connected with a thread—as the original example says—consider the system consisting of two particles connected with a spring (Point **12**). Let us first ignore the spring. Assume that the two particles are at rest relative to K, one at the origin, the other at the point d, where $d = L$, the equilibrium length of the spring when it is at rest. It follows from (24)–(26) that the Lorentz boosted system corresponds to two particles moving at constant velocity v, such that their motions satisfy the following conditions:

$$\begin{aligned} t_1^\star &= 0 \\ t_2^\star &= \frac{\frac{v}{c^2}d}{\sqrt{1-\frac{v^2}{c^2}}} \\ r_1^{new}(0) &= 0 \\ r_2^{new}\left(\frac{\frac{v}{c^2}d}{\sqrt{1-\frac{v^2}{c^2}}}\right) &= \frac{d}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned} \tag{27}$$

However, the corresponding new solution of the equations of motion does not “know” about how the system was set into motion and/or how the state of the system corresponding to the above conditions comes about. Consider the following possible scenarios:

Example 1

The two particles are at rest; the distance between them is d (see Fig. 2). Then, particle 1 starts its motion at constant velocity v at $t = 0$ from the point of coordinate 0 (the last two dimensions are omitted); particle 2 start its motion at velocity v from the point of coordinate d with a delay at time t''. Meanwhile particle 1 moves closer to particle 2 and the distance between them is $d'' = d\sqrt{1-v^2/c^2}$, in accordance with the Lorentz contraction. Now, one can say that the two particles are in collective motion at velocity v relative to the original system K—or, equivalently, they are collectively at rest relative to K'—for times $t > t_2^\star = vd/\left(c^2\sqrt{1-v^2/c^2}\right)$. In this particular case they have actually been moving in this way since t''. Before that time, however, the particles moved relative to each other, in other words, the system underwent deformation.

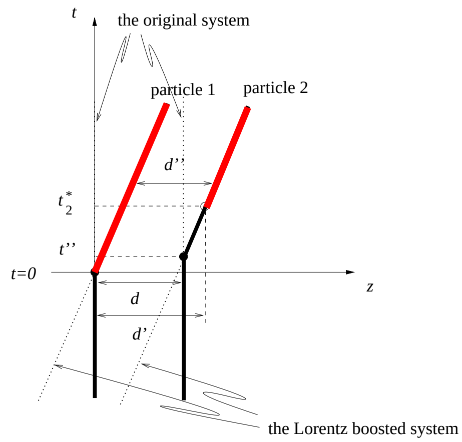


Figure 2. *Both particles are at rest. Then particle 1 starts its motion at* $t = 0$. *The motion of particle 2 is such that it goes through the point* $(t_2^\star, d')$, *where* $d' = d/\sqrt{1 - v^2/c^2}$, *consequently it started from the point of coordinate* d *at* $t'' = d\left(v/\left(c^2\sqrt{1 - v^2/c^2}\right) - \left(1 - \sqrt{1 - v^2/c^2}\right)/\left(v\sqrt{1 - v^2/c^2}\right)\right)$. *The distance between the particles at* t'' *is* $d'' = d\sqrt{1 - v^2/c^2}$, *in accordance with the Lorentz contraction.*

Example 2

Both particles started at $t = 0$, but particle 2 was previously moved to the point of coordinate $d\sqrt{1 - v^2/c^2}$ and starts from there. (Fig. 3)

Example 3

Both particles started at $t = 0$ from their original places. The distance between them remains d (Fig. 4). They are in collective motion at velocity v, although this motion is not described by the Lorentz boost.

Example 4

If, however, they are connected with the spring (Fig. 5), then the spring (when moving at velocity v) first finds itself in a non-equilibrium state of length d, then it relaxes to its equilibrium state (when moving at velocity v) and—assuming that the equilibrium properties of the spring satisfy the relativity principle, which we will argue for later on—its length (the distance of the particles) would relax to $d\sqrt{1 - v^2/c^2}$, according to the Lorentz boost.

17 . We have seen from these examples that the relationship between the Lorentz boost—the motion determined by the conditions $\Lambda_v^{-1}(\psi_0')$—and the systems being in collective motion—determined by ψ_v—is not so trivial. In Examples 1 and 2—although, at least for large t, the system is identical with the one obtained through the Lorentz boost—it would be entirely counter intuitive

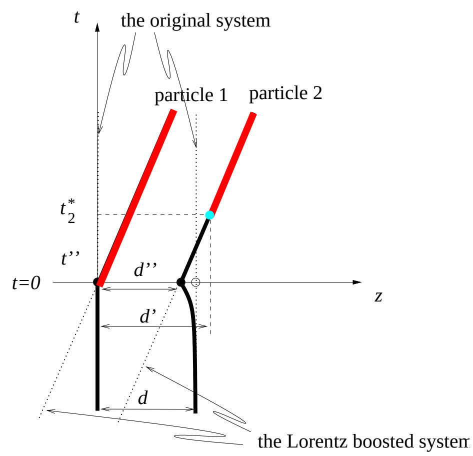


Figure 3. *Both particles start at* $t = 0$. *Particle 2 is previously moved to the point of coordinate* $d'' = d\sqrt{1 - v^2/c^2}$.

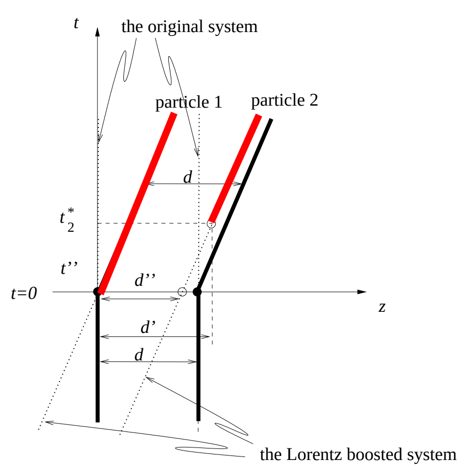


Figure 4. *Both particles start at* $t = 0$ *from the original places. The distance between the particles does not change.*

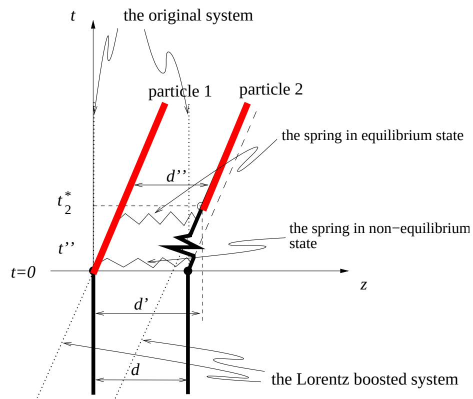


Figure 5. *The particles are connected with a spring (and, say, the mass of particle 1 is much larger)*

to say that we simply set the system in collective motion at velocity v, because we first distorted it: in Example 1 the particles were set into motion at different moments of time; in Example 2, before we set them in motion, one of the particles was relocated relative to the other. In contrast, in Examples 3 and 4 we are entitled to say that the system was set into collective motion at velocity v. But, in Example 3 the system in collective motion is different from the Lorentz boosted system (for all t), while in Example 4 the moving system is indeed identical with the Lorentz boosted one, at least for large t, after the relaxation process.

Thus, as Bell rightly pointed out:

> Lorentz invariance alone shows that for any state of a system at rest there is a corresponding 'primed' state of that system in motion. But it does not tell us that if the system is set anyhow in motion, it will actually go into the 'primed' of the original state, rather than into the 'prime' of some *other* state of the original system. (Bell 1987, p. 75)

18 . However, neither Bell's paper nor the preceding discussion of the "two rockets problem" provide proper explanation of this fact. For instance, after the above passage Bell continues:

> In fact, it will generally do the latter. A system set brutally in motion may be bruised, or broken, or heated or burned. For the simple classical atom similar things could have happened if the nucleus, instead of being moved smoothly, had been *jerked*. The electron could be left behind completely. Moreover, a given acceleration is or is not sufficiently gentle depending on the orbit in question. An electron in a small, high frequency, tightly bound orbit, can follow closely a nucleus that an electron in a more remote orbit

> – or in another atom – would not follow at all. Thus we can only assume the Fitzgerald contraction, etc., for a coherent dynamical system whose configuration is determined essentially by internal forces and only little perturbed by gentle external forces accelerating the system as a whole. (*Ibid.*, p. 75)

The possible "damage" of the system due to "brutal" acceleration is a completely different issue (to which we will return in Point **26**) which obscures a more essential problem. As the above examples show,[1] gentle acceleration in itself does not guarantee that the Lorentz boosted solution describes the original system gently accelerated from K to K'.

19 . Before I proceed to formulate my thesis about this question, let me give one more example.

Example 5

Consider a rod at rest in K. The length of the rod is l. At a given moment of time t_0 we take a record about the positions and velocities of all particles of the rod:

$$r_i(t=t_0) = R_i \tag{28}$$

$$\left.\frac{dr_i(t)}{dt}\right|_{t=t_0} = w_i \tag{29}$$

Then, forget this system, and imagine another one which is initiated at moment $t = t_0$ with the initial condition (28)–(29). No doubt, the new system will be identical with a rod of length l, that continues to be at rest in K.

Now, imagine that the new system is initiated at $t = t_0$ with the initial condition

$$r_i(t=t_0) = R_i \tag{30}$$

$$\left.\frac{dr_i(t)}{dt}\right|_{t=t_0} = w_i + v \tag{31}$$

instead of (28)–(29). No doubt, in a very short interval of time $(t_0, t_0 + \Delta t)$ this system is a rod of length l, moving at velocity v; the motion of each particle is a superposition of its original motion, according to (28)–(29), and the collective translation at velocity v. In other words, it is a rod co-moving with the reference frame K'. Still, its length is l, contrary to the principle of relativity, according to which the rod should be of length $l\sqrt{1 - v^2/c^2}$—as a consequence of $l' = l$.

[1] In our examples we omitted the acceleration period—symbolised by a black point on the figures—for the sake of simplicity.

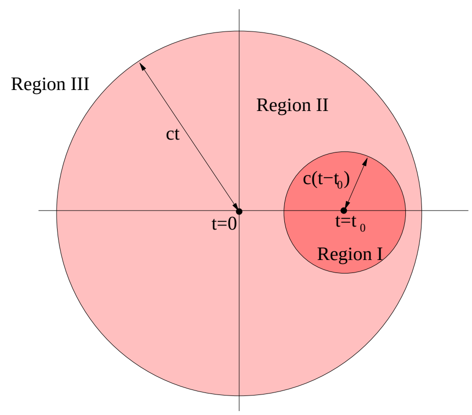


Figure 6. *Scheme of regions I, II and III*

The restricted relativity principle as a principle of thermodynamics

20 . The resolution of this "contradiction" is that the system initiated in state (30)–(31) at time t_0 finds itself in a non-equilibrium state and then, due to certain dissipations, it *relaxes* to the *new* equilibrium state. What such a new equilibrium state is like, depends on the details of the dissipation/relaxation process. It is, in fact, a *thermodynamical* question. The concept of "gentle acceleration" not only means that the system does not go irreversibly far apart from its equilibrium state, but, more essentially, it incorporates the assumption that there is such a dissipation/relaxation phenomenon.

Without entering into the quantum mechanics of solid state systems, a good way to picture it is imagine that the system is radiating during the relaxation period. This process can be followed in details by looking at one single point charge accelerated from K to K' (see Jánossy 1971, pp. 208-210). Suppose the particle is at rest for $t < 0$, the acceleration starts at $t = 0$ and the particle moves with constant velocity v for $t \geq t_0$. Using the retarded potentials, we can calculate the field of the moving particle at some time $t > t_0$. We find three zones in the field (see Fig. 6). In Region I, surrounding the particle, we find the "Lorentz-transformed Coulomb field" of the point charge moving at constant velocity (see (71)–(76) in Point **44**). This is the solution we usually find in textbooks. In Region II, surrounding Region I, we find a radiation field travelling outwards which was emitted by the particle in the period $0 < t < t_0$ of acceleration. Finally, outside Region II, the field is produced by the particle at times $t < 0$. The field in Region III is therefore the Coulomb field of the charge at rest (Point **44** eqs. (65)–(70)). Thus, the principle of relativity *never* holds exactly. Although, the region where "the principle holds" (Region I) is blowing up at the speed of light. In this way the whole configuration relaxes to a solution which is identical with the one derived from the principle of relativity.

21 . Thus, we must draw the conclusion that, in spite of the Lorentz covariance of the equations, whether or not the solution determined by the condition $\Lambda_v^{-1}(\psi'_0)$ is identical with the solution belonging to the condition ψ_v, in other words, whether or not the relativity principle holds, depends on the details of the dissipation/relaxation process in question, *given that 1) there is dissipation in the system at all and, 2) the physical quantities in question, to which the relativity principle applies, are equilibrium quantities characterising the equilibrium properties of the system.* For instance, in Example 5, the relativity principle does not hold for all dynamical details of all particles of the rod. The reason is that many of these details are sensitive to the initial conditions. The principle holds only for some macroscopic equilibrium properties of the system, like the length of the rod. It is a typical feature of a dissipative system that it unlearns the initial conditions; some of the properties of the system in equilibrium state, after the relaxation, are independent from the initial conditions. The limiting ($t \to \infty$) electromagnetic field of the moving charge and the equilibrium length of a solid rod are good examples. These equilibrium properties are completely determined by the equations themselves *independently of the initial conditions.* If so, the Lorentz covariance of the equations in itself guarantees the satisfaction of the principle of relativity *with respect to these properties*: Let X be the value of such a physical quantity—characterising the equilibrium state of the system in question, fully determined by the equations independently of the initial conditions—ascertained by the measuring devices at rest in K. Let X' be the value of the same quantity of the same system when it is in equilibrium and at rest relative to the moving reference frame K', ascertained by the measuring devices co-moving with K'. If the equations are Lorentz covariant, then $X = X'$. We must recognise that whenever in relativistic physics we derive correct results by applying the principle of relativity, we apply it for such particular equilibrium quantities. *But the relativity principle, in general, does not hold for the whole dynamics of the systems in relativity theory,* in contrast to classical mechanics.

22 . When claiming that relativity principle, in general, does not hold for the whole dynamics of the system, a lot depends on what we mean by the system set into uniform motion. One has to admit that this concept is still vague. As we pointed out, it was not clearly defined in Einstein's formulation of the principle either. By leaving this concept vague, Einstein tacitly assumes that these details are irrelevant. However, they can be irrelevant only if the system has dissipations and the principle is meant to be valid only for some equilibrium properties with respect to which the system unlearns the initial conditions. So the best thing we can do is to keep the classical definition of ψ_v: Consider a system of particles the motion of which satisfies the following "initial" conditions:[2]

$$\begin{array}{rcl} \mathbf{r}_i(t=t_0) & = & \mathbf{R}_{i0} \\ \frac{d\mathbf{r}_1}{dt}\Big|_{t=t_0} & = & \mathbf{V}_{i0} \end{array} \tag{32}$$

[2] A condition like (32) does not necessarily mean either that $t_0 = 0$ nor that the solution in question describes the motion only for $t \geq t_0$, it just fixes a particular solution by prescribing the state of the particle at a given moment of time.

The system is set in collective motion at velocity $\mathbf{v}$ at the moment of time t_0 if its motion satisfies

$$\begin{aligned} \mathbf{r}_i(t = t_0) &= \mathbf{R}_{i0} \\ \left.\frac{d\mathbf{r}_1}{dt}\right|_{t=t_0} &= \mathbf{V}_{i0} + \mathbf{v} \end{aligned} \tag{33}$$

I have basically two arguments for such a choice:

(a) The first is a methodological one. The usual Einsteinian derivation of Lorentz transformation, simultaneity in K', etc., starts with the declaration of the relativity principle. In order to formulate the principle, we need the concept of a physical system in uniform motion relative to K. This concept, therefore, must logically precede relativity theory. (See also Point **??**)

(b) The second support comes from what Bell calls "Lorentzian pedagogy".

> Its special merit is to drive home the lesson that the laws of physics in any one reference frame account for all physical phenomena, including the observations of moving observers. And it is often simpler to work in a single frame, rather than to hurry after each moving objects in turn. (Bell 1987, p. 77.)

In reference frame K, the concept of setting a system of state (32) in collective motion at velocity $\mathbf{v}$ in turn means nothing but setting it in state (33).

23 . Thus, we have seen that in classical mechanics the principle of relativity is, indeed, a universal principle. It holds, without any restriction, for *all* dynamical details of *all* possible systems described by classical mechanics. In contrast, in relativistic physics this is not the case:

1. The principle of relativity is not a universal principle. It does not hold for the whole range of validity of the Lorentz covariant laws of relativistic physics, but only for the equilibrium quantities characterising the equilibrium states of dissipative systems. Since dissipation, relaxation and equilibrium are thermodynamical conceptions *par excellence*, the special relativistic principle of relativity is actually a thermodynamical principle, rather than a general principle satisfied by all dynamical laws of physics describing all physical processes in details. One has to recognise that the special relativistic principle of relativity is experimentally confirmed only in such restricted sense.

2. The satisfaction of the principle of relativity in such restricted sense is indeed guaranteed by the Lorentz covariance of those physical equations that determine, independently of the initial conditions, the equilibrium quantities for which the principle of relativity holds. In general, however, Lorentz covariance of the laws of physics does not guarantee the satisfaction of the relativity principle.

3. It is an experimentally confirmed fact of nature that some laws of physics are *ab ovo* Lorentz covariant. However, since relativity principle is not a universal principle, it does not entitle us to infer that Lorentz covariance is a fundamental symmetry of physics.

4. The fact that the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames can be connected through the Lorentz transformation is compatible with our general observation that the principle of relativity only holds for such equilibrium quantities as the length of a solid rod or the characteristic periods of a clock-like system.

The fact that relativity principle is not a universal principle throws new light upon the discussion of how far the Einsteinian special relativity can be regarded as a principle theory relative to the other (constructive) approaches (cf. Einstein 1969, p. 57; Bell 1992; Brown and Pooley 2001; Brown 2001; 2003). It can also be interesting from the point of view of other reflections on possible violations of Lorentz covariance (see, for example, Kostelecký and Samuel 1989).

It must be emphasised that the physical explanation of this more complex picture is rooted in the physical deformations of moving measuring-rods and moving clocks by which the space and time tags are defined in moving reference frames. In Einstein's words:

> A Priori it is quite clear that we must be able to learn something about the physical behaviour of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. (Einstein 1920, p. 35)

Since therefore Lorentz transformation itself is not merely a mathematical concept without contingent physical content, we must not forget the real physical content of Lorentz covariance and relativity principle.

Comments

24 . It is sometimes thought that the Lorentz transformations, and the relativity principle, say nothing about what happens when a physical system that is at rest in reference frame K is accelerated in such a way that it becomes at rest in another reference frame K'. They are only about the relations between systems that already were at rest in K and K', respectively; and that are in the same conditions as judged from their respective rest frames.

In this view, however, beyond the vagueness of the concept of "a system being at rest in a given reference frame" which has been our main concern so far, there also appears a methodological nonsense. How can our physical theories, including the Lorentz transformation rules and the relativity principle, be empirically confirmed scientific theories, if we have no empirical knowledge about the systems' behaviours when they are accelerated from one reference frame into the other? How can we identify systems of the same kind, "living" in different reference frames K and K', without having experience about a system, say, in K accelerated in such a way that it becomes a system moving together with the other reference frame K'? How can we ascertain they identical states? How can we transfer the standard measuring equipments from one frame to the other, if we have no empirical information about their behaviours when they are moving? Or, if it is taken that we have independent standard equipments in every reference frames, existing there from eternity, how can we identify these different standard measuring equipments and how can we identify the different physical quantities defined in terms of these independent *etalons?* How can our physical world view be consistent if a "system already moving at constant velocity v relative to K" has nothing to do with the "same system having been (gently) accelerated to velocity v relative to K" and if the latter has nothing to do with the "same system being at rest in the frame K' moving at velocity v relative to K"—whatever these phrases mean.

On the contrary, as we pointed out in Point **5**, the empirically confirmed laws of physics in any one reference frame K must describe—and, actually, do describe—the behaviour of all physical systems performing arbitrary motions, including acceleration relative to K. Applying these laws, we can determine the results of measurements obtainable by means of measuring equipments co-moving with K' on various systems including those which are co-moving with K'. Whether or not these results, in comparison with the similar results of measurements obtainable by means of measuring equipments at rest relative to K, satisfy the Lorentz transformation rules and/or the relativity principle is a contingent fact of nature inscribed in the physical laws in question in K. If so, then the Lorentz transformation rules and/or the relativity principle describe nothing but the physical behaviours of the (measuring and measured) systems in question performing various motions relative to K.

25 . Another source of confusion is the widespread view that accelerated systems, especially accelerated observers, are always problematic within the context of the principle of special relativity; by definition, such things fall outside of the scope of the relativity principle. It must be clear, however, that only accelerated *reference frames* fall outside the scope of the relativity principle—in the sense that the principle asserts that the corresponding physical laws take the same form in all *inertial* frames—but not accelerated physical *objects*.

Moreover, note that an accelerated reference frame falls outside of the scope of the relativity principle only as the *subject* of the principle, but not as its *object*. For, in any inertial reference frame K the special relativistic laws of physics must account for the behaviour of all physical objects, including both *accelerated* measuring equipments and the other physical objects (of arbitrary motion) to be measured. Therefore the Lorentz covariant special relativistic laws must account for how the things look like even in an arbitrary accelerated frame $\mathcal{K}$. For example, if the description is correct, it must reflect the fact that relativity principle does not hold for the reference frames of relative acceleration.

Moreover, relativity principle also holds—in the usual restricted sense—for these descriptions. For imagine another inertial frame K' moving at velocity v relative to K. The laws of physics in K' also account for what an observer observes in $\mathcal{K}$. The relativity principle relates two such descriptions in the following sense: Let the described phenomenon be <how the things look like in $\mathcal{K}$>. Let things$_v$ symbolically denote the same things when they are in collective motion at velocity v relative to K, and similarly let $\mathcal{K}_v$ be a frame which performs the same accelerating motion as G in superposition with a translation at velocity v relative to K. (Of course, these all are vague concepts, as usual.) Now, according to the relativity principle the <how the things$_v$ look like in $\mathcal{K}_v$> expressed in the terms of the results of measurements obtained by means of measuring-rods, clocks, etc. co-moving with K' takes the same form as the <how the things look like in $\mathcal{K}$>, expressed in terms of the measurements with the devices at rest in K.

26 . Another reason why accelerated systems are eyed with suspicion is that brutal acceleration may damage the physical object in question. As I pointed out in Point **18**, this problem is different from what has been our main concern that the relativity principle has only limited validity in relativistic physics, simply because the principle can fail even if the system is gently accelerated. Let us now examine this difference in more details.

Recall first what the relativity principle says in classical physics. It asserts that equations (9)–(10) hold for initial conditions like (32)–(33):

$$\psi_0 = \begin{cases} \mathbf{r}_i(t=t_0) & = \mathbf{R}_{i0} \\ \left.\frac{d\mathbf{r}_1}{dt}\right|_{t=t_0} & = \mathbf{V}_{i0} \end{cases} \tag{34}$$

$$\psi_v = \begin{cases} \mathbf{r}_i(t=t_0) & = \mathbf{R}_{i0} \\ \left.\frac{d\mathbf{r}_1}{dt}\right|_{t=t_0} & = \mathbf{V}_{i0} + \mathbf{v} \end{cases} \tag{35}$$

That is, $G_v(\psi_v) = \psi'_0$, no matter how brutally the system is set in state ψ_v. The point is that the principle is about the comparison of the system's behaviour initiated from the sate (34) with the system's behaviour initiated from state (35). The only difference between the two states is that the latter contains a collective motion of all particles at velocity $\mathbf{v}$. In other words, if (35) describes the sate of the system right after it was brutally accelerated to co-moving with K', then (34) describes the sate of the system right after it was brutally accelerated to co-moving with K. The principle has nothing to do with the difference between the states before and after the brutal acceleration.

Let me illustrate this with an example. Imagine a system of interacting particles in state

$$\psi_- = \begin{cases} \mathbf{r}_i(t=t_-) & = \mathbf{R}_{i-} \\ \left.\frac{d\mathbf{r}_1}{dt}\right|_{t=t_-} & = \mathbf{V}_{i-} \end{cases}$$

at time t_-. Then at time $t_0 - \Delta t$ the system is exploded, and right after the explosion its state is

$$\psi_0 = \begin{cases} \mathbf{r}_i(t = t_0) & = & \mathbf{R}_{i0} \\ \frac{d\mathbf{r}_1}{dt}\Big|_{t=t_0} & = & \mathbf{V}_{i0} \end{cases}$$

Now, imagine that the system is exploded in a slightly different way, such that a very strong but homogeneous gravitational field is turned on during the explosion, so all particles obtain an additional velocity $\mathbf{v} = \mathbf{a} \cdot \Delta t$. Therefore the system's state at t_0 will be

$$\psi_v = \begin{cases} \mathbf{r}_i(t = t_0) & = & \mathbf{R}_{i0} \\ \frac{d\mathbf{r}_1}{dt}\Big|_{t=t_0} & = & \mathbf{V}_{i0} + \mathbf{v} \end{cases}$$

As a result, the system of state ψ_- is set in collective motion at velocity $\mathbf{v}$ relative to K in the most brutal way. Of course, the principle tells nothing about the differences either between the states ψ_0 and ψ_- or between ψ_v and ψ_-. But, in spite of the brutality of the state preparation, in classical physics, the relativity principle always holds: $G_v(\psi_v) = \psi_0'$.

Now, as we have seen, the same is not true in relativistic physics. Namely, even if the laws of physics satisfy condition (11), $\Lambda_v(\psi_v) \neq \psi_0'$ in general—no matter how brutal or gentle was the change from ψ_- to ψ_0/ψ_v.

Does Special Relativity Theory Tell Us Anything New About Space and Time?

Prolog

27 . Consider the following definitions of electrodynamical quantities:

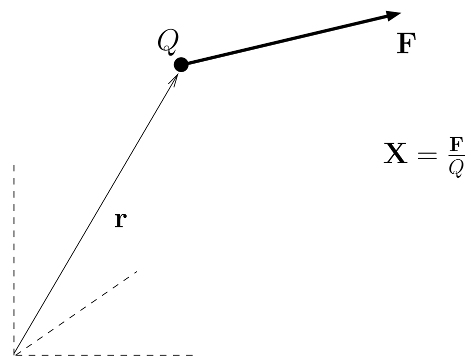


Figure 7. **X** *is defined as the force felt by the unit test charge*

X (**r**) Locate a test charge Q at point **r** and measure the force **F** felt by the charge. $\mathbf{X}(\mathbf{r}) = \frac{\mathbf{F}}{Q}$ (Fig 7).

Y (**r**) Locate two contacting metal plates of area A at point **r**. Separate them and measure the influence charge Q on one of the plates. $Y(\mathbf{r}) = \frac{Q}{A}$. The direction of $\mathbf{Y}(\mathbf{r})$ is determined by the normal vector of the plates, when the charge separation is maximal (Fig 8).

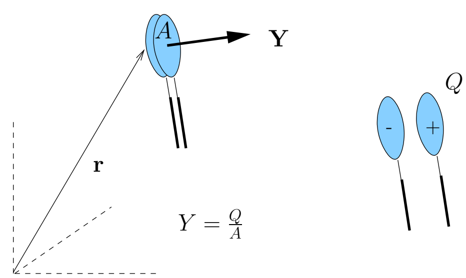


Figure 8. **Y** *is defined by means of the influence charge divided by the surface*

It is a well known empirical fact that these quantities are not independent of each other. For the sake of simplicity, assume the simplest material equation

$$\mathbf{Y} = \varepsilon \mathbf{X} \tag{36}$$

where ε, called dielectric constant, is a scalar field characterising the medium.

Traditionally, in phenomenological electrodynamics, physical quantity **X** is called 'electric field strength' and denoted by **E**, and **Y** is called 'electric displacement' and denoted by **D**. Due to the material equation (36) one can eliminate one of the field variables.

28 . Imagine a text book (I shall refer to it as the "old" one), which only uses **E**. The equations of electrostatics are written as follows:

$$\text{div } \varepsilon \mathbf{E} = \rho \tag{37}$$
$$\text{rot } \mathbf{E} = 0 \tag{38}$$

For example, the book contains the following exercise and solution:

Exercise Consider the static electric field around a point charge q located at the border of two materials of dielectric constant ε_1 and ε_2. Is the electric field strength spherically symmetric, or not?

Solution (see Fig 9)

$$\mathbf{E}_1 = \frac{1}{2\pi(\varepsilon_1+\varepsilon_2)}\frac{q}{r^3}\mathbf{r} \tag{39}$$
$$\mathbf{E}_2 = \frac{1}{2\pi(\varepsilon_1+\varepsilon_2)}\frac{q}{r^3}\mathbf{r} \tag{40}$$

Consequently,

(S1) The electric field strength is spherically symmetric.

29 . Now, imagine a new electrodynamics text book which is non-traditional in the following sense: it uses only field variable **Y** (traditionally called 'electric displacement' and denoted by **D**), but it systematically calls **Y** 'electric field strength' and denotes it by **E**. Accordingly, the equations of electrostatics are written as follows:

$$\text{div } \mathbf{E} = \rho \tag{41}$$
$$\text{rot } \frac{\mathbf{E}}{\varepsilon} = 0 \tag{42}$$

This new book also contains the above exercise, but with the following solution:

Solution (see Fig 10)

$$\mathbf{E}_1 = \frac{\varepsilon_1}{2\pi(\varepsilon_1+\varepsilon_2)}\frac{q}{r^3}\mathbf{r} \tag{43}$$
$$\mathbf{E}_2 = \frac{\varepsilon_2}{2\pi(\varepsilon_1+\varepsilon_2)}\frac{q}{r^3}\mathbf{r} \tag{44}$$

Consequently,

(S2) The electric field strength is not spherically symmetric.

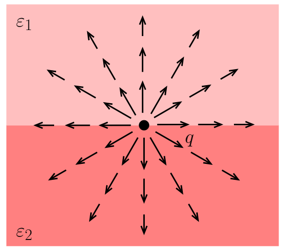


Figure 9. *The 'electric field strength' of the static electric field around a point charge q located at the border of two materials of dielectric constants ε_1 and ε_2*

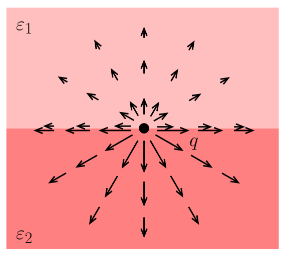


Figure 10. *The 'electric field strength' of the static electric field around a point charge q located at the border of two materials of dielectric constants ε_1 and ε_2*

Now, does sentence (S2) of the new book contradict to sentence (S1) of the old book? Is it true that the theory described in the new book is a *new theory* of electromagnetism? Of course, not. Seemingly the two sentences contradict to each other, on the level of the words. However, in order to clarify the meaning of sentence (S1) and (S2), one has to go back to the first pages of the corresponding book and clarify the definition of the physical quantity called 'electric field strength'. And it will be clear that the term 'electric field strength' stands for two different physical quantities in the two books. Moreover, both text books provide complete descriptions of electromagnetic phenomena. Therefore, although the theory in the old book does not use the field variable $\mathbf{Y}$, it is capable to account for the physical phenomena by which physical quantity $\mathbf{Y}$ is empirically defined. It is capable to determine the influence charge on the separated plates (by calculating εEA). In other words, it is capable to determine the value of $\mathbf{Y}$, that is, the value of what the new book calls 'electric field strength'. And vice versa, on the basis of the theory described in the new book one can calculate the force felt by a unit test charge (by calculating $\frac{\mathbf{E}}{\varepsilon}$), that is, one can predict the value of $\mathbf{X}$, what the old book calls 'electric field strength'. And both, the theory in the old book and the theory in the new book have the same predictions for both, $\mathbf{X}$ and $\mathbf{Y}$. That is to say, although they use different terminology, the two text books contain the same electrodynamics, they provide the same description of physical reality.

What will be challenged

30 . It is widely believed that the principal difference between Einstein's special relativity and its contemporary rival Lorentz theory was that while the Lorentz theory[3] was also capable of "explaining away" the null result of the Michelson–Morley experiment and other experimental findings by means of the distortions of moving measuring-rods and moving clocks, special relativity revealed more fundamental new facts about the geometry of space-time behind these phenomena. According to this widespread view, special relativity theory has radically changed our conceptions about space and time by claiming that space-time is not like an $\mathbb{E}^3 \times \mathbb{E}^1$ space, as was believed in classical physics, but it is a four dimensional Minkowski space $\mathbb{M}^4$. One can express this revolutionary change by the following logical schema: Earlier we believed in $G_1(M)$, where M stands for space-time and G_1 denotes some predicate (like $\mathbb{E}^3 \times \mathbb{E}^1$). Then we discovered that $\neg G_1(M)$ but $G_2(M)$, where G_2 denotes a predicate different from G_1 (something like $\mathbb{M}^4$).

Contrary to this common view, our first thesis will be the following:

Thesis 1. *In comparison with the pre-relativistic Galileo-invariant conceptions, special relativity tells us nothing new about the geometry of space-time. It simply calls something else "space-time", and this something else has different properties. All statements of special relativity about those features of reality that correspond to the original meaning of the terms "space" and "time" are identical with the corresponding traditional pre-relativistic statements.*

Thus the only new factor in the special relativistic account of space-time is the decision to designate something else "space-time". In other words: Earlier we believed in $G_1(M)$. Then we discovered for some $\widetilde{M} \neq M$ that $\neg G_1\left(\widetilde{M}\right)$ but $G_2\left(\widetilde{M}\right)$. Consequently, it still holds that $G_1(M)$.

31 . So the real novelty in special relativity is some $G_2\left(\widetilde{M}\right)$. As we will see, this is nothing but the description of the physical behaviour of moving measuring-rods and clocks. It will be also argued, however, that $G_2\left(\widetilde{M}\right)$ does not contradict to what Lorentz theory claims. More exactly, as our second thesis asserts, both theories claim that $G_1(M)\,\&G_2\left(\widetilde{M}\right)$:

Thesis 2. *Special relativity and Lorentz theory are completely identical in both senses, as theories about space-time and as theories about the behaviour of*

[3] I use the term "Lorentz theory" as classification to refer to the similar approaches of Lorentz, FitzGerald, and Poincaré, that save the classical Galilei covariant conceptions of space and time by explaining the null result of the Michelson–Morley experiment and other similar experimental findings through the physical distortions of moving objects (first of all of moving measuring-rods and clocks), no matter whether these physical distortions are simply hypothesised in the theory, or prescribed by some "principle" like Lorentz's principle, or they are constructively derived from the behaviour of the molecular forces. From the point of view of my recent concerns what is important is the logical possibility of such an alternative theory. Although, Lorentz's 1904 paper is very close to be a good historic example.

moving physical objects.

On the meaning of the question "What is space-time like?"

32 . A theory *about* space-time describes a certain group of objective features of physical reality, which we call (the structure of) space-time. According to classical physics, the geometry of space-time $\mathbb{E}^3 \times \mathbb{E}^1$, where $\mathbb{E}^3$ is a three-dimensional Euclidean space for space, and $\mathbb{E}^1$ is a one-dimensional Euclidean space for time, with two independent invariant metrics corresponding to the space and time intervals. In contrast, special relativity claims that the geometry of space-time—understood as the same objective features of physical reality—is different: it is a Minkowski geometry.

Physics describes objective features of reality by means of physical quantities. Our scrutiny will therefore start by clarifying how classical physics and relativity theory define the space and time tags assigned to an arbitrary event. It will be seen that these empirical definitions are different.

The empirical definition of a physical quantity requires an *etalon* measuring equipment and a precise description of the operation how the quantity to be defined is measured. For example, assume we choose, as the *etalon* measuring-rod, the meter stick that is lying in the International Bureau of Weights and Measures (BIPM) in Paris. Also assume—this is another convention—that "time" is defined as a physical quantity measured by the standard clock also sitting in the BIPM. When I use the word "convention" here, I mean the semantical freedom we have in the use of the uncommitted signs "distance" and "time"—a freedom what Grünbaum (1974, p. 27) calls "trivial semantical conventionalism".

33 . Now we are going to describe the empirical definitions of the space and time tags of an arbitrary event A, relative to the reference frame K in which the the *etalons* are at rest, and to another reference fame K' which is moving (at constant velocity v) relative to K. For the sake of simplicity consider only one space dimension and assume that the origin of both K and K' is at the BIPM at the initial moment of time.

(D1) Time tag in K according to classical physics

> Take a synchronised copy of the standard clock at rest in the BIPM, and slowly[4] move it to the locus of event A. The time tag $\hat{t}^K(A)$ is the reading of the transfered clock when A occurs.[5]

[4]"Slowly" means that we move the clock from one place to the other over a long period of time, according to the reading of the clock itself. The reason is to avoid the loss of phase accumulated by the clock during its journey.

[5]With this definition we actually use the standard "$\varepsilon = \frac{1}{2}$-synchronisation". I do not want to enter now into the question of the conventionality of simultaneity, which is a hotly debated problem, in itself. (See Point **67**.)

(D2) Space tag in K according to classical physics

The space tag $\hat{x}^K(A)$ of event A is is the distance from the origin of K of the locus of A along the x-axis[6] measured by superposing the standard measuring-rod, being always at rest relative to K.

(D3) Time tag in K according to special relativity

Take a synchronised copy of the standard clock at rest in the BIPM, and slowly move it to the locus of event A. The time tag $\widetilde{t}^K(A)$ is the reading of the transfered clock when A occurs.

(D4) Space tag in K according to special relativity

The space tag $\widetilde{x}^K(A)$ of event A is the distance from the origin of K of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K.

(D5) Time tag of an event in K' according to classical physics

The time tag of event A relative to the frame K' is

$$\hat{t}^{K'}(A) := \hat{t}^K(A) \tag{45}$$

(D6) Space tag of an event in K' according to classical physics

The space tag of event A relative to the frame K' is

$$\hat{x}^{K'}(A) := \hat{x}^K(A) - v\hat{t}^K(A) \tag{46}$$

where $v = \hat{v}^K(K')$ is the velocity of K' relative to K in the sense of definition (D9).

(D7) Time tag in K' according to special relativity

Take a synchronised copy of the standard clock at rest in the BIPM, gently accelerate it from K to K' and set it to show 0 when the origins of K and K' coincide. Then slowly (relative to K') move it to the locus of event A. The time tag $\widetilde{t}^{K'}(A)$ is the reading of the transfered clock when A occurs.

(D8) Space tag in K' according to special relativity

The space tag $\widetilde{x}^{K'}(A)$ of event A is the distance from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest.

[6] The straight line is defined by a light beam.

(D9) Velocities in the different cases

Velocity is a quantity derived from the above defined space and time tags:

$$\hat{v}^K = \frac{\Delta \hat{x}^K}{\Delta \hat{t}^K}$$
$$\widetilde{v}^K = \frac{\Delta \widetilde{x}^K}{\Delta \widetilde{t}^K}$$
$$\hat{v}^{K'} = \frac{\Delta \hat{x}^{K'}}{\Delta \hat{t}^{K'}}$$
$$\widetilde{v}^{K'} = \frac{\Delta \widetilde{x}^{K'}}{\Delta \widetilde{t}^{K'}}$$

34 . With these empirical definitions, in every inertial frame we define four different quantities for each event, such that:

$$\hat{x}^K(A) \equiv \widetilde{x}^K(A) \tag{47}$$
$$\hat{t}^K(A) \equiv \widetilde{t}^K(A) \tag{48}$$
$$\hat{x}^{K'}(A) \not\equiv \widetilde{x}^{K'}(A) \tag{49}$$
$$\hat{t}^{K'}(A) \not\equiv \widetilde{t}^{K'}(A) \tag{50}$$

where $\equiv$ denotes the identical empirical definition.

In spite of the different empirical definitions, it could be a *contingent* fact of nature that $\hat{x}^{K'}(A) = \widetilde{x}^{K'}(A)$ and/or $\hat{t}^{K'}(A) = \widetilde{t}^{K'}(A)$ for every event A. Let me illustrate this with an example. The inertial mass m_i and gravitational mass m_g are two quantities having different experimental definitions. But, it is a contingent fact of nature (experimentally proved by Eötvös around 1900) that, for any object, the two masses are equal, $m_i = m_g$. A little reflection reveals, however, that this is not the case here. It follows from special relativity that $\widetilde{x}^K(A), \widetilde{t}^K(A)$ are related with $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ through the Lorentz transformation, while $\hat{x}^K(A), \hat{t}^K(A)$ are related with $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ through the corresponding Galilean transformation, therefore, taking into account identities (47)–(48), $\hat{x}^{K'}(A) \neq \widetilde{x}^{K'}(A)$ and $\hat{t}^{K'}(A) \neq \widetilde{t}^{K'}(A)$, if $v \neq 0$.

Thus, our first partial conclusion is that *different physical quantities are called "space" tag, and similarly, different physical quantities are called "time" tag in special relativity and in classical physics.*[7] In order to avoid further confusion, from now on $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags will mean the physical quantities defined in (D1), (D2), (D5), and (D6)—according to the usage of the terms in classical physics—, and "space" and "time" in the sense of the relativistic definitions (D3), (D4), (D7) and (D8) will be called $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$.

Special relativity theory makes *different* assertions about somethings which are *different* from $\widehat{\text{space}}$ and $\widehat{\text{time}}$. In our symbolic notation, classical physics

[7]This was first recognised by Bridgeman (1927, p. 12), although he did not investigate the further consequences of this fact.

claims $G_1\left(\hat{M}\right)$ about $\hat{M}$ and relativity theory claims $G_2\left(\widetilde{M}\right)$ about some other features of reality $\widetilde{M}$. The question is what special relativity and classical physics say when they are making assertions about the same things.

Special relativity does not tell us anything new about space and time

35 . Classical physics calls "space" and "time" what we denoted by $\widehat{\text{space}}$ and $\widehat{\text{time}}$. So relativity theory would tell us something new if it accounted for physical quantities $\hat{x}$ and $\hat{t}$ differently. If there were any event A and any inertial frame of reference $K^\star$ in which the $\widehat{\text{space}}$ or $\widehat{\text{time}}$ tag assigned to the event by special relativity, $\left[\hat{x}^{K^\star}(A)\right]_{relativity}$, $\left[\hat{t}^{K^\star}(A)\right]_{relativity}$, were different from the similar tags assigned by classical physics, $\left[\hat{x}^{K^\star}(A)\right]_{classical}$, $\left[\hat{t}^{K^\star}(A)\right]_{classical}$. If, for example, there were any two events $\widehat{\text{simultaneous}}$ in relativity theory which were not $\widehat{\text{simultaneous}}$ according to classical physics, or vice versa—to touch on a sore point. But a little reflection shows that this is not the case. Taking into account empirical identities (47)–(48), one can calculate the relativity theoretic prediction for the outcomes of the measurements described in (D1), (D2), (D5), and (D6), that is, the relativity theoretic prediction for $\hat{x}^{K'}(A)$:

$$\left[\hat{x}^{K'}(A)\right]_{relativity} = \widetilde{x}^{K}(A) - \widetilde{v}^{K}(K')\widetilde{t}^{K}(A) \tag{51}$$

the value of which is equal to

$$\hat{x}^{K}(A) - \hat{v}^{K}(K')\hat{t}^{K}(A) = \left[\hat{x}^{K'}(A)\right]_{classical} \tag{52}$$

Similarly,

$$\left[\hat{t}^{K'}(A)\right]_{relativity} = \widetilde{t}^{K}(A) = \hat{t}^{K}(A) = \left[\hat{t}^{K'}(A)\right]_{classical} \tag{53}$$

This completes the proof of Thesis 1.

Lorentz theory and special relativity are completely identical theories

36 . Since Lorentz theory adopts the classical conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$, it does not differ from special relativity in its assertions about $\widehat{\text{space}}$ and $\widehat{\text{time}}$. What about the other claim—$G_2\left(\widetilde{\mathcal{M}}\right)$—about $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$? In order to prove what Thesis 2 asserts, that is to say the complete identity of Lorentz theory and of special relativity, we also have to show that the two theories have identical assertions about $\widetilde{x}$ and $\widetilde{t}$, that is,

$$\begin{aligned}\left[\widetilde{x}^{K'}(A)\right]_{relativity} &= \left[\widetilde{x}^{K'}(A)\right]_{LT} \\ \left[\widetilde{t}^{K'}(A)\right]_{relativity} &= \left[\widetilde{t}^{K'}(A)\right]_{LT}\end{aligned}$$

According to relativity theory, the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags in K' and in K are related through the Lorentz transformations. From (47)–(48) we have

$$\left[\widetilde{t}^{K'}(A)\right]_{relativity} = \frac{\hat{t}^K(A) - \frac{v\,\hat{x}^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{54}$$

$$\left[\widetilde{x}^{K'}(A)\right]_{relativity} = \frac{\hat{x}^K(A) - v\,\hat{t}^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{55}$$

37 . On the other hand, taking the assumptions of Lorentz theory that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K', one can directly calculate the $\widetilde{\text{space}}$ tag $\widetilde{x}^{K'}(A)$ and the $\widetilde{\text{time}}$ tag $\widetilde{t}^{K'}(A)$, following the descriptions of operations in (D7) and (D8).

First, let us calculate the reading of the clock slowly transported in K' from the origin to the locus of an event A. The clock is moving with a varying velocity[8]

$$\hat{v}_C^K(\hat{t}^K) = v + \hat{w}^K(\hat{t}^K)$$

where $\hat{w}^K(\hat{t}^K)$ is the velocity of the clock relative to K', that is, $\hat{w}^K(0) = 0$ when it starts at $\hat{x}_C^K(0) = 0$ (as we assumed, $\hat{t}^K = 0$ and the transported clock shows 0 when the origins of K and K' coincide) and $\hat{w}^K(\hat{t}_1^K) = 0$ when the clock arrives at the place of A. The reading of the clock at the time $\hat{t}_1^K$ will be

$$T = \int_0^{\hat{t}_1^K} \sqrt{1 - \frac{\left(v + \hat{w}^K(\hat{t})\right)^2}{c^2}}\, d\hat{t} \tag{56}$$

Since $\hat{w}^K$ is small we may develop in powers of $\hat{w}^K$, and we find from (56) when neglecting terms of second and higher order

$$T = \frac{\hat{t}_1^K - \frac{\left(\hat{t}_1^K v + \int_0^{\hat{t}_1^K} \hat{w}^K(\hat{t})\, d\hat{t}\right) v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\hat{t}^K(A) - \frac{\hat{x}^K(A) v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{57}$$

[8] For the sake of simplicity we continue to restrict our calculation to one space dimension. For the general calculation of the phase shift suffered by moving clocks, see Jánossy 1971, pp. 142–147.

(where, without loss of generality, we take $\hat{t}_1^K = \hat{t}^K(A)$). Thus, according to the definition of $\widetilde{t}$, we have

$$\left[\widetilde{t}^{K'}(A)\right]_{LT} = \frac{\hat{t}^K(A) - \frac{v\,\hat{x}^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{58}$$

which is equal to $\left[\widetilde{t}^{K'}(A)\right]_{relativity}$ in (54).

Now, taking into account that the length of the co-moving meter stick is only $\sqrt{1-\frac{v^2}{c^2}}$, the distance of event A from the origin of K is the following:

$$\hat{x}^K(A) = \hat{t}^K(A)v + \widetilde{x}^{K'}(A)\sqrt{1-\frac{v^2}{c^2}} \tag{59}$$

and thus

$$\left[\widetilde{x}^{K'}(A)\right]_{LT} = \frac{\hat{x}^K(A) - v\,\hat{t}^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} = \left[\widetilde{x}^{K'}(A)\right]_{relativity}$$

This completes the proof. The two theories make completely identical assertions not only about the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}, \hat{t}$ but also about the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\tilde{x}, \tilde{t}$.

38 . Consequently, there is full agreement between the Lorentz theory and special relativity theory in the following statements:

(a) $\widetilde{\text{Velocity}}$—which is called "velocity" by relativity theory—is not an additive quantity,

$$\widetilde{v}^{K'}(K''') = \frac{\widetilde{v}^{K'}(K'') + \widetilde{v}^{K''}(K''')}{1 + \frac{\widetilde{v}^{K'}(K'')\widetilde{v}^{K''}(K''')}{c^2}}$$

while $\widehat{\text{velocity}}$—that is, what we traditionally call "velocity"—is an additive quantity,

$$\hat{v}^{K'}(K''') = \hat{v}^{K'}(K'') + \hat{v}^{K''}(K''')$$

where K', K'', K''' are arbitrary three frames. For example,

$$\hat{v}^{K'}(light\,signal) = \hat{v}^{K'}(K'') + \hat{v}^{K''}(light\,signal)$$

(b) The $\left(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}\right)$-map of the world can be conveniently described through a Minkowski geometry, such that the $\widetilde{t}$-simultaneity can be described through the orthogonality with respect to the 4-metric of the Minkowski space, etc.

(c) The $\left(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}\right)$-map of the world, can be conveniently described through a traditional "space-time geometry" like $\mathbb{E}^3 \times \mathbb{E}^1$.

(d) The $\widehat{\text{velocity}}$ of light is not the same in all inertial frames of reference.

(e) The $\widetilde{\text{velocity}}$ of light is the same in all inertial frames of reference.

(f) $\widehat{\text{Time}}$ and $\widehat{\text{distance}}$ are invariant, the reference frame independent concepts, $\widetilde{\text{time}}$ and $\widetilde{\text{distance}}$ are not.

(g) $\hat{t}$-simultaneity is an invariant, frame-independent concept, while $\widetilde{t}$-simultaneity is not.

(h) For arbitrary K' and K'', $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ can be expressed by $\hat{x}^{K''}(A), \hat{t}^{K''}(A)$ through a suitable Galilean transformation

(i) For arbitrary K' and K'', $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ can be expressed by $\widetilde{x}^{K''}(A), \widetilde{t}^{K''}(A)$ through a suitable Lorentz transformation.

$\vdots$

Moreover, in all cases when it holds, they will agree in the relativity principle:

(j) The behaviour of similar systems co-moving as a whole with different inertial frames, expressed in terms of the results of measurements obtainable by means of co-moving measuring-rods and clocks (that is, in terms of quantities $\widetilde{x}$ and $\widetilde{t}$) is the same in every inertial frame of reference.

Combining this with (i),

(k) The laws of physics, expressed in terms of $\widetilde{x}$ and $\widetilde{t}$, must be given by means of Lorentz covariant equations.

Finally, they agree that

(l) All facts about $\widetilde{x}$ and $\widetilde{t}$ (and, consequently, all facts about $\hat{x}$ and $\hat{t}$) can be derived *backward* from (e) and (j).

To sum up symbolically, Lorentz theory and and special relativity theory have identical assertions about both $\hat{M}$ and $\widetilde{M}$: they unanimously claim that $G_1\left(\hat{M}\right) \& G_2\left(\widetilde{M}\right)$.

39 . Finally, note that in an arbitrary inertial frame K' for every event A the tags $\hat{x}_1^{K'}(A)$, $\hat{x}_2^{K'}(A)$, $\hat{x}_3^{K'}(A)$, $\hat{t}^{K'}(A)$ can be expressed in terms of $\widetilde{x}_1^{K'}(A)$, $\widetilde{x}_2^{K'}(A)$, $\widetilde{x}_3^{K'}(A)$, $\widetilde{t}^{K'}(A)$ and *vice versa*. Consequently, we can express the laws of physics—as is done in special relativity—equally well in terms of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ instead of the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$. On the other hand, we should emphasise that the one-to-one correspondence between $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ and $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ also entails that the laws of physics (so called "relativistic" laws included) can be equally well expressed in terms of the (traditional) $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ instead of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$. In brief, physics could manage equally well with the classical Galileo-invariant conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$.

Comments

40 . In a strict logical sense we have fished the argumentation for our two theses in Point **30**. We proved that special relativity and Lorentz theory are completely identical theories. Nevertheless, the following comments may aid the reader in arriving at his own appraisal.

Are relativistic deformations real physical changes?

41 . Many believe that it is an essential difference between the two theories that relativistic deformations like the Lorentz–FitzGerald contraction and the time dilatation are real physical changes in Lorentz theory, but there are no similar physical effects in special relativity. Let us examine two typical argumentations.

According to the first argument the "Lorentz contraction/dilatation" of a rod cannot be an objective physical deformation in relativity theory, because it is a frame-dependent fact whether "the rod is shrinking or expanding". Consider a rod accelerated from the sate of rest in reference frame K' to the state of rest in reference frame K''. According to relativity theory, "the rod shrinks in frame K' and, at the same time, expands in frame K''". But this is a contradiction, the argument says, if the deformation was a real physical change. (In contrast, the argument says, Lorentz's theory claims that "the length of a rod" is a frame-independent concept. Consequently, in Lorentz's theory, "the contraction/dilatation of a rod" can indeed be an objective physical change.)

However, we have already clarified, that the terms "distance" and "time" have different meanings in relativity theory and Lorentz's theory. Due to the difference between $\widehat{\text{length}}$ and $\widetilde{\text{length}}$, we must also differentiate $\widehat{\text{dilatation}}$ from $\widetilde{\text{dilatation}}$, $\widehat{\text{contraction}}$ from $\widetilde{\text{contraction}}$, and so on. For example, consider the reference frame of the *etalons* K and another frame K' moving relative to K. The following statements are true about the "length" of a rod accelerated from the sate of rest in reference frame K ($state_1$) to the state of rest in reference frame K' ($state_2$):

$$\widehat{l}^K(state_1) > \hat{l}^K(state_2) \qquad \widehat{\text{contraction}} \text{ in } K \tag{60}$$

$$\hat{l}^{K'}(state_1) > \hat{l}^{K'}(state_2) \qquad \widehat{\text{contraction}} \text{ in } K' \tag{61}$$

$$\tilde{l}^K(state_1) > \tilde{l}^K(state_2) \qquad \widetilde{\text{contraction}} \text{ in } K \tag{62}$$

$$\tilde{l}^{K'}(state_1) < \tilde{l}^{K'}(state_2) \qquad \widetilde{\text{dilatation}} \text{ in } K' \tag{63}$$

And there is no difference between relativity theory and Lorentz's theory: *all* of the four statements (60)–(63) are true *in both theories*. If, in Lorentz's theory, facts (60)–(61) provide enough reason to say that there is a real physical change, then the same facts provide enough reason to say the same thing in relativity theory. And *vice versa*, if (62)–(63) contradicted to the existence of real physical change of the rod in relativity theory, then the same holds for Lorentz's theory.

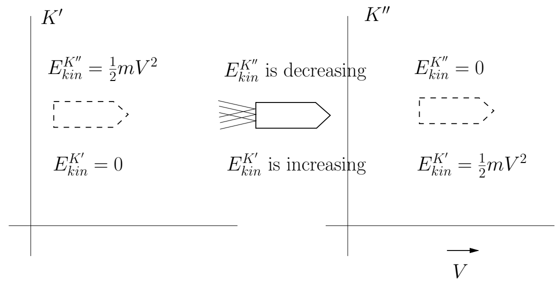


Figure 11. One and the same objective physical process is traced in the increase of kinetic energy of the spaceship relative to frame K', while it is traced in the decrease of kinetic energy relative to frame K''

42 . It should be mentioned, however, that there is no contradiction between (62)–(63) and the existence of real physical change of the rod. Relativity theory and Lorentz's theory unanimously claim that $\widetilde{\text{length}}$ is a relative physical quantity. It is entirely possible that one and the same objective physical change is traced in the increase of the value of a relative quantity relative to one reference frame, while it is traced in the decrease of the same quantity relative to another reference frame (Fig 11). (What is more, both, the value relative to one frame and the value relative to the other frame, reflect objective features of the objective physical process in question.)

43 . According to the other wide-spread argument the relativistic deformations cannot be real physical effects since they can be observed by an observer also if the object is at rest but the observer is in motion at constant velocity. And these "relativistic deformations" cannot be explained as real physical deformations of the object at rest—the argument says.

There is, however, a triple misunderstanding behind such an argument:

- Of course, no real distortion is suffered by an object which is continuously at rest relative to a reference frame K', and, consequently, which is continuously in motion at a constant velocity relative to another frame K''. None of the observers can observe such a distortion. For example,

$$\begin{aligned}\widetilde{l}^{K'}\left(\begin{array}{c}\text{distortion free}\\ \text{rod at } \widetilde{t}_1\end{array}\right) &= \widetilde{l}^{K'}\left(\begin{array}{c}\text{distortion free}\\ \text{rod at } \widetilde{t}_2\end{array}\right)\\ \widetilde{l}^{K''}\left(\begin{array}{c}\text{distortion free}\\ \text{rod at } \widetilde{t}_1\end{array}\right) &= \widetilde{l}^{K''}\left(\begin{array}{c}\text{distortion free}\\ \text{rod at } \widetilde{t}_2\end{array}\right)\end{aligned}$$

- It is surely true for any $\widetilde{t}$ that

$$\widetilde{l}^{K'}\left(\begin{array}{c}\text{distortion}\\ \text{free rod at } \widetilde{t}\end{array}\right) \neq \widetilde{l}^{K''}\left(\begin{array}{c}\text{distortion}\\ \text{free rod at } \widetilde{t}\end{array}\right) \tag{64}$$

This fact, however, does not express a contraction of the rod—neither a real nor an apparent contraction.

- On the other hand, inequality (64) is a *consequence* of the real physical distortions suffered by the measuring equipments—with which the space and time tags are empirically defined—when they are transfered from the BIPM to the other reference frame in question.[9]

44 . Finally, let me give an example for a well known physical phenomenon which is of exactly the same kind as the relativistic deformations, but nobody would question whether it is a real physical change. Consider the electromagnetic field of a point charge q. One can easily solve the Maxwell equations when the particle is at rest in a given K'). The result is the familiar spherically symmetric Coulomb field (Fig. 12):

$$\left.\widetilde{E}_1^{K'}\right|_{\substack{at\,rest\\ in\,K'}} = \frac{q\widetilde{x}_1^{K'}}{\left(\left(\widetilde{x}_1^{K'}\right)^2+\left(\widetilde{x}_2^{K'}\right)^2+\left(\widetilde{x}_3^{K'}\right)^2\right)^{\frac{3}{2}}} \tag{65}$$

$$\left.\widetilde{E}_2^{K'}\right|_{\substack{at\,rest\\ in\,K'}} = \frac{q\widetilde{x}_2^{K'}}{\left(\left(\widetilde{x}_1^{K'}\right)^2+\left(\widetilde{x}_2^{K'}\right)^2+\left(\widetilde{x}_3^{K'}\right)^2\right)^{\frac{3}{2}}} \tag{66}$$

$$\left.\widetilde{E}_3^{K'}\right|_{\substack{at\,rest\\ in\,K'}} = \frac{q\widetilde{x}_3^{K'}}{\left(\left(\widetilde{x}_1^{K'}\right)^2+\left(\widetilde{x}_2^{K'}\right)^2+\left(\widetilde{x}_3^{K'}\right)^2\right)^{\frac{3}{2}}} \tag{67}$$

$$\left.\widetilde{B}_1^{K'}\right|_{\substack{at\,rest\\ in\,K'}} = 0 \tag{68}$$

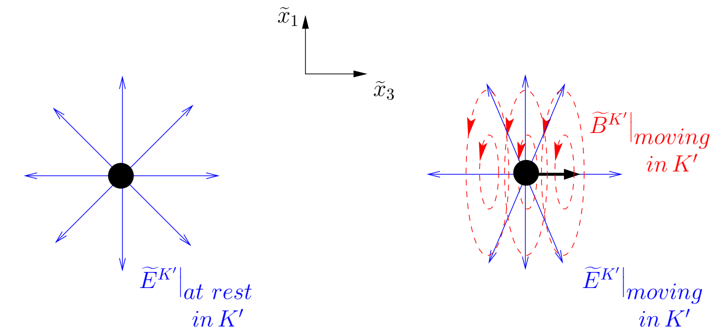
$$\left.\widetilde{B}_2^{K'}\right|_{\substack{at\,rest\\ in\,K'}} = 0 \tag{69}$$

$$\left.\widetilde{B}_3^{\prime K'}\right|_{\substack{at\,rest\\ in\,K'}} = 0 \tag{70}$$

How does this field change if we set the charge in motion at constant velocity $\widetilde{v}$ along the $\widetilde{x}_3$ axis? Maxwell's equations can also answer this question. First we solve the Maxwell equations for arbitrary time-depending sources. Then, from the retarded potentials such obtained, we derive the Lienart-Wiechert potentials, from which we can determine the field. (See, for example, Feynman, Leighton and Sands 1963, Vol. 2.) Here is the result:

$$\left.\widetilde{E}_1^{K'}\right|_{\substack{moving\\ in\,K'}} = \frac{q\widetilde{x}_1^{K'}\left(1-\frac{\widetilde{v}^2}{c^2}\right)^{-\frac{1}{2}}}{\left(\left(\widetilde{x}_1^{K'}\right)^2+\left(\widetilde{x}_2^{K'}\right)^2+B^2\right)^{\frac{3}{2}}} \tag{71}$$

[9]For further details of what a moving observer can observe by means of his or her distorted measuring equipments, see Bell 1983, pp. 75–76.



Electromagnetic field of a point charge at rest

Electromagnetic field of a point charge moving in $\tilde{x}_3$-direction

Figure 12. The electric field of a point charge

$$\widetilde{E}_2^{K}\Big|_{\substack{moving\\ in\,K'}} = \frac{q\widetilde{x}_2^{K}\left(1-\frac{\widetilde{v}^2}{c^2}\right)^{-\frac{1}{2}}}{\left(\left(\widetilde{x}_1^{K'}\right)^2+\left(\widetilde{x}_2^{K'}\right)^2+B^2\right)^{\frac{3}{2}}} \tag{72}$$

$$\widetilde{E}_3^{K'}\Big|_{\substack{moving\\ in\,K'}} = \frac{qB}{\left(\left(\widetilde{x}_1^{K'}\right)^2+\left(\widetilde{x}_2^{K'}\right)^2+B^2\right)^{\frac{3}{2}}} \tag{73}$$

$$\widetilde{B}_1^{K'}\Big|_{\substack{moving\\ in\,K'}} = -\frac{\widetilde{v}}{c}\widetilde{E}_2^{K'} \tag{74}$$

$$\widetilde{B}_2^{K'}\Big|_{\substack{moving\\ in\,K'}} = \frac{\widetilde{v}}{c}\widetilde{E}_1^{K} \tag{75}$$

$$\widetilde{B}_3'^{K'}\Big|_{\substack{moving\\ in\,K'}} = 0 \tag{76}$$

where

$$B = \frac{\widetilde{x}_3^{K'} - \widetilde{X}_3^{K'}\left(\widetilde{t}\right)}{\sqrt{1-\frac{\widetilde{v}^2}{c^2}}}$$

and $\widetilde{X}_3^{K'}\left(\widetilde{t}\right)$ is the $\widetilde{\text{position}}$ of the charge at $\widetilde{\text{time}}$ $\widetilde{t}$.

So, the electromagnetic field of the charge *changed*: earlier it was like (65)–(70), then it *changed* for the one described by (71)–(76). *There appeared* a magnetic field (turning the magnetic needle, for example) and the electric field *flattened* in the direction of motion (Fig. 12). No physicist would say that this is not a real physical change in the electromagnetic field of the charge, only because we can express the new electromagnetic field of the moving charge in terms of the variables relative to the co-moving reference frame K'',

$$\left.\widetilde{E}_1^{K''}\right|_{\substack{moving\\ in\, K'}} = \frac{q\widetilde{x}_1^{K''}}{\left(\left(\widetilde{x}_1^{K''}\right)^2+\left(\widetilde{x}_2^{K''}\right)^2+\left(\widetilde{x}_3^{K''}\right)^2\right)^{\frac{3}{2}}} \tag{77}$$

$$\left.\widetilde{E}_2^{K''}\right|_{\substack{moving\\ in\, K'}} = \frac{q\widetilde{x}_2^{K''}}{\left(\left(\widetilde{x}_1^{K''}\right)^2+\left(\widetilde{x}_2^{K''}\right)^2+\left(\widetilde{x}_3^{K''}\right)^2\right)^{\frac{3}{2}}} \tag{78}$$

$$\left.\widetilde{E}_3^{K''}\right|_{\substack{moving\\ in\, K'}} = \frac{q\widetilde{x}_3^{K''}}{\left(\left(\widetilde{x}_1^{K''}\right)^2+\left(\widetilde{x}_2^{K''}\right)^2+\left(\widetilde{x}_3^{K''}\right)^2\right)^{\frac{3}{2}}} \tag{79}$$

$$\left.\widetilde{B}_1^{K''}\right|_{\substack{moving\\ in\, K'}} = 0 \tag{80}$$

$$\left.\widetilde{B}_2^{K''}\right|_{\substack{moving\\ in\, K'}} = 0 \tag{81}$$

$$\left.\widetilde{B}_3^{K''}\right|_{\substack{moving\\ in\, K'}} = 0 \tag{82}$$

and it has the same form as the old electromagnetic field, when the charge was at rest in K', expressed in the terms of the variables relative to K'.

45 . Thus, relativistic deformations are real physical deformations also in special relativity theory. One has to emphasise this fact because it is an important part of the physical content of relativity theory. It must be clear, however, that this conclusion is independent of our main concern. What is important is the following: Lorentz's theory and special relativity have identical assertions about $\overset{\frown}{\text{length}}$ and $\overset{\smile}{\text{length}}$, $\overset{\frown}{\text{duration}}$ and $\overset{\smile}{\text{duration}}$, $\overset{\frown}{\text{shrinking}}$ and $\overset{\smile}{\text{shrinking}}$, etc. Consequently, whether or not these facts provide enough reason to say that the deformations are real physical changes, the conclusion is common to both theories.

The intuition behind the definitions

46 . Before entering into the discussion of the intuitions behind definitions (D1)–(D9), I would like to emphasise that, from the point of view of our main concern, it is not important how the different definitions are justified and whether these justifications are correct or not. What is important is the mere fact of the terminological confusion that the "space" and "time" tags mean *different* physical quantities in classical physics and relativity theory.

The basic difference between the intuitions behind the classical and relativistic definitions is the following. As we have seen, both Lorentz theory and special relativity "know" about the distortions of measuring-rods and clocks when they are transfered from the BIPM to the moving (relative to the BIPM) reference frame K'. In the relativistic definitions, (D7) and (D8), we *ignore* this fact and define the space and time tags as they are measured by means of the distorted equipments. In contrast, as it follows from the whole tradition of classical physics, in definitions (D5)–(D6) we *take into account* the distortions of the measuring equipments. That is why the space and time tags in K' are defined through the original space and time data, measured by the original distortion free measuring-rod and clock, which are at rest relative to the BIPM.

47 . In order to see this "compensatory view" of the classical definition in a more explicit form, it worth while to mention possible alternative definitions instead of (D5) and (D6). We know that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K'. Therefore, according to the compensatory view, if we measure a distance and the result is X, then the "real distance" is $X\sqrt{1-\frac{v^2}{c^2}}$. Similarly, taking into account the phase shift suffered by a moving clock, we know from (57) that if the reading of the clock is T then the "real time" is

$$\frac{T+X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Accordingly, the alternative definitions are the following:

(D6') Space tag of an event in K' according to classical physics

> Let X be the "distance" from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest. The space tag $\breve{x}^{K'}(A)$ of event A is
>
> $$\breve{x}^{K'}(A) := X\sqrt{1-\frac{v^2}{c^2}} \tag{83}$$

(D5') Time tag of an event in K' according to classical physics

> Take a synchronised copy of the standard clock at rest in the BIPM, gently accelerate it from K to K' and set it to show 0 when the origins of K and K' coincide. Then slowly (relative to K') move it to the locus of event A. Let T be the reading of the transfered clock when A occurs. The time tag $\widetilde{t}^{K'}(A)$ is
>
> $$\breve{t}^{K'}(A) := \frac{T+X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{84}$$

Since X and T are nothing but $\widetilde{x}^{K'}(A)$ and $\widetilde{t}^{K'}(A)$, it follows from (58) and (59) that

$$\begin{aligned} \check{x}^{K'}(A) &= \hat{x}^{K'}(A) \\ \check{t}^{K'}(A) &= \hat{t}^{K'}(A) \end{aligned}$$

On the null result of the Michelson–Morley experiment

48 . Consider the following passage from Einstein:

> A ray of light requires a perfectly definite time T to pass from one mirror to the other and back again, if the whole system be at rest with respect to the aether. It is found by calculation, however, that a slightly different time T^1 is required for this process, if the body, together with the mirrors, be moving relatively to the aether. And yet another point: it is shown by calculation that for a given velocity v with reference to the aether, this time T^1 is different when the body is moving perpendicularly to the planes of the mirrors from that resulting when the motion is parallel to these planes. Although the estimated difference between these two times is exceedingly small, Michelson and Morley performed an experiment involving interference in which this difference should have been clearly detectable. But the experiment gave a negative result — a fact very perplexing to physicists. (Einstein 1920, p. 49)

The "calculation" that Einstein refers to is based on the Galilean "kinematics", that is, on the invariance of "time" and "simultaneity", on the invariance of "distance", on the classical addition rule of "velocities", etc. That is to say, "distance", "time", and "velocity" in the above passage mean the classical $\widehat{\text{distance}}$, $\widehat{\text{time}}$, and $\widehat{\text{velocity}}$ defined in (D1), (D2), (D5), and (D6). The negative result was "very perplexing to physicists" because their expectations were based on traditional concepts of $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and they could not imagine other that if the $\widehat{\text{speed}}$ of light is c relative to one inertial frame then the $\widehat{\text{speed}}$ of the same light signal cannot be the same c relative to another reference frame.

49 . On the other hand, Einstein continues this passage in the following way:

> Lorentz and FitzGerald rescued the theory from this difficulty by assuming that the motion of the body relative to the aether produces a contraction of the body in the direction of motion, the amount of contraction being just sufficient to compensate for the difference in time mentioned above. Comparison with the discussion in Section 11 shows that also from the standpoint of the theory of relativity this solution of the difficulty was the right one. But on the basis of the theory of relativity the method of interpretation is incomparably more satisfactory. According to this theory there is no such thing as a "specially favoured" (unique) co-ordinate system to occasion the

> introduction of the aether-idea, and hence there can be no aether-drift, nor any experiment with which to demonstrate it. Here the contraction of moving bodies follows from the two fundamental principles of the theory, without the introduction of particular hypotheses; and as the prime factor involved in this contraction we find, not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point. Thus for a co-ordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a co-ordinate system which is at rest relatively to the sun. (Einstein 1920, p. 49)

What "rescued" means here is that—within the framework of the classical space-time theory and Galilean kinematics—Lorentz and FitzGerald proved that if the assumed deformations of moving bodies exist then the expected result of the Michelson–Morley experiment is the null effect. On the other hand, we have already clarified, what Einstein also confirms in the above quoted passage, that these deformations also derive from the two basic postulates of special relativity. Putting all these facts together (see Schema 1), we must say that the null result of the Michelson–Morley experiment simultaneously confirms *both*, the classical rules of Galilean kinematics for $\hat{x}$ and $\hat{t}$, and the violation of these rules (Lorentzian kinematics) for the space and time tags $\tilde{x}, \tilde{t}$. It confirms the classical addition rule of velocities, on the one hand, and, on the other hand, it also confirms that velocity of light is the same in all frames of reference.

This actually holds for all other experimental confirmations of special relativity. That is why the only difference Einstein can mention in the quoted passage is that special relativity does not refers to the aether. (As a historical fact, this difference is true. Although, as we will see in Points **55**–**56** and **59**–**61**, the concept of aether can be entirely removed from the recent logical reconstruction of the Lorentz theory.)

50 . Finally, it is no surprise that the deformations can be "derived" from the Lorentz kinematics. The *physical* information about the deformations suffered by objects accelerated from one state of motion to another, say from the state of rest relative to K' to the state of rest relative to K'', is inbuilt into the relationship between the tags $\tilde{x}^{K'}(A), \tilde{t}^{K'}(A)$ and $\tilde{x}^{K''}(A), \tilde{t}^{K''}(A)$. For these relations are determined by the *physical behaviour* of measuring rods and clocks during the acceleration and relaxation process, as Einstein warns us (see the quotation in Point **23**).

The conventionalist approach

51 . According to the conventionalist thesis,[10] Lorentz's theory and Einstein's special relativity are two alternative scientific theories which are equivalent on

[10] Friedman 1983, p. 293; Einstein 1983, p. 35. (see Point **??**)

$$\left.\begin{array}{c}\underbrace{\begin{array}{c}\text{[deformations]}\\ \&\\ \left[\begin{array}{c}\text{Galilean kin}\widehat{\text{ematics}}\\ \text{for } \hat{x}, \hat{t} \text{ (the } \widehat{\text{speed}}\\ \text{of light is NOT}\\ \text{the same in all}\\ \text{inertial frame)}\end{array}\right]\end{array}}\end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{the result of the}\\ \text{Michelson-Morley}\\ \text{experiment must}\\ \text{be the null effect}\end{array}\right]$$

$$\Updownarrow$$

$$\left.\begin{array}{c}\overbrace{\left[\begin{array}{c}\text{Lorentz kin}\widetilde{\text{ematics}}\\ \text{for } \tilde{x}, \tilde{t} \text{ (the } \widetilde{\text{speed}}\\ \text{of light IS the same}\\ \text{in all inertial frame)}\end{array}\right]}\\ \Downarrow\\ \text{[deformations]}\end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{the result of the}\\ \text{Michelson-Morley}\\ \text{experiment must}\\ \text{be the null effect}\end{array}\right]$$

Schema 1: The null result of the Michelson–Morley experiment simultaneously confirms both, the classical rules of Galilean kin$\widehat{\text{ematics}}$ for $\hat{x}$ and $\hat{t}$, and the violation of these rules (Lorentzian kin$\widetilde{\text{ematics}}$) for the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\widetilde{x}, \widetilde{t}$.

empirical level. Due to the empirical underdeterminacy, the choice between these alternative theories is based on external aspects.[11] Following Poincaré's similar argument about the relationship between geometry, physics, and the empirical facts, the conventionalist thesis asserts the following relationship between Lorentz theory and special relativity:

$$\begin{bmatrix}\text{classical}\\ \text{space-time}\\ \mathbb{E}^3\times\mathbb{E}^1\end{bmatrix} + \begin{bmatrix}\text{physical}\\ \text{content of}\\ \text{Lorentz}\\ \text{theory}\end{bmatrix} = \begin{bmatrix}\text{empirical}\\ \text{facts}\end{bmatrix}$$

$$\begin{bmatrix}\text{relativistic}\\ \text{space-time}\\ \mathbb{M}^4\end{bmatrix} + \begin{bmatrix}\text{special}\\ \text{relativistic}\\ \text{physics}\end{bmatrix} = \begin{bmatrix}\text{empirical}\\ \text{facts}\end{bmatrix}$$

Continuing the symbolic notations we used in the Introduction, denote Z those objective features of physical reality that are described by the alternative physical theories P_1 and P_2 in question. With these notations, the logical schema of the conventionalist thesis can be described in the following way: We cannot distinguish by means of the available experiments whether $G_1(M)\,\&P_1(Z)$ is true about the objective features of physical reality $M\cup Z$, or $G_2(M)\,\&P_2(Z)$ is true about the *same* objective features $M\cup Z$. Schematically,

$$[G_1(M)]+[P_1(Z)] = \begin{bmatrix}\text{empirical}\\ \text{facts}\end{bmatrix}$$

$$[G_2(M)]+[P_2(Z)] = \begin{bmatrix}\text{empirical}\\ \text{facts}\end{bmatrix}$$

52 . However, it is clear from the previous sections that the terms "space" and "time" have different meanings in the two theories. Lorentz theory claims $G_1\left(\hat{M}\right)$ about $\hat{M}$ and relativity theory claims $G_2\left(\widetilde{M}\right)$ about some other features of reality $\widetilde{M}$. Of course, this terminological confusion also appears in the physical assertions. Let us symbolise with $\hat{Z}$ the objective features of physical reality, such as the $\widehat{\text{length}}$ of a rod, etc., described by physical theory P_1. And let $\widetilde{Z}$ denote some (partly) different features of reality described by P_2, such as the $\widetilde{\text{length}}$ of a rod, etc. Now, as we have seen, both theories actually claim that $G_1\left(\hat{M}\right)\&G_2\left(\widetilde{M}\right)$. It is also clear that, for example, within Lorentz's theory, we can legitimately query the $\widetilde{\text{length}}$ of a rod. For Lorentz's theory has complete description of the behaviour of a moving rigid rod, as well as the behaviour of a moving clock and measuring-rod. Therefore, it is no problem in Lorentz's theory to predict the result of a measurement of the "length" of the rod, if the measurement is performed with a co-moving measuring equipments, according to empirical definition (D8). This prediction will be exactly the same as the

[11] Cf. Zahar 1973; Grünbaum 1974; Friedman 1983; Brush 1999; Janssen 2002.

prediction of special relativity. And vice versa, special relativity would have the same prediction for the $\widehat{\text{length}}$ of the rod as the prediction of the Lorentz theory. That is to say, the physical contents of Lorentz's theory and special relativity also are identical: both claim that $P_1\left(\hat{Z}\right)\&P_2\left(\widetilde{Z}\right)$. So we have the following:

$$\left[G_1\left(\hat{M}\right)\&G_2\left(\widetilde{M}\right)\right]+\left[P_1\left(\hat{Z}\right)\&P_2\left(\widetilde{Z}\right)\right] \quad = \quad \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right]$$

$$\left[G_1\left(\hat{M}\right)\&G_2\left(\widetilde{M}\right)\right]+\left[P_1\left(\hat{Z}\right)\&P_2\left(\widetilde{Z}\right)\right] \quad = \quad \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right]$$

In other words, since there are no two different theories, there is *no choice*, based neither on internal nor on external aspects.

Methodological remarks

53 . It worth while emphasising that my argument is based on the following very weak "operationalist" premise: physical terms, assigned to measurable physical quantities, have different meanings if they have different empirical definitions. This premise is one of the fundamental pre-assumptions of Einstein's 1905 paper and is widely accepted among physicists. Without clear empirical definition of the measurable physical quantities a physical theory cannot be empirically confirmable or disconfirmable. In itself, this premise is not yet equivalent to operationalism or verificationalism. It does not generally imply that a statement is necessarily meaningless if it is neither analytic nor empirically verifiable. However, when the physicist assigns time and space tags to an event, relative to a reference frame, (s)he is already after all kinds of metaphysical considerations about "What is space and what is time?" and means definite physical quantities with already settled empirical meanings.

54 . In saying that the meanings of the words "space" and "time" are different in relativity theory and in classical physics, it is necessary to be careful of a possible misunderstanding. I am talking about something entirely different from the incommensurability thesis of the relativist philosophy of science.[12] How is it that relativity makes any assertion about classical $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and vice versa, how can Lorentz's theory make assertions about quantities which are not even defined in the theory? As we have seen, each of the two theories is sufficiently complete account of physical reality to make predictions about those features of reality that correspond—according to the empirical definitions—to the variables used by the other theory, and we can *compare* these predictions. For example, within Lorentz's theory, we can legitimately query the reading of a clock slowly transported in K' from one place to another. That exactly is what we calculated in section **??**. Similarly, in relativity theory, we can legitimately query the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags of an event in the reference frame of the *etalons* and then apply formulas (46)–(45). This is a fair calculation, in spite of the fact

[12]See Kuhn 1970, Chapter X; Feyerabend 1970.

that the result so obtained is not explicitly mentioned and named in the theory. This is what we actually did. And the conclusion was that not only are the two theories commensurable, but they provide completely identical accounts of the same physical reality.

Privileged reference frame

55 . Due to the popular/textbook literature on relativity theory, there is a widespread aversion to a privileged reference frame. However, like it or not, there is a privileged reference frame in both special relativity and classical physics. It is the frame of reference in which the *etalons* are at rest. This privileged reference frame, however, has nothing to do with the concepts of "absolute rest" or the aether, and it is not privileged by nature, but it is privileged by the trivial semantical convention providing meanings for the terms "distance" and "time", by the fact that of all possible measuring-rod-like and clock-like objects floating in the universe, we have chosen the ones floating together with the International Bureau of Weights and Measures in Paris. In Bridgman's words:

> It cannot be too strongly emphasised that there is no getting away from preferred operations and unique standpoint in physics; the unique physical operations in terms of which interval has its meaning afford one example, and there are many others also. (Bridgman 1936, p. 83)

56 . Many believe that one can avoid a reference to the *etalons* sitting in a privileged reference frame by defining, for example, the unit of $\widetilde{\text{time}}$ for an arbitrary (moving) frame of reference K' through a cesium clock, or the like, co-moving with K'. In this way, one needs not to refer to a standard clock accelerated from the reference frame of the *etalons* into reference frame K'. But further thought reveals that such a definition has several difficulties. For if this operation is regarded as a convenient way of *measuring* $\widetilde{\text{time}}$, then we still have $\widetilde{\text{time}}$ in the theory, together with the privileged reference frame of the *etalons*. If, however, this operation is regarded as the empirical *definition* of a physical quantity, then it must be clear that this quantity is not $\widetilde{\text{time}}$ but a new physical quantity, say $\widetilde{\widetilde{\text{time}}}$. In order to establish any relationship between $\widetilde{\widetilde{\text{time}}}$ tags belonging to different reference frames, it is a must to use an "*etalon* cesium clock" as well as to refer to its behaviour when accelerated from one inertial frame into the other.

The physics of moving objects

57 . Although special relativity does not tell us anything new about space and time, both special relativity and Lorentz theory enrich our knowledge of the physical world with *the physics of objects moving at constant velocities*—in accordance with the title of Einstein's original 1905 paper. The essential physical content of their discoveries is that physical objects suffer distortions when they are accelerated from one inertial frame to the other, and that these distortions satisfy some uniform laws.

FitzGerald, Lorentz[13] and Poincaré derived these laws from the requirement that the deformations must explain the null result of the Michelson–Morley experiment. They arrived to the conclusion that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K'. As we have shown in Point **37**, this claim is equivalent with the assertion that the space and time tags $\tilde{x}^{K''}(A), \tilde{t}^{K''}(A)$ measured by the co-moving distorted equipments can be expressed from the similar tags $\tilde{x}^{K'}(A), \tilde{t}^{K'}(A)$ by a suitable Lorentz transformation.

The general laws of deformations apply to both the measuring-equipment and the object to be measured. Therefore, it is no surprise that the "length" of a moving, consequently distorted, rod measured by co-moving, consequently distorted, measuring-rod and clock, that is the length of the rod, is the same as the length of the corresponding stationary rod measured with stationary measuring-rod and clock. The duration of a slowed down process in a moving object measured with a co-moving, consequently slowed down, clock will be the same as the duration of the same process in a similar object at rest, measured with the original distortion free clock at rest. These and similar observations lead Lorentz and Poincaré to conclude with the general validity of the relativity principle.[14] In his 1905 paper Einstein showed how to derive the same rules from the assumption that relativity principle generally holds and (or consequently) the velocity of a light signal is the same in all inertial reference frames. These historic differences are, however, not important from the point of view of our main concern. What is important is that in both ways one can derive exactly the same laws of deformations, exactly the same rules for $\hat{x}$ and $\hat{t}$, and exactly the same rules for $\tilde{x}$ and $\tilde{t}$.

58 . The relativity principle together with the Lorentz transformation of space and time provide the general description of the behaviour of moving physical systems. Using similar notations we introduced in Point **8**, let $\mathcal{E}'$ be a set of differential equations describing the behaviour of the system in question in an arbitrary reference frame K'. Let ψ'_0 denote a set of (initial) conditions, such that the solution determined by ψ'_0 describes the behaviour of the system when it is, as a whole, at rest relative to K'. Let $\psi'_{\widetilde{v}}$ be a set of conditions which corresponds to the solution describing the same system in uniform motion at velocity $\widetilde{v}$ relative to K'. To be more exact, $\psi'_{\widetilde{v}}$ corresponds to a solution of $\mathcal{E}'$ that describes the same behaviour of the system as ψ'_0 but in superposition

[13] FitzGerald and Lorentz also made an attempt to understand how these deformations actually come about from the molecular forces. (See Bell 1992; Brown and Pooley 2001; Brown 2001; 2003.)

[14] Whether or not relativity principle generally holds in relativistic physics is a more complex question. See Szabó 2004.

with a collective translation at velocity $\widetilde{v}$. Denote $\mathcal{E}''$ and ψ_0'' the equations and conditions obtained from $\mathcal{E}'$ and ψ_0' by substituting every $\widetilde{x}^{K'}$ with $\widetilde{x}^{K''}$ and $\widetilde{t}^{K'}$ with $\widetilde{t}^{K''}$. Denote $\Lambda_{\widetilde{v}}(\mathcal{E}'), \Lambda_{\widetilde{v}}(\psi_{\widetilde{v}}')$ the set of equations and conditions expressed in terms of the double-primed variables, applying the Lorentz transformations. Now, what the relativity principle (statement (j) in Section **??**) states is that the laws of physics describing the behaviour of moving objects are such that they satisfy the following relationships:

$$\Lambda_{\widetilde{v}}(\mathcal{E}') = \mathcal{E}'' \tag{85}$$

$$\Lambda_{\widetilde{v}}(\psi_{\widetilde{v}}') = \psi_0'' \tag{86}$$

To make more explicit how this principle provides a useful method in the description of the deformations of physical systems when they are accelerated from one inertial frame K' into some other K'', consider the following situation: Assume we know the relevant physical equations and know the solution of the equations describing the physical properties of the object in question when it is at rest in K': $\mathcal{E}', \psi_0'$. We now inquire as to the same description of the object when it is moving at a given constant velocity relative to K'. If (85)–(86) is true, then we can solve the problem in the following way. Simply take $\mathcal{E}'', \psi_0''$—by putting one more prime on each variable—and express $\psi_{\widetilde{v}}'$ from (86) by means of the inverse Lorentz transformation: $\psi_{\widetilde{v}}' = \Lambda_{\widetilde{v}}^{-1}(\psi_0'')$. Now, according to the standard views, the solution belonging to condition $\psi_{\widetilde{v}}'$ describes the same object when it is moving at a given constant velocity relative to K'. This is the way we usually solve problems such as the electromagnetic field of a moving point charge, the Lorentz contraction of a rigid body, the loss of phase suffered by a moving clock, the dilatation of the mean life of a cosmic ray μ-meson, etc. (As we have seen in Points **10–11**, the situation is, in fact, much more complex. Whether or not the solution thus obtained is correct depends on the details of the relaxation process after the acceleration of the system.)

The aether

59 . Many of those, like Einstein himself (see Point **49**), who admit the "empirical equivalence" of Lorentz's theory and special relativity argue that the latter is "incomparably more satisfactory" because it has no reference to the aether. As it is obvious from the previous sections, we did not make any reference to the aether in the logical reconstruction of Lorentz's theory. It is however a historic fact that Lorentz did. In this section, I want to clarify that the concept of aether is merely a verbal decoration in Lorentz theory, which can be interesting for the historians, but negligible from the point of view of recent logical reconstructions.

60 . One can find various verbal formulations of the relativity principle and Lorentz-covariance. In order to compare these formulations, let us introduce the following notations:

$A(K', K'') :=$ The laws of physics in inertial frame K' are such that the laws describing a physical system co-moving with frame K'' are obtainable by solving the problem for the similar physical system at rest relative to K' and perform the following substitutions:

$$\begin{aligned} \widetilde{x}_1^{K'} &\mapsto \alpha_1 = \widetilde{x}_1^{K'} \\ \widetilde{x}_2^{K'} &\mapsto \alpha_2 = \widetilde{x}_2^{K'} \\ \widetilde{x}_3^{K'} &\mapsto \alpha_3 = \frac{\widetilde{x}_3^{K'} - \widetilde{v}\widetilde{t}^{K'}}{\sqrt{1-\frac{\widetilde{v}^2}{c^2}}} \\ \widetilde{t}^{K'} &\mapsto \tau = \frac{\widetilde{t}^{K'} - \frac{\widetilde{v}}{c^2}\widetilde{x}_3^{K'}}{\sqrt{1-\frac{\widetilde{v}^2}{c^2}}} \end{aligned} \tag{87}$$

$B(K', K'') :=$ The laws of physics in K' are such that the mathematically introduced variables $\alpha_1, \alpha_2, \alpha_3, \tau$ in (87) are equal to $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, that is, the "space" and "time" tags obtained by means of measurements in K'', performed with the same measuring-rods and clocks we used in K' after that they were transfered from K' into K'', ignoring the fact that the equipments undergo deformations during the transmission.

$C(K', K'') :=$ The laws of physics in K' are such that the laws of physics empirically ascertained by an observer in K'', describing the behaviour of physical objects co-moving with K'', expressed in variables $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in in K', describing the similar physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transfered from K' to K'', ignoring the fact that the equipments undergo deformations during the transmission.

It is obvious that

$$A(K', K'') \,\&\, B(K', K'') \Rightarrow C(K', K'')$$

So, let us restrict our considerations on the more fundamental

$$A(K', K'') \,\&\, B(K', K'')$$

Taking this statement, the usual Einsteinian formulation of the relativity principle is the following:

$$\left[\begin{array}{c}\text{Einstein's}\\ \text{Relativity}\\ \text{Principle}\end{array}\right] = (\forall K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right]$$

Many believe that this version of relativity principle is essentially different from the similar principle of Lorentz, since Lorentz's principle makes explicit reference to the motion relative to the aether. Using the above introduced notations, it says the following:

$$\begin{bmatrix}\text{Lorentz's}\\ \text{Principle}\end{bmatrix} = (\forall K'')\left[A\left(\text{aether}, K''\right) \,\&\, B\left(\text{aether}, K''\right)\right]$$

It must be clearly seen, however, that Lorentz's aether hypothesis is logically independent from the actual physical content of his theory. In fact, as a little reflection reveals, *Lorentz's principle and Einstein's relativity principle are logically equivalent to each other.* It is trivially true that

$$\begin{aligned}\begin{bmatrix}\text{Einstein's}\\ \text{Relativity}\\ \text{Principle}\end{bmatrix} &= (\forall K')(\forall K'')\left[A\left(K', K''\right) \,\&\, B\left(K', K''\right)\right]\\ &\Rightarrow (\forall K'')\left[A\left(\text{aether}, K''\right) \,\&\, B\left(\text{aether}, K''\right)\right]\\ &= \begin{bmatrix}\text{Lorentz's}\\ \text{Principle}\end{bmatrix}\end{aligned}$$

It follows from the meaning of $A\left(K', K''\right)$ and $B\left(K', K''\right)$ that

$$\begin{aligned}&(\exists K')(\forall K'')\left[A\left(K', K''\right) \,\&\, B\left(K', K''\right)\right]\\ \Rightarrow\; &(\forall K')(\forall K'')\left[A\left(K', K''\right) \,\&\, B\left(K', K''\right)\right]\end{aligned}$$

Consequently,

$$\begin{aligned}\begin{bmatrix}\text{Lorentz's}\\ \text{Principle}\end{bmatrix} &= (\forall K'')\left[A\left(\text{aether}, K''\right) \,\&\, B\left(\text{aether}, K''\right)\right]\\ &\Rightarrow (\exists K')(\forall K'')\left[A\left(K', K''\right) \,\&\, B\left(K', K''\right)\right]\\ &\Rightarrow (\forall K')(\forall K'')\left[A\left(K', K''\right) \,\&\, B\left(K', K''\right)\right]\\ &= \begin{bmatrix}\text{Einstein's}\\ \text{Relativity}\\ \text{Principle}\end{bmatrix}\end{aligned}$$

Thus, it is Lorentz's principle itself—the verbal formulation of which refers to the aether—that renders any claim about the aether a logically separated hypothesis outside of the scope of the factual content of both Lorentz theory and special relativity. The role of the aether could be played by anything else. As both theories claim, it follows from the empirically confirmed laws of physics that physical systems undergo deformations when they are transferred from one inertial frame K' to another frame K''. One could say, these deformations are caused by the transmission of the system from K' to K''. You could say they are caused by the "wind of aether". By the same token you could say, however, that they are caused by "the wind of *anything*", since if the physical system is transfered from K' to K'' then its state of motion changes relative to an arbitrary third frame of reference.

61 . On the other hand, it must be mentioned that special relativity does not exclude the existence of the aether.[15] Neither does the Michelson–Morley experiment. If special relativity/Lorentz theory is true then there must be no indication of the motion of the interferometer relative to the aether. Consequently, the fact that we do not observe indication of this motion is not a challenge for the aether theorist. Thus, the hypothesis about the existence of aether is logically independent of both Lorentz theory and special relativity.

Symmetry principle and heuristic value

62 . Finally, it worth while mentioning that Lorentz's theory and special relativity, as completely identical theories, offer the same symmetry principles and heuristic power. As we have seen, both theories claim that quantities $\widetilde{x}^{K'}, \widetilde{t}^{K'}$ in an arbitrary K' and the similar quantities $\widetilde{x}^{K''}, \widetilde{t}^{K''}$ in another arbitrary K'' are related through a suitable Lorentz transformation. This fact in conjunction with the relativity principle (within the scope of validity of the principle) implies that laws of physics are to be described by Lorentz covariant equations, if they are expressed in terms of variables $\widetilde{x}$ and $\widetilde{t}$, that is, in terms of the results of measurements obtainable by means of the corresponding co-moving equipments—which are distorted relative to the *etalons*. There is no difference between the two theories that this $\widetilde{\text{space}}$-$\widetilde{\text{time}}$ symmetry provides a valuable heuristic aid in the search for new laws of nature.

63 . With these comments I have completed the argumentation for my basic claim that special relativity and Lorentz theory are completely identical in both senses, as theories about space-time and as theories about the behaviour of moving physical objects. Consequently, in comparison with the classical Galileo-invariant conceptions, special relativity theory does not tell us anything new about space and time. As we have seen, the longstanding belief that it does is the result of a simple but subversive terminological confusion.

[15] Not to mention that already in 1920 Einstein himself argues for the existence of some kind of aether. (See Reignier 2000)

Absolute Theory of Space and Time

64 . Definitions (D1)–(D8) in Point **33**, faithfully reflecting how "space" and "time" tags are understood in classical physics and relativity theory, answered the purpose of demonstrating that Einstein's special relativity has exactly the same claims about space and time as classical physics and Lorentz's theory. However, neither the classical nor the relativistic definitions are trouble free. They are based on several pre-assumptions about contingent facts of nature which cannot be known or even formulated prior to the definitions of space and time tags.

Let us focus on what is common to both the classical and relativistic approaches, definitions (D1)–(D4). The first difficulty is caused by the usage of measuring rod for the definition of distance. The problem I mean is different from the one proposed by Reichenbach (1958), namely that the length of the rod may be altered by some universal forces when the rod is transported from one place to the another. This is no problem from logical/operational point of view, as long as this method provides an unambiguous definition of space tags. In accordance with Reichenbach's final conclusion, I believe that the Newtonian concept of "absolute length" (see Point **67**) of the rod, independent of operational definition of "distance", is meaningless or at least is outside of the scope of physics. If space tags are defined according to (D2) then the length of the measuring rod is—by definition—constant, no matter what is our metaphysical pre-assumption about the length of the rod *ansich.* There are, however, real circularities here that appear at the very operational level. The operations described in (D2) and (D4) rest on the concept of a measuring rod at rest relative to a given reference frame. However, we encounter the following difficulties:

(a) We have seen in Point **19** that the concept of a rod "at rest" relative to a reference frame is problematic in itself.

(b) One might think that this is no problem if the measuring rod is always in equilibrium state when we are measuring with it. It must be clear however that the equilibrium state of the rod cannot be ascertained prior to the definition of its length, that is, prior to the definition of distance.

(c) The concept of rest relative to a reference frame is problematic not only for the measuring rod, as a whole, but even for one single particle of the rod. The reason is that we are missing a prior definition of velocity relative to a given reference frame.

(d) Throughout definitions (D1)–(D9) we nonchalantly used the term "reference frame". Of course, it is no problem to give the usual meaning to this term *after* having defined space and time tags of events; when we already have the concepts of simultaneity, the distance of simultaneous events, dimensions, straight lines, etc. But the term "reference frame" has no meaning prior to the space and time tags. We encounter this wrong circularity in definitions (D2) and (D4): we ought to superpose the measuring-rod along a straight line, such that the rod is always at rest relative to the reference frame.

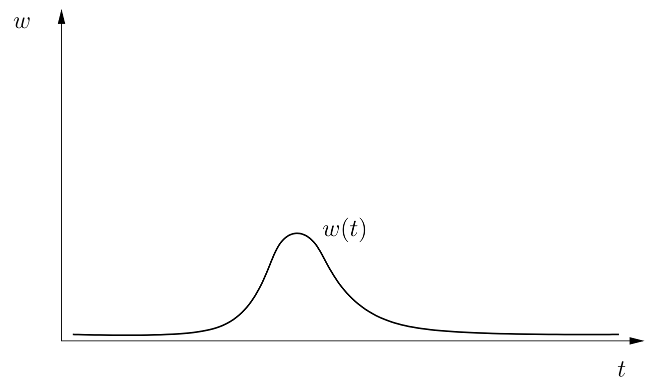


Figure 13. Velocity may vary such that the clock's journey takes very long time, nevertheless the integral in (88) is less than t

(e) We also used the term "inertial" frame of reference. This is another term that has no meaning without a previous definition of space and time tags.

65 . Another source of circularities is the "slow transportation" of the standard clock in definitions (D1) and (D3). The reason why the transportation must be slow is that the clock may accumulate a loss of phase during its journey. From (56) we can express this phase shift:

$$\Delta T = t - \int_0^t \sqrt{1 - \frac{w(\tau)^2}{c^2}}\, d\tau \tag{88}$$

where $w(t)$ is the clock's velocity during its journey. Of course, $\Delta T \to 0$ if $w(t)$ tends to zero in some uniform sense, for instance if $max\ |w(t)| \to 0$. One might think that this condition can be provided without any vicious circularity by moving the standard clock from its original place to the locus of the event in question over a very long period of time, according to the reading of the clock itself. This is however not the case. If function $w(t)$ is something like as shown in Fig. 13 then the clock's journey takes very long time, nevertheless the loss of phase in (88) does not vanish. Yet one might also think that this does not cause a vicious circularity in the operational definition of time tags, because we can include the loss of phase into the definition, just like in the relativistic definition (D6).[16] However, this operation could not provide an unambiguous definition of time tags. The reason is that the phase shift (consequently, the reading) of the clock depends on the concrete shape of function $w(t)$. To keep $w(t)$ controlled—either in order to avoid ambiguity, or in order to provide the condition $max\ |w(t)| \to 0$—we must be able to ascertain the clock's instantaneous velocity relative to reference frame K, throughout

[16] In definition (D6), the time tag is simply defined by the reading of the clock, disregarding the loss of phase accumulated during its journey. This phase shift—calculated in Point **37**—is, for example, the origin of the difference between $\hat{t}$-simultaneity and $\widetilde{t}$-simultaneity.

its journey. And this leads to the same vicious circularities we mentioned in Point **64** (c) and (d).

66 . The upshot of these considerations is that, in order to avoid the circularities mentioned above and to minimise the conventional elements in the empirical foundation of our physical theory of space and time, we must avoid using standard measuring rod in the definition of distance and using slow transportation of the standard clock in the definition of time tags. We must also abstain from relying on the concept of reference frame and inertial motion.

Instead, we will use one standard clock and light signals. A light signal should not be understood as a "light ray" or a "light beam", that is, we should not assume—in advance—that the light signal propagates along a "straight line".

Empirical Definition of Space and Time Tags

67 . First we chose an *etalon* clock. That is to say, we chose a system (a sequence of phenomena) floating somewhere in the universe. Let the *etalon* clock be the clock in the Paris International Bureau of Weights and Measures. We do not assume that this is a clock measuring “proper time”. We do not assume that it is “running uniformly”. Neither we assume that it is “at rest” relative to anything, nor that it is of “inertial motion”. None of these concepts is defined yet.

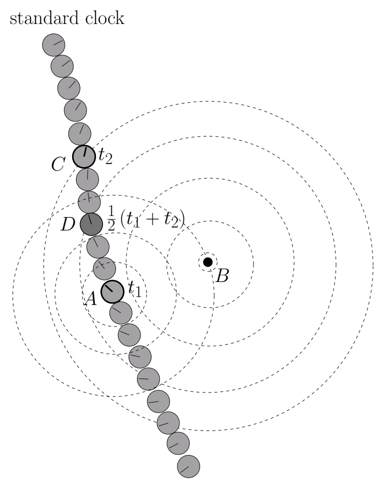


Figure 14. Operational definition of time tags

Consider the experimental arrangement in Fig. 14. The standard clock emits a radio signal at clock-reading t_1 (event A). The signal is received by another equipment which, immediately, emits another signal (event B). This “reflected” signal is detected by the standard clock at t_2 (event C).

Definition (A1) The *absolute time* tag of event B is the following:

$$\tau(B) := t_1 + \frac{1}{2}(t_2 - t_1) \tag{89}$$

The definition is about event B consisting in the “reflection” of the radio signal emitted by the standard clock. That is to say, we assigned an absolute time tag to a definite physical phenomenon which we called “event”. It is far from obvious, however, what must be regarded as an event in general—prior to the concepts of time and distance—, and how one can extend the definition for the physical events of other kinds. (See Brown 2005, pp. 11-14.) We do not dwell on this problem here. The reader can easily imagine various operational solutions of how to use the B-type “reflection” events for marking other physical events/phenomena. So we will assume that definition (A1) is extended for all physical events.

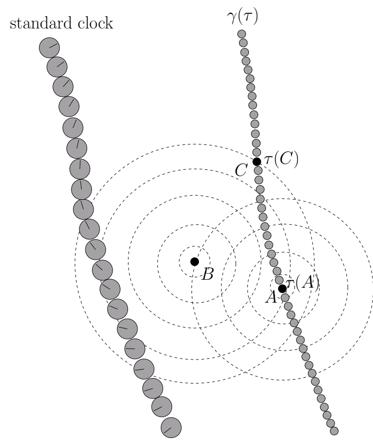


Figure 15. Clock-like time sequence

68 . At this point, one might think that we are ready to define the distance between simultaneous events in the usual way. Surely, we can define the distance between the simultaneous events D and B as $\frac{1}{2}(t_2 - t_1)\,c$, where the value of c is taken as a convention. However, as a little reflection reveals, in this way we can define only the distance from the standard clock. But there is no way to extend this definition for arbitrary pair of simultaneous events. In order to define the distance between arbitrary simultaneous evens we need further preparations.

Denote S_τ the set of simultaneous events with time tag τ.

Definition (A2) A one-parameter family of events $\gamma(\tau)$ is called *time sequence* if $\gamma(\tau) \in S_\tau$ for all τ.

One has to recognise that a time sequence is a clock-like process. For every event, one can define a time-like tag in the same way as (A1): Event A (Fig. 15) is marked with the emission of a radio signal at time $\tau(A)$. The signal is reflected at event B. Event C is the detection of the reflected signal at time $\tau(C)$. We define the following time-like tag for event B:

$$\tau^\gamma(B) := \tau(A) + \frac{1}{2}\left(\tau(C) - \tau(A)\right)$$

It is an empirical fact that $\tau^\gamma(B) \neq \tau(B)$ in general. It is another empirical observation however that for some particular cases $\tau^\gamma(B) = \tau(B)$.

Definition (A3) A time sequence $\gamma(\tau)$ is a *synchronised copy of the standard clock* if for every event B $\tau^\gamma(B) = \tau(B)$.

Whether or not there exist synchronised copies of the standard clock is an empirical question. Observations confirm the following statement:

Empirical fact (E1) For any event A there exists a unique synchronised copy of the standard clock $\gamma(\tau)$ such that

$$A = \gamma\left(\tau(A)\right)$$

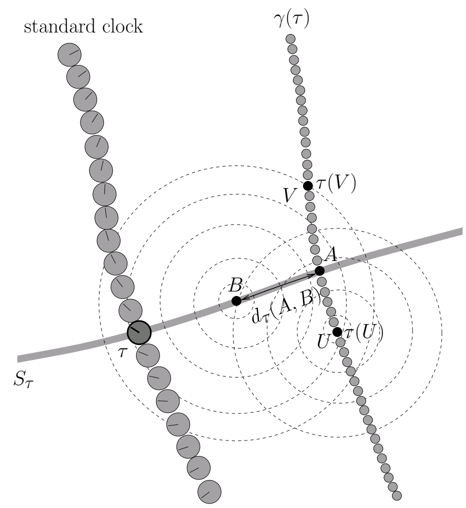


Figure 16. The distance between two simultaneous events

69 . Now we are ready to define the distance between simultaneous events.

Definition (A4) The *absolute distance* between two simultaneous evens $A, B \in S_\tau$ is operationally defined in the following way. Take a synchronised copy of the standard clock γ such that $A = \gamma(\tau)$. (See Fig. 16) Let $U = \gamma(\tau(U))$ is an event marked with the emission of a radio signal at absolute time $\tau(U)$, such that the signal is received and reflected at event B. The detection of the reflected signal marks event $V = \gamma(\tau(V))$ of time tag $\tau(V)$. The absolute distance is

$$d_\tau(A, B) := \frac{1}{2}(\tau(V) - \tau(U))\, c \tag{90}$$

where $c = 300\,000\,000 \frac{m}{s}$ by convention.

70 . Although they seem to be evident, the following facts cannot be known *a priori*:

Empirical fact (E2) For all $A, B, C \in S_\tau$

$$\begin{aligned} d_\tau(A, B) &\geq 0 & (91)\\ d_\tau(A, A) &= 0 & (92)\\ d_\tau(A, B) + d_\tau(B, C) &\geq d_\tau(A, C) & (93) \end{aligned}$$

The following propositions are however derivable from the definitions.

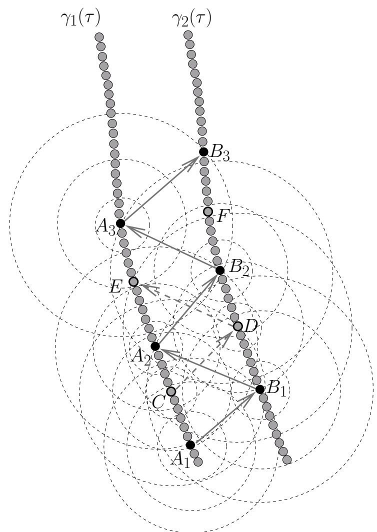


Figure 17. Synchronised copies of the standard clock keep the distance between each other

Lemma 1 Consider two synchronised copies of the standard clock γ_1 and γ_2 (Fig. 17). For any moment of absolute time τ_0

$$d_{\tau_0}\left(\gamma_1(\tau_0), \gamma_2(\tau_0)\right) = d_{\tau_0}\left(\gamma_2(\tau_0), \gamma_1(\tau_0)\right) \tag{94}$$

and

$$d_{\tau_0}\left(\gamma_1(\tau_0), \gamma_2(\tau_0)\right) = d_{\tau_0+T}\left(\gamma_1(\tau_0+T), \gamma_2(\tau_0+T)\right) \tag{95}$$

where

$$T = \frac{d_{\tau_0}\left(\gamma_1(\tau_0), \gamma_2(\tau_0)\right)}{c}$$

Proof Let $\gamma_1(\tau_0)$ be event A_2. Consider the following events: a radio signal is emitted at A_1, then reflected at B_1, then it is reflected again at A_2 and reflected again at B_2, and so on. Let $\tau(E) = \tau(B_2)$ and $\tau(C) = \tau(B_1)$. Taking into account that both γ_1 and γ_2 are synchronised copies of the standard clock, we have the following equations:

$$\begin{aligned}
\tau(A_2) &= \frac{\tau(B_2) + \tau(B_1)}{2} \\
\tau(B_2) &= \frac{\tau(A_3) + \tau(A_2)}{2} \\
\tau(B_1) &= \frac{\tau(A_2) + \tau(A_1)}{2}
\end{aligned}$$

From the above three equations we have

$$\tau(A_3) - \tau(A_2) = \tau(A_2) - \tau(A_1) \tag{96}$$

and

$$\tau(B_2) - \tau(B_1) = \tau(A_2) - \tau(A_1) \tag{97}$$

Therefore,

$$\tau(E) - \tau(C) = \tau(A_2) - \tau(A_1) = \tau(B_2) - \tau(B_1)$$

Imagine now a radio signal emitted from C, reflected at D and detected at E. Taking into account that

$$\frac{\tau(E) + \tau(C)}{2} = \tau(D) = \tau_0 = \frac{\tau(B_2) + \tau(B_1)}{2}$$

we have

$$\begin{aligned} d_{\tau_0}(\gamma_1(\tau_0), \gamma_2(\tau_0)) &= \frac{\tau(E) - \tau(C)}{2} c \\ &= \frac{\tau(B_2) - \tau(B_1)}{2} c \\ &= d_{\tau_0}(\gamma_2(\tau_0), \gamma_1(\tau_0)) \end{aligned}$$

Taking into account this symmetry, (95) immediately follows from (96).

■

In other words, as it follows from (94), for any $A, B \in S_\tau$

$$d_\tau(A, B) = d_\tau(B, A) \tag{98}$$

One has to recognise that a function $S_\tau \times S_\tau \to \mathbb{R}$ with properties (91)–(93) and (98) is what the mathematician calls metric on S_τ. Thus, we can stipulate that (S_τ, d_τ) is a metric space for every moment of absolute time τ.

71 . Having metric defined on S_τ, we can define the concept of a straight line in S_τ (Fig. 18).

Definition (A5) *A* subset $\sigma \subset S_\tau$ is called (straight) *line* if satisfies the following conditions:

1. for any $A, B, C \in \sigma$ $d_\tau(A, C) + d_\tau(C, B) = d_\tau(A, B)$ or $d_\tau(A, B) + d_\tau(B, C) = d_\tau(A, C)$ or $d_\tau(B, A) + d_\tau(A, C) = d_\tau(B, C)$.

2. σ is maximal for property 1.

Empirical fact (E3) For every $A, B \in S_\tau$ there exists a unique line containing A and B.

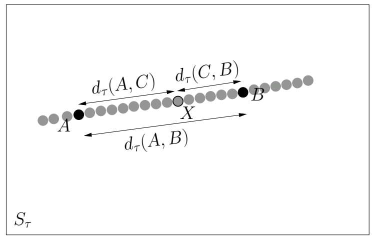


Figure 18. Line segment

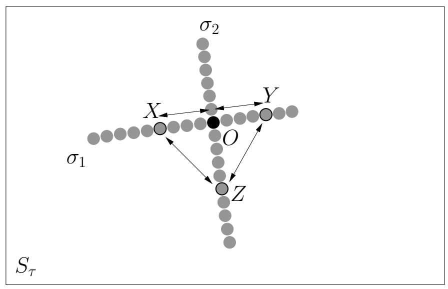


Figure 19. Orthogonal line segments

Definition (A6) Let σ_1 and σ_2 two lines in S_τ such that $\sigma_1 \cap \sigma_2 = \{O\}$ (see Fig. 19). σ_2 is *orthogonal* to σ_1 if for every $Z \in \sigma_2$ and for every $X, Y \in \sigma_1$

$$d_\tau(X,O) = d_\tau(O,Y) \Leftrightarrow d_\tau(X,Z) = d_\tau(Y,Z)$$

Empirical fact (E4) If σ_1 is orthogonal to σ_2 then σ_2 is orthogonal to σ_1.

Empirical fact (E5) For every $O \in S_\tau$ there exist three lines σ_1,σ_2 and σ_3 such that they are pairwise orthogonal and $\sigma_1 \cap \sigma_2 \cap \sigma_3 = \{O\}$.

Empirical fact (E6) Let $O \in S_\tau$ an arbitrary event and three lines σ_1,σ_2 and σ_3 such that they are pairwise orthogonal and $\sigma_1 \cap \sigma_2 \cap \sigma_3 = \{O\}$. There is no line $\sigma \subset S_\tau$ orthogonal to each of σ_1,σ_2 and σ_3, such that $\sigma_1 \cap \sigma_2 \cap \sigma_3 \cap \sigma = \{O\}$.

We usually express this fact by saying that space is three dimensional.

Empirical fact (E7) Let $A \in S_\tau$ be an arbitrary event and $\sigma_1 \subset S_\tau$ and arbitrary line. There always exists a line σ_2 orthogonal to σ_1.

Definition (A7) Using the notations in (E7), let $\sigma_1 \cap \sigma_2 = \{O\}$. Distance of $d_\tau(A,O)$ is called *the distance of A from* σ_1.

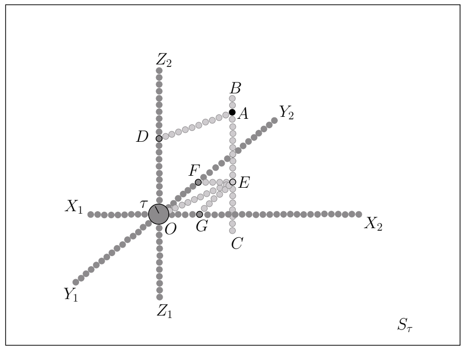


Figure 20. Cartesian coordinates in S_τ

Definition (A8) Let $\sigma_1 \subset S_\tau$ be a line. A line σ_2 is *parallel* to σ_1 if for all $X \in \sigma_2$ the distance of X from σ_1 is the same.

Empirical fact (E8) Let $\sigma_1 \subset S_\tau$ be a line and let $C \in S_\tau$ an arbitrary event. There exists exactly one line σ_2 such that $C \in \sigma_2$ and σ_2 is parallel to σ_1.

Definition (A9) Let $A, B \in \sigma$ two events on line σ. *Line segment* between events $A, B \in S_\tau$ is the following subset of σ:

$$\sigma(A,B) := \{ X \in \sigma |\, d_\tau(A,X) + d_\tau(X,B) = d_\tau(A,B)\} \tag{99}$$

72 . Now, we have everything at hand to define the usual Cartesian coordinates in S_τ. First we need a 3-frame.

Definition (A10) A *3-frame* in S_τ consists of three pairwise orthogonal line segments , $\sigma(Y_1, Y_2)$, $\sigma(Z_1, Z_2)$, such that

$$\sigma(X_1, X_2) \cap \sigma(Y_1, Y_2) \cap \sigma(Z_1, Z_2) = \{O\}$$

where O is the origin of the frame (Fig. 20).

The end points play marginal role, but we do not assume that these segments have "infinite" length. The segments are supposed to be long enough for the purposes of the empirical coordination of the physical events in question. The origin of the 3-frame is arbitrary, although it could be a nature choice to take the "τ-event" of the standard clock as origin.

In the following definition we give the operational definition of the three absolute space tags of an event $A \in S_\tau$.

Definition (A11) Take a line segment $\sigma(B, C) \ni A$ parallel to $\sigma(Z_1, Z_2)$. Take another line segment $\sigma(A, D)$ orthogonal to $\sigma(Z_1, Z_2)$ such that $D \in \sigma(Z_1, Z_2)$. Let $\sigma(O, E)$ be a line segment parallel to $\sigma(A, D)$ such that $E \in \sigma(B, C)$. Finally, take the line segments $\sigma(E, F)$ and $\sigma(E, G)$ such that $\sigma(E, F)$ is parallel to $\sigma(X_1, X_2)$ and $F \in \sigma(Y_1, Y_2)$, and $\sigma(E, G)$ is parallel to $\sigma(Y_1, Y_2)$ and $G \in \sigma(X_1, X_2)$. Now, the space tags are defined as follows:

$$x_\tau(A) := \begin{cases} d_\tau(G, O) & \text{if} \quad G \in \sigma(O, X_2) \\ -d_\tau(G, O) & \text{if} \quad G \in \sigma(O, X_1) \end{cases}$$

$$y_\tau(A) := \begin{cases} d_\tau(F, O) & \text{if} \quad F \in \sigma(O, Y_2) \\ -d_\tau(F, O) & \text{if} \quad F \in \sigma(O, Y_1) \end{cases}$$

$$z_\tau(A) := \begin{cases} d_\tau(D, O) & \text{if} \quad D \in \sigma(O, Z_2) \\ -d_\tau(D, O) & \text{if} \quad D \in \sigma(O, Z_1) \end{cases}$$

73 . It must be emphasised that with the above definitions we only defined the space tags in a given set of simultaneous events S_τ. Yet, we have no connection whatsoever between two S_τ and $S_{\tau'}$ if $\tau \neq \tau'$. In principle, there exist infinitely many possible bijections between the different S_τ's, but without any natural physical meaning. This is true, even if we prescribe that the bijection must be an isomorphism preserving distances.

According to some vague intuition, a time sequence $\gamma(\tau)$ satisfying that

$$x_\tau(\gamma(\tau)) = \text{const.} \tag{100}$$

$$y_\tau(\gamma(\tau)) = \text{const.} \tag{101}$$

$$z_\tau(\gamma(\tau)) = \text{const.} \tag{102}$$

corresponds to a localised physical object being at rest. "At rest" – relative to what? The actual behaviour described by these equations very much depends on how the different 3-frames are chosen in the different S_τ's. One might think that an object is at rest if equations (100)–(102) hold in one and the same 3-frame in all S_τ. But, what does it mean that "one and the same 3-frame in all S_τ"? When can we say that a line segment $\sigma(X_1', X_2')$ in $S_{\tau'}$ is *the same* 3-frame axis as $\sigma(X_1, X_2)$ in S_τ? When can we say that an event A' is in the same place in $S_{\tau'}$ as event A in S_τ? In asking these questions, it is necessary to be careful of a possible misunderstanding. Although they are close to each other, the problem we are addressing here is different from the problem of persistence of physical objects. What we would like to define is the identity of two locuses of space at two different times, and not the genidentity of the physical objects occupying them. One might think that some definition of genidentity of physical objects must be prior to our operational definition of space and time tags, at least in the case of the standard clock. This is, however, not necessarily the case. The standard clock is just an ordered (ordered by the clock readings) sequence of physical events, but without any further metaphysical assumption that these

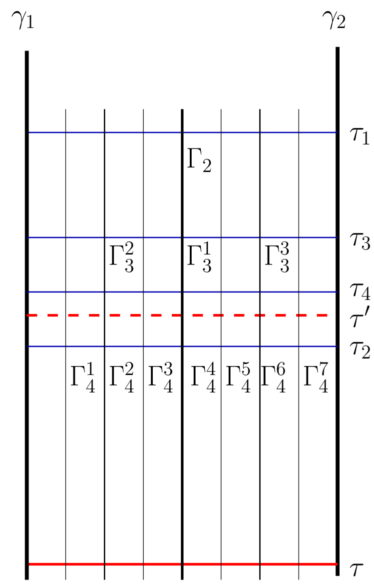


Figure 21. Proof of Lemma 2

events belong to the same physical object. (We definitely do not have such an assumption in the case of a synchronised copy of the standard clock.)

74 . In order to establish connection between arbitrary two sets of simultaneous events we need some preparations.

Lemma 2 Let γ_1 and γ_2 be arbitrary two synchronised copies of the standard clock. For any two moments of absolute time τ and τ'

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) = d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau')) \tag{103}$$

Proof The proof will be based on (95). Let us assume that $\tau < \tau'$. Denote T the period in (95), that is

$$T = \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{c}$$

First we will prove that

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \geq d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau'))$$

Let n be the smallest integer such that $\tau' < \tau + nT =: \tau_1$ (Fig. 21). It follows from (95) that

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) = d_{\tau_1}(\gamma_1(\tau_1), \gamma_2(\tau_1))$$

Let $\tau_2 := \frac{\tau_1+\tau}{2}$. Consider the synchronised copy of the standard clock Γ_2 that goes through the middle point of line segment $\sigma\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)$. Taking into account that $\tau_2 = \tau + m_2\frac{T}{2}$ for some integer m_2 (namely, $m_2 = n$), and also that $\frac{T}{2}c = \frac{d_\tau(\gamma_1(\tau),\gamma_2(\tau))}{2}$, one can apply (95) for the synchronised copies of the standard clock γ_1 and Γ_2. Therefore,

$$d_{\tau_2}\left(\gamma_1\left(\tau_2\right),\Gamma_2\left(\tau_2\right)\right) = d_\tau\left(\gamma_1\left(\tau\right),\Gamma_2\left(\tau\right)\right) = \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{2}$$

The same argument can be repeated for γ_2 and Γ_2. Therefore,

$$d_{\tau_2}\left(\Gamma_2\left(\tau_2\right),\gamma_2\left(\tau_2\right)\right) = d_\tau\left(\Gamma_2\left(\tau\right),\gamma_2\left(\tau\right)\right) = \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{2}$$

It follows from (93) that

$$d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right) \geq d_{\tau_2}\left(\gamma_1\left(\tau_2\right),\gamma_2\left(\tau_2\right)\right)$$

Assume that $\tau' > \tau_2$. Therefore, take $\tau_3 := \frac{\tau_2+\tau_1}{2}$. Again, consider the synchronised copies of the standard clock Γ_3^1, Γ_3^2, Γ_3^3 dividing line segment $\sigma\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)$ into 4 pieces of equal length. Taking into account that $\tau_3 = \tau + m_3\frac{T}{4}$ for some integer m_3 and also that $\frac{T}{4}c = \frac{d_\tau(\gamma_1(\tau),\gamma_2(\tau))}{4}$, one can apply (95) for the synchronised copies of the standard clock γ_1 and Γ_3^1. Therefore,

$$d_{\tau_3}\left(\gamma_1\left(\tau_3\right),\Gamma_3^1\left(\tau_3\right)\right) = d_\tau\left(\gamma_1\left(\tau\right),\Gamma_3^1\left(\tau\right)\right) = \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{4}$$

Similarly,

$$\begin{aligned} d_{\tau_3}\left(\Gamma_3^1\left(\tau_3\right),\Gamma_3^2\left(\tau_3\right)\right) &= \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{4} \\ d_{\tau_3}\left(\Gamma_3^2\left(\tau_3\right),\Gamma_3^3\left(\tau_3\right)\right) &= \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{4} \\ d_{\tau_3}\left(\Gamma_3^3\left(\tau_3\right),\gamma_2\left(\tau_3\right)\right) &= \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{4} \end{aligned}$$

Consequently, from (93),

$$d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right) \geq d_{\tau_3}\left(\gamma_1\left(\tau_3\right),\gamma_2\left(\tau_3\right)\right)$$

Assume $\tau' < \tau_3$. Therefore, take $\tau_4 := \frac{\tau_3+\tau_2}{2}$. Again, consider the synchronised copies of the standard clock $\Gamma_4^1, \Gamma_4^2, \Gamma_4^3, \ldots \Gamma_4^7$ dividing line segment $\sigma\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)$ into 8 pieces of equal length. Taking into account that $\tau_4 = \tau + m_4\frac{T}{8}$ for some integer m_4 and also that $\frac{T}{8}c = \frac{d_\tau(\gamma_1(\tau),\gamma_2(\tau))}{8}$, one can apply (95) for the synchronised copies of the standard clock γ_1 and Γ_4^1. Therefore,

$$d_{\tau_4}\left(\gamma_1\left(\tau_4\right),\Gamma_4^1\left(\tau_4\right)\right) = d_\tau\left(\gamma_1\left(\tau\right),\Gamma_4^1\left(\tau\right)\right) = \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{8}$$

Similarly,

$$\begin{aligned} d_{\tau_4}\left(\Gamma_4^1(\tau_4), \Gamma_4^2(\tau_4)\right) &= \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{8} \\ d_{\tau_4}\left(\Gamma_4^2(\tau_4), \Gamma_4^3(\tau_4)\right) &= \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{8} \\ &\vdots \\ d_{\tau_4}\left(\Gamma_4^7(\tau_4), \gamma_2(\tau_4)\right) &= \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{8} \end{aligned}$$

Consequently, from (93),

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \geq d_{\tau_4}(\gamma_1(\tau_4), \gamma_2(\tau_4))$$

And so on and so forth,

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \geq d_{\tau_i}(\gamma_1(\tau_i), \gamma_2(\tau_i))$$

On the other hand,

$$\lim_{i\to\infty} \tau_i = \tau'$$

therefore

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \geq d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau'))$$

Exactly in the same way one can prove that

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \leq d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau'))$$

One simply has to change the roles of τ and τ'. Denote T', this time, the period

$$T' = \frac{d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau'))}{c}$$

Let n' be the smallest integer such that $\tau > \tau' - n'T' =: \tau_1'$ Then, it follows from (95) that

$$d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau')) = d_{\tau_1'}(\gamma_1(\tau_1'), \gamma_2(\tau_1'))$$

Let $\tau_2' := \frac{\tau_1'+\tau'}{2}$. Consider the synchronised copy of the standard clock Γ_2' that goes through the middle point of line segment $\sigma(\gamma_1(\tau'), \gamma_2(\tau'))$. Taking into account that $\tau_2' = \tau' - m_2'\frac{T}{2}$ for some integer m_2, and also that $\frac{T}{2}c = \frac{d_{\tau'}(\gamma_1(\tau'),\gamma_2(\tau'))}{2}$, one can apply (95) for the synchronised copies of the standard clock γ_1 and Γ_2'. Therefore,

$$\begin{aligned} d_{\tau_2'}(\gamma_1(\tau_2'), \Gamma_2'(\tau_2')) &= d_{\tau'}(\gamma_1(\tau'), \Gamma_2'(\tau')) \\ &= \frac{d_{\tau'}(\gamma_1(\tau'), \gamma_2(\tau'))}{2} \end{aligned}$$

Similarly,

$$d_{\tau_2'}\left(\Gamma_2'\left(\tau_2'\right), \gamma_2\left(\tau_2'\right)\right) = \frac{d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)}{2}$$

Therefore,

$$d_{\tau_2'}\left(\gamma_1\left(\tau_2'\right), \gamma_2\left(\tau_2'\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)$$

And so on and so forth,

$$d_{\tau_i'}\left(\gamma_1\left(\tau_i'\right), \gamma_2\left(\tau_i'\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)$$

At the same time,

$$\lim_{i\to\infty} \tau_i' = \tau$$

Consequently,

$$d_{\tau}\left(\gamma_1\left(\tau\right), \gamma_2\left(\tau\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)$$

■

75 . The following isomorphism can be regarded as a natural one.

Definition (A12)

$$\begin{aligned} \mathbb{T}_{\tau}^{\tau'} : S_{\tau} &\rightarrow S_{\tau'} \\ A &\mapsto \mathbb{T}_{\tau}^{\tau'}(A) = \gamma(\tau') \end{aligned}$$

where γ is a synchronised copy of the standard clock such that $A = \gamma(\tau)$. Let us call $\mathbb{T}_{\tau}^{\tau'}$ the *time shift* between S_{τ} and $S_{\tau'}$.

It follows from (E1) and Lemma 2 that this definition is sound and $\mathbb{T}_{\tau}^{\tau'}$ is a bijection preserving distances.

76 . Now we have everything at hand to define the space tags of events.

Definition (A13) Let A be an arbitrary event. The *absolute space tags* of A are defined as follows:

$$\begin{aligned} \xi_1(A) &:= x_0\left(\mathbb{T}_{\tau(A)}^{0}(A)\right) \\ \xi_2(A) &:= y_0\left(\mathbb{T}_{\tau(A)}^{0}(A)\right) \\ \xi_3(A) &:= z_0\left(\mathbb{T}_{\tau(A)}^{0}(A)\right) \end{aligned}$$

Thus we have defined four absolute space-time tags for every event: $\tau(A), \xi_1(A), \xi_2(A), \xi_3(A)$.

Comments

77 . I call $\tau(A)$ "absolute time" not in the sense of what Newton called "absolute, true and mathematical time", that is independent of any empirical definition (see Scholium II in chapter "Definitions" of the *Principia.*), but in the sense of what the 20th century physics calls absolute time, that is "independent of the position and the condition of motion of the system of co-ordinates" (Einstein 1920, p. 51). The space-time tags $\tau(A), \xi_1(A), \xi_2(A), \xi_3(A)$ are *absolute* in the sense that they are not relative to a reference frame but prior to any reference frame (actually the concept of "reference frame" is still not defined).

Our concepts of absolute time and space tags are, of course, "relative" to the trivial semantical convention by which we define the meaning of the terms. Namely, they are "relative" to the *etalon* clock-like process we have chosen in the universe. *This kind of "relativism" is however common to all physical quantities having empirical meaning.* (Beyond the choice of the *etalon* clock, the space tags $\xi_1(A), \xi_2(A), \xi_3(A)$ have some additional conventional element; they also are relative to the chosen 3-frame in S_0. This additional conventionality is, however, of marginal importance; it is nothing more than what we would call in our usual language "the choice of a 3-coordinate basis in a given reference frame".)

78 . As it was already mentioned in Point **33** (Footnote 5), there has been a long discussion in the literature about the conventionality of simultaneity. (See, for example, Reichenbach 1956; Bridgeman 1965; Grünbaum 1974; Salmon 1977; Malament 1977; Friedman 1983; Ben-Yami 2006.) Without entering in the details of the various arguments, the following facts must be pointed out here.

As it is obvious from (89), we chose the standard "$\varepsilon = \frac{1}{2}$-synchronisation". (Of course, it could be a contingent fact of nature that $t_2 = t_1$ in Fig. 14. In that case the choice of the value of ε would not matter.) This choice was entirely conventional; it was *a part of the trivial semantical convention* defining the term "absolute time tag". This choice is prior to any claims about the one-way or even round-trip speed of electromagnetic signals, because there is no such a concept as "speed" prior to the definition of time and space tags; it is, of course, prior to "the metric of Minkowski space-time", in particular to the "light-cone structure of the Minkowski space-time", because we have no words to tell this structure prior to the space-time tags; and it is prior to the causal order of physical events, because—even if we could know this causal order prior to temporality—we cannot know in advance how causal order is related with temporal order (which we have defined here). It is actually prior to any discourse about two locuses in space, because there is no "space" prior to definiton (A1) and there is no concept of a "persistent space locus" prior to definiton (A12).

79 . A remark is in order on the empirical facts (E1)–(E8) to which we refer in constructing space-time tags. In claiming these statements as empirical facts I mean that they ought to be true according to our ordinary physical theories. The ordinary physical theories are however based on the ordinary, problematic, space-time conceptions, relaying on "reference frames realised by rigid bodies" and the like, without proper, non-circular, empirical definitons. Thus, especially in the context of defining the two most fundamental physical quantities, distance and time, we must not regard our ordinary physical theories as empirically meaningful and empirically confirmed claims about the world. Whether these statements are true or not is, therefore, an empirical question, and it is far from obvious whether they would be completely confirmed if the corresponding experiments were performed with higher precision, similar to the recent GPS measurements, especially for larger distances. Strangely enough, according to my knowledge, these very fundamental facts have never been tested experimentally; no textbook or monograph on space-time physics refers to such experimental results; actually, they do not even attempt to provide a clear, non-circular empirical definition of "time" and "distance".

So, the best we can do is to *believe* that our physical theories based on the usual sloppy formulation of space-time concepts are true (in some sense) and to consider the predictions of these theories as empirical facts. However, as the following analysis reveals, it is far from obvious whether the predictions of the believed theories really imply (E1)–(E8).

80 . Throughout the definition of space-time tags, we avoided the term "inertial", and because of a good reason. First of all, if "inertial" is regarded as a kinematical notion based on the concept of straight line and constancy of velocity, then it cannot be antecedent to the concept of space-time tags. If, on the other hand, it is understood as a manner of existence of a physical object in the universe, when the object is undergoing a free floating, in other words, when it is "free from forces", then the concept is even more problematic. The reason is that "force" is a concept defined through the deviation from the trajectory of inertial motion (first circularity), and neither the inertial trajectory nor the measure of deviation from it can be expressed without spatiotemporal concepts, that is, they cannot be antecedent to the definition of space-time tags (second circularity). So there is *no* precise, non-circular definition of inertial motion.

81 . According to our believed special relativistic physical theory, space-time is a 4-dimensional Minkowski space and inertial trajectory is a time-like straight line in the Minkowski space. Since we are prior to the empirical definitions of the basic spatiotemporal quantities, we cannot regard this claim as an empirically confirmed physical theory. Nevertheless, let us assume for a moment that our special relativistic theory is the true description of the world "from God's point of view". It is straightforward to check that all the facts (E1)–(E8) are true if 1) the standard clock moves along an inertial world line in the Minkowski space-time and 2) it reads the proper time, that is, it measures the length of its own word line, according to the Minkowski metric. However, we human beings can know neither whether the standard clock (chosen by us) is of inertial motion in God's Minkowskian space-time nor whether it reads the proper time. What if these conditions fail? What does special relativistic kinematics say about (E1)–(E8) if the standard clock is accelerated and/or it does not read the proper time?

In order to answer this question, we have to follow up the operational definitions (D1), (D2),... and *calculate* whether statements (E1), (E2),... are true or not if the standard clock moves along a given world line γ and the "time" it reads is, say, a given function of the Minkowskian coordinate time, $\chi(t)$. Although the task is straightforward, the calculation is too complex to give a general answer by analytic means. But the problem can be efficiently solved by computer. One finds the following—perhaps surprising—results.

For the sake of the contrast, let me first mention that one obtains a very misguiding result if, for the sake of simplicity, the calculation is made in a *2-dimensional* Minkowski space-time: *No matter if the standard clock moves along a non-inertial world line γ, no matter if it reads a time $\chi(t)$ which is an arbitrary monotonic function of the Minkowskian coordinate time, different from the proper time along its world line, facts (E1)–(E8) are always true.*

If this 2-dimensional result were the final truth one would conclude that no spatiotemporal measurement can ascertain whether the standard clock moves inertially or not; the very concept of "inertial" motion would remain a purely conventional one; facts (E1)–(E8) would always be true, independently of the "objective" fact of how the standard clock moves in God's Minkowski space-time.

In contrast, the real *4-dimensional* calculation leads to the following results:

(A) *Facts (E1)–(E8) are always true if the standard clock moves along an inertial world line, no matter if the clock reads a time $\chi(t)$ which is an arbitrary monotonic function of the Minkowskian coordinate time, different from the proper time along its world line.*

(B) *If the standard clock moves along a non-inertial world line γ, facts (E1)–(E8) are never true, no matter if the clock reads the proper time or not.*

The whole thing hinges on (E1); there are no synchronised copies of the standard clock if the standard clock moves non-inertially.

82 . There are remarkable consequences of the above results:

1. Result (A) implies that no objective meaning can be assigned to the concept of "proper time". "Time" is what the *etalon* clock reads, by definition.

2. Contrary to the misguiding 2-dimensional result, (B) shows that the notion of "inertial motion" is not entirely conventional. In accord with our intuition based on the believed physical theories, we can give an objective meaning to "inertial motion" by means of correct—neither logically nor operationally circular—experiments: *the standard clock is of inertial motion if statements (E1)–(E8) are true.* Assuming that the standard clock is inertial, one can extend the concept for an arbitrary time sequence $\gamma(\tau)$ of events: $\gamma(\tau)$ corresponds to an inertial motion if the absolute space tags $\xi_1(\gamma(\tau)), \xi_2(\gamma(\tau)), \xi_3(\gamma(\tau))$ are linear functions of the absolute time tag τ.

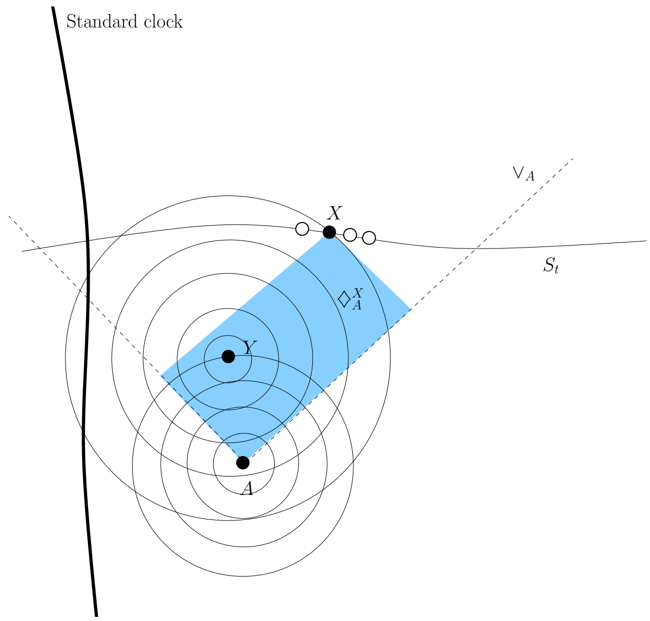


Figure 22. The test of inertiality

3. On the basis of our believed physical theories, one cannot, however, predict whether (E1)–(E8) are true or false. It is still an open *empirical* question.

4. Imagine that (E1)–(E8) are not satisfied. It not only means that the standard clock we have chosen is non-inertial but it also means that the chosen clock is inappropriate for the definition of space-time tags. More exactly, one has to stop at definition (D1). One can define the time tags but cannot define the spatial notions, in particular the distances between simultaneous evens.

5. Consequently, it is meaningless to talk about "non-inertial reference frame", "space-time coordinates (tags) defined/measured by an accelerated observer", and the likes.

83 . In the light of these consequences, it is an intriguing question whether the standard clock contemporary physical laboratories use for coordination of physical events satisfies conditions (E1)–(E8), in particular (E1). It is quite implausible that it does—taking into account the Earth's rotation, the Earth's motion around the Sun, the Solar System's motion in our Galaxy, etc.

Consider first what in fact has to be tested (Fig. 22). (E1) would require the existence of a unique synchronised copy of the standard clock through every event. Let therefore A be an arbitrary event with absolute time tag $\tau(A)$.

Introduce the following notations:

$$\begin{aligned} \vee_A &:= \left\{ X \,\middle|\, \begin{array}{c}\text{Radio signal from } A\\ \text{is received at } X.\end{array} \right\} \\ \wedge^A &:= \left\{ X \,\middle|\, \begin{array}{c}\text{Radio signal from } X\\ \text{is received at } A.\end{array} \right\} \\ \diamondsuit_A^B &:= \vee_A \cap \wedge^B \end{aligned}$$

Consider the following quantity:

$$N := \max_{t,A} \left\{ \begin{array}{lll} \min\limits_{X \in \vee_A \cap S_t} & \max\limits_{Y \in \diamondsuit_A^X} \left| \tau(Y) - \frac{\tau(A)+\tau(X)}{2} \right| & t > \tau(A) \\ \min\limits_{X \in \wedge^A \cap S_t} & \max\limits_{Y \in \diamondsuit_X^A} \left| \tau(Y) - \frac{\tau(A)+\tau(X)}{2} \right| & t < \tau(A) \end{array} \right.$$

$N = 0$ is a necessary condition of inertiality of the standard clock. In this case, for every event A there exists a unique synchronised copy of the standard clock. That is, for every time $t > \tau(A)$ there is a unique event $X \in \vee_A \cap S_t$ such that $\tau(Y) = \frac{\tau(A)+\tau(X)}{2}$ for all $Y \in \diamondsuit_A^X$ and for every time $t < \tau(A)$ there is a unique event $X \in \wedge^A \cap S_t$ such that $\tau(Y) = \frac{\tau(A)+\tau(X)}{2}$ for all $Y \in \diamondsuit_X^A$.

84 . Let us outline how the experimental test could be realised. Our standard clock is transmitting, say in every few nanoseconds, a radio signal encoding the actual clock reading (Fig. 23). We need a huge number of little devices $e_1, e_2, \ldots e_i, \ldots$ with the following functions:

1. They continuously receive the regular time signals from the standard clock.
2. They can transmit radio signals containing the following information: a) an ID code of the device and information about the standard clock reading, so from the signal they send it always can be known which device was the transmitter and what was the standard clock reading received by the transmitter at the moment of the emission of the signal, b) information about the type of event on the occasion of which the signal was transmitted.
3. They can receive the signals transmitted by the others.

We install these devices everywhere in a certain region of the universe. Now, the following events will happen.

1. Assume that e_3 is programed such that it transmits a radio signal (event A) when receives the time signal of t_1 from the standard clock. Let us call it A-signal. The A-signal will arrive back to the standard clock at time t_2.
2. The A-signal sweeps through the whole region and triggers the other devices to transmit a B-signal. For example, event B_i consists in that e_i receives the A-signal from e_3 and emits its own B_i-signal with the needed information. B_j is a similar event for e_j, etc.

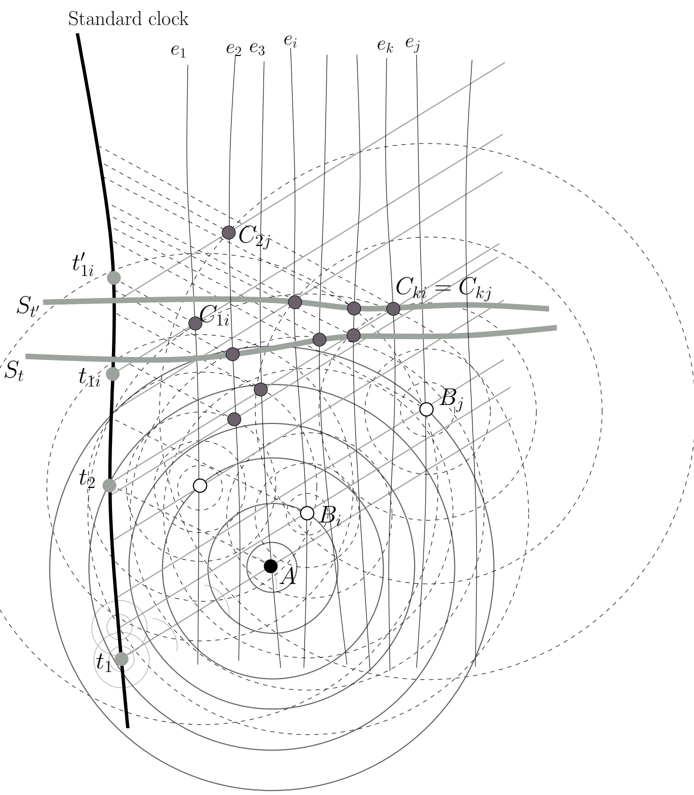


Figure 23. The sketch of a realistic measurement to decide whether the standard clock is inertial or not

3. The B-signals will be received by some other devices. For example, C_{1i} is the event when e_1 receives the B_i-signal transmitted by e_i and sends out his own signal (C_{1i}-signal) with the corresponding information. This information arrives back to the standard clock at time t_{1i}.

In this way, a huge amount of data is recorded, from which we can ascertain the absolute time tags of all events in question. We can determine $\diamondsuit_A^{C_{lm}}$ for every C_{lm}. For example, say, it turns out that $C_{ki} = C_{kj}$ and, therefore, $B_i, B_j \in \diamondsuit_A^{C_{ki}}$, etc. One also can determine the sets of simultaneous events. Now, the standard clock is inertial only if in every S_t there is a unique $C_{lm} \in S_t$ such that for every event $B_i \in \diamondsuit_A^{C_{lm}}$

$$\tau(B_i) = \frac{\tau(A) + \tau(C_{lm})}{2}$$

A matematikai elméletek — fizikai elméletek

The metaphysical basis of logic and mathematics (A physicalist approach)

"after sufficient clarification of the concepts in question it will be possible to conduct these discussions with mathematical rigor and that the result then will be that (under certain assumptions which can hardly be denied [in particular the assumption that there exists at all something like mathematical knowledge] **the platonistic view is the only one tenable**" (Gödel: *Some basic theorems on the foundations of mathematics and their implications*, 1951)

Question: **What if I am not a Platonist but I am a physicalist?**

Physicalism:

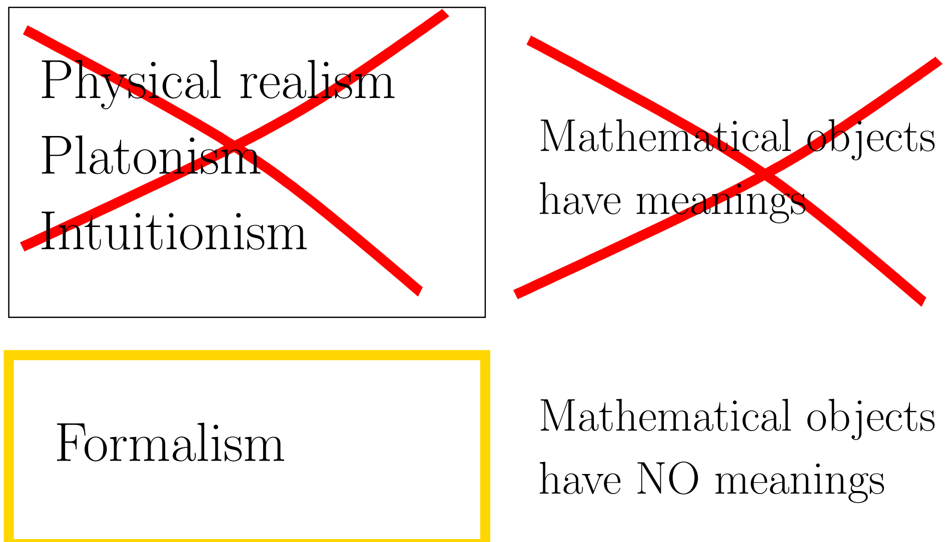
Empiricism: Genuine information about the world must be acquired by *a posteriori* means.

+

Physicalist account of the mental: Experiencing itself, as any other mental phenomena, including the mental processing the experiences, can be wholly explained in terms of physical properties, states, and events in the physical world.

Standard schools in philosophy of mathematics

Physical realism Platonism Intuitionism	Mathematical objects have meanings
Formalism	Mathematical objects have NO meanings



Mathematical objects have no meanings

Thesis **Mathematical "statements" are formulas of a formal language. They are not linguistic objects, consequently they *carry no meanings and Tarskian truths.***

The argument will be based on the **Truth-Condition Theory of Meaning**:

> A meaning for a sentence is something that determines the conditions under which the sentence is true or false.(David Lewis: *General Semantics,* 1972)

In order to determine this "something" one has to follow up how the sentence can be confirmed or refuted.

Consider electrodynamics. What will the physicist answer to the following questions:

Why is $F = k\frac{Q_1 Q_2}{r^2}$ (Coulomb law) true?

How do we know that $F = k\frac{Q_1 Q_2}{r^2}$ is true?

How could you convince me that $F = k\frac{Q_1 Q_2}{r^2}$ is true?

How do you mean that $F = k\frac{Q_1 Q_2}{r^2}$ is true?

How can we verify that $F = k\frac{Q_1 Q_2}{r^2}$ is true?

Answer: $F = k\frac{Q_1 Q_2}{r^2}$ is true in the sense that the force measured between small charged particles is indeed equal to $k\frac{Q_1 Q_2}{r^2}$. We can test/confirm this fact by means of laboratory experiments.

Consider group theory:

Aphabet

variables	$x, y, z, \ldots$	
individual constant	e	(identity)
function symbols	i, p	(inverse, product)
predicate symbol	$=$	
punctuation	$($, $)$, ,	
logical symbols	$\forall, \neg \rightarrow$	

Axioms

(G1)	$p(p(x, y), z) = p(x, p(y, z))$	(associative law)
(G2)	$p(e, x) = x$	(left identity)
(G3)	$p(i(x), x) = e$	(left inverse)

What will the mathematician answer to the following questions:

Why is $p(e, p(e, e)) = e$ is true?

How do we know that $p(e, p(e, e)) = e$ is true?

How could you convince me that $p(e, p(e, e)) = e$ is true?

How do you mean hat $p(e, p(e, e)) = e$ is true?

How can we verify that $p(e, p(e, e)) = e$ is true?

Answer:

The mathematician never refers to the physical/platonic/mental realm and the corresponding epistemic faculties! The mathematician's final argument always is that $p(e, p(e, e)) = e$ is **proved** from the axioms of group theory:

(1)	$p(e, x) = x$	(G2)
(2)	$(\forall x)(p(e, x) = x)$	Gen.
(3)	$(\forall x)(p(e, x) = x) \rightarrow p(e, e) = e$	PC
(4)	$p(e, e) = e$	(2), (3), MP
(5)	$(\forall x)(p(e, x) = x) \rightarrow p(e, p(e, e)) = p(e, e)$	PC
(6)	$p(e, p(e, e)) = p(e, e)$	(2), (5), MP
(7)	$p(e, e) = e \rightarrow p(e, p(e, e)) = p(e, e) \rightarrow p(e, p(e, e)) = e$	PC($=$)
(8)	$p(e, p(e, e)) = p(e, e) \rightarrow p(e, p(e, e)) = e$	(4), (7), MP
(9)	$p(e, p(e, e)) = e$	(6), (8), MP

In Dummett's words:

> Like the empiricist view, the platonist one fails to do justice to the role of proof in mathematics. For, presumably, the supra-sensible realm is as much God's creature as is the sensible one; if so, conditions in it must be as contingent as in the latter. [...] [W]e do not seek, in order to refute or confirm a [mathematical] hypothesis, a means of refining our intuitive faculties, as astronomers seek to improve their instruments. Rather, **if we suppose the hypothesis true, we seek for a *proof* of it**, and it remains a mere hypothesis, whose assertion would therefore be unwarranted, until we find one. (Dummett: *What Is Mathematics About?* (1994), p. 13.)

Partial conclusion:

$p(e, p(e, e)) = e$ does not have meaning; it does not refer to anything and cannot be true or false in the ordinary semantical sense. It is actually not a linguistic object, it is just a brick in a formal system.

The meaningful sentences are like "$\{\text{Group}\} \vdash p(e, p(e, e)) = e$" instead of "$p(e, p(e, e)) = e$". The "$\Sigma \vdash X$" sentences do have meanings and can be true or false—in what sense, it will be clear later on.

RemarkA typical misinterpretation of the formalist "$\Sigma \vdash X$":

"If Σ (is true) then X (is true)"

The essential difference between mathematical truth and semantical truth in a scientific theory describing something in the world

A **physical theory** P is a formal system L + a semantics S pointing to the empirical world. Normally, L is a (first-order) system with

- some *logical axioms and the derivation rules* (usually the first-order predicate calculus with identity)
- the axioms of certain *mathematical theories*
- some *physical axioms*.

A sentence A in physical theory P can be true in two different senses:

Truth$_1$: A is a theorem of L, that is, $\vdash_L A$ (which is a mathematical truth within the formal system L, a fact of the formal system L).

Truth$_2$: According to the semantics S, A refers to an empirical fact (about the physical system described by P).

Example:"The electric field strength of a point charge is $\frac{kQ}{r^2}$" is a theorem of Maxwell's electrodynamics. On the other hand, according to the semantics relating the symbols of the Maxwell theory to the empirical terms, this sentence corresponds to an empirical fact (about the point charges).

Truth$_1$ and Truth$_2$ are independent concepts – one does not automatically imply the other Assume that

- Γ is a set of true$_2$ sentences in L
- and $\Gamma \vdash_L A$

It does not automatically follow that A is true$_2$. **Whether A is true$_2$ is again an empirical question:**

If so, then it is new empirically obtained information about the world, confirming the validity of the ***whole*** physical theory $P = L + S$.

If not, then this information disconfirms the physical theory, ***as a whole***. That is to say, one has to think about *revising* ***one of the*** *constituents* of P.

The physicalist ontology of formal systems

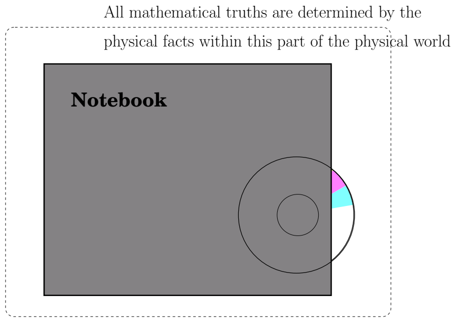
> [N]o philosophy can possibly be sympathetic to a mathematician which does not admit, in one manner or the other, the immutable and unconditional validity of mathematical truth. Mathematical theorems are true or false; their truth or falsity is absolute and independent of our knowledge of them. In *some* sense, mathematical truth is a part of objective reality. (Hardy: *Mathematical Proof*, 1929)

Now we determine what this objective reality actually is.

Thesis:

The objective fact expressed by a mathematical proposition is a fact of a particular part of the physical world: it is a fact of the formal system itself, that is, a fact about the physical system consisting of the signs and the mechanical rules according to which the signs can be combined.

Arguments



Taking into account that the only means of obtaining reliable knowledge about this fact is mathematical proof, ***it must be a fact of the realm inside of the scope of formal derivations***.

Of course, from physicalist point of view it does not matter whether the formal system is embodied in a computer, in a human brain, in brain+paper+hand+pen, etc.

"$p(e, p(e, e)) = e$"	This *is not* a linguistic object!
actually means that	the usual formalist step
"{Group} $\vdash p(e, p(e, e)) = e$"	This is a linguistic object!
which is nothing but	the physicalist step
The assertion that there exists a proof-process, the result of which is $p(e, p(e, e)) = e$	This is a usual scientific assertion, just like $2H_2 + O_2 \rightarrow 2H_2O$

In this way, a mathematical truth **has contingent factual content, as any similar scientific assertion**. It is

- expressing **objective fact** of the physical world
- **synthetic**
- ***a posteriori***
- **not necessary** and **not certain**
- **true before anybody can prove it**

Abstraction is a move from the concrete to the concrete

Many from the formalist school admit that

> **...** in order to think of a formal system at all we must think of it as represented somehow.
>
> (Haskell Curry: *Outlines of a Formalist Philosophy of Mathematics*, 1951)

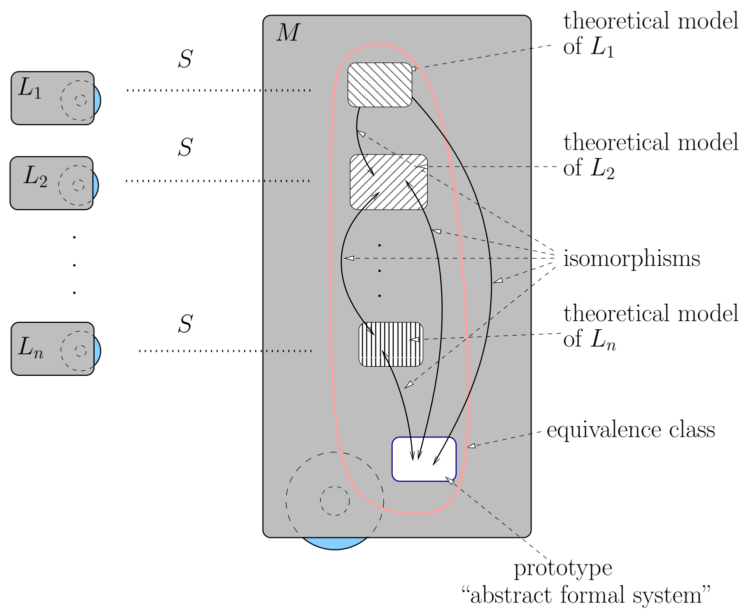
But, Curry continues this passage as follows:

> **...** in order to think of a formal system at all we must think of it as represented somehow. **But when we think of it *as* formal system we abstract from all properties peculiar to the representation.**
>
> (Haskell Curry: *Outlines of a Formalist Philosophy of Mathematics*, 1951)

What does such an "abstraction" actually mean?

What do we obtain if we abstract from some unimportant, peculiar properties of a physical system L_1 (which is a "representation of a formal system") ? We obtain a theory $P = L_2 + S$ about L_1, that is, a formal system L_2 with a semantics S relating the elements of L_2 to the important empirical facts of L_1. That is, **instead of an "abstract structure"** we obtain another **flesh and blood** formal system L_2.

By the same token, one cannot obtain an "abstract structure" as an "equivalence class of isomorphic formal systems". **Such things as "isomorphism", "equivalence", "equivalence class" are living in a formal system "represented somehow"**, that is, **in a flesh and blood formal system**:



This is no attack on scientific realism When a *physical theory* claims that a physical object has a certain property adequately described by means of a formal system, then **this reflects a real feature of physical reality**.

This is not nominalism When many different physical objects display a similar property that is describable by means of the same (equivalent) elements of one common formal system, this will be a true *general* feature of the group.

But, **this realist commitment does not entitle us to claim that "abstract structures" exist *over and above* the real formal systems of physical existence**.

Epistemological status of meta-mathematical theories

We follow **Hilbert's** careful distinction:

mathematics – a system of meaningless signs

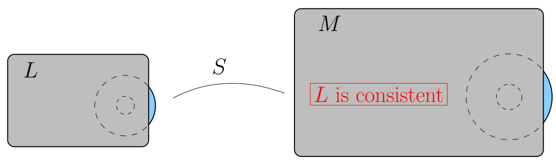
meta-mathematics – meaningful statements *about* mathematics

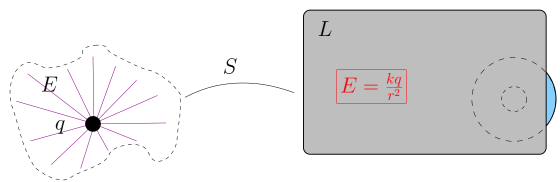
+ physicalism:

formal system – a physical system L

meta-mathematical theory – a *physical* theory (M, S)

All the truths that a meta-mathematical theory can tell us about its object are of the type **Truth$_2$**. This means that **no feature of a formal system can be "proved" mathematically: Genuine information about a formal system must be acquired by *a posteriori* means, that is, by observation of the formal system and, as in physics in general, by inductive generalisation.**





Consequently, all **meta-mathematical "proofs" are questionable**!

- When I say "questionable" I do not mean that I don't believe that, for example, the sentence calculus is consistent. I only mean that **I believe in it just as I believe in the Coulomb law or in the conservation of energy, or any other physical laws, which are acquired by *a posteriori* means.**
- To be sure, both truth$_1$ and truth$_2$ of a formula of M, like L is consistent are known by *a posteriori* means. But,
  - $\vdash_M L$ is consistent is known by observation of the formal system M
  - L is consistent (is true$_2$) is confirmed by observations of the formal system L.

ExampleConsider the following meta-mathematical statements:

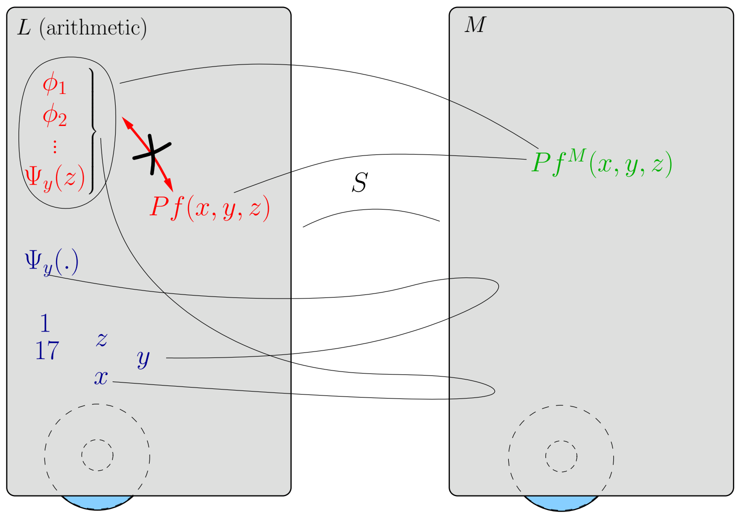
$Pf^M(x,y)$ — x is the Gödel number of a sequence of formulas constituting a proof of the formula of Gödel number y.

$Pf^M(x,y,z)$ — x is the Gödel number of a proof of the formula obtained from the formula of Gödel number y by substituting its only free variable with number z.

Representation:

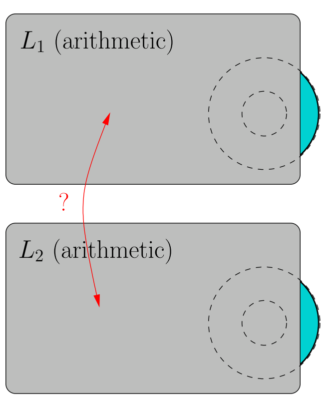
$$\begin{array}{lcl}\{\text{arithmetic}\} & \vdash & Pf(x,y,z) \text{ if } Pf^M(x,y,z) \text{ is true}_2 \\ \{\text{arithmetic}\} & \vdash & \neg Pf(x,y,z) \text{ if } Pf^M(x,y,z) \text{ is false}_2\end{array} \quad (104)$$

Problem:(104) is not "formally proved". It is known by *a posteriori* means!

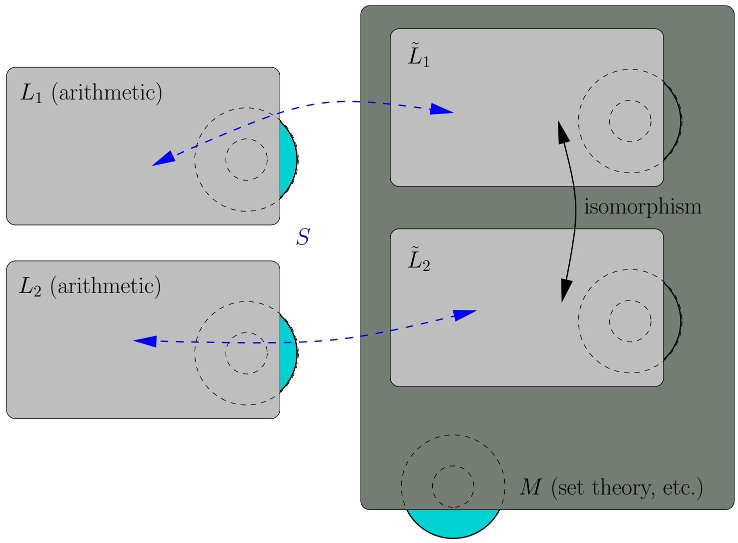


Hogyan lehet megragadni két formális rendszer közötti struktúláis hasonlóságot?

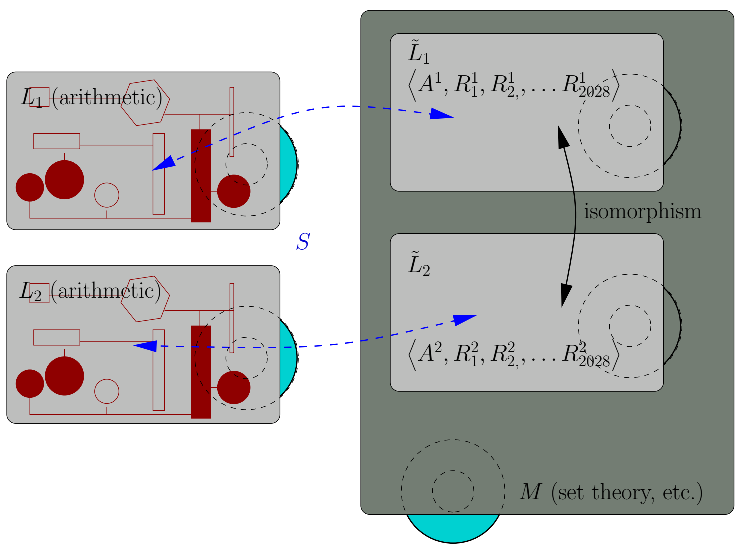
I.



II.


L_1 (arithmetic)
L_2 (arithmetic)
S
$\tilde{L}_1$
isomorphism
$\tilde{L}_2$
M (set theory, etc.)

III.


L_1 (arithmetic)
L_2 (arithmetic)
S
$\tilde{L}_1$
$\langle A^1, R^1_1, R^1_2, \ldots R^1_{2028} \rangle$
isomorphism
$\tilde{L}_2$
$\langle A^2, R^2_1, R^2_2, \ldots R^2_{2028} \rangle$
M (set theory, etc.)

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