A téridő fizikájától a tér és idő metafizikájáig

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What Exactly Does Relativity Principle Say?

The relativity principle

1. The first formulation of the relativity principle appeared in the following passage of Galilei's *Dialogue*:

> ... the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also. (Galilei 1953, p. 187)

In Einstein's formulation it is the following:

> If, relative to K, K' is a uniformly moving co-ordinate system devoid of rotation, then natural phenomena run their course with respect to K' according to exactly the same general laws as with respect to K. (Einstein 1920, p. 16)

Finally, in a typical text book formulation, relativity principle is the following assertion:

> All the laws of physics take the same form in any inertial frame.

Let us try to unpack what this principle actually asserts. First of all it must be clear that the *same law* of physics must take the same form in all inertial frames. What are the same laws of physics in different inertial frames? Of course, the laws of physics can be identified by means of the physical phenomena they describe. If so, then one can think that the same physical phenomenon must be described by the same solution of the same equations in all frames. This is however obviously not the case. For example, the motion of the plasma of the same solar flare is described differently by two observers in two different inertial frames. Thus, the opposite must be true: *different* physical phenomena are described by the same solutions of the same equations in different inertial frames. So, our first task will be to clarify what are those different physical phenomena the description of which must have the same form in all inertial frame.

2. The second problem is how the phrase "same form" should be understood. For, in terms of different variables, one and the same physical law in one and the same inertial frame of reference can be expressed in different forms. Therefore we have to add to the principle that the physical laws must be expressed in terms of the same physical quantities. This immediately raises the next question of how the physical quantities defined in different inertial frames are identified. Obviously, we identify those physical quantities that have identical empirical definitions. It is however far from obvious how these identical empirical definitions are actually understood.

The empirical/operational definitions require *etalon* measuring equipments. But how do the observers in different reference frames share these *etalon* measuring equipments? Do they all base their definitions on the same *etalon* measuring equipments? They must do something like that, otherwise any comparison between their observations would be meaningless. But, is principle of relativity really understood in this way? Is it true that the laws of physics in K and K', which ought to take the same form, are expressed in terms of physical quantities defined/measured with one and the same standard measuring equipments? Not exactly! “The cause of all these correspondences of effects is the fact that *the ship's motion is common to all the things* [italics mine] contained in it”—Galilei writes in the above quoted passage. Or, consider how Einstein applies the principle:

> Let there be given a stationary rigid rod; and let its length be l as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod. We now inquire as to the length of the moving rod, and imagine its length to be ascertained by the following two operations:
>
> (a) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way *as if all three were at rest* [italics mine].
>
> (b) By means of stationary clocks set up in the stationary system and synchronizing in accordance with [the light-signal synchronization], the observer ascertains at what points of the stationary system the two ends of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is at rest, is also a length which may be designated “the length of the rod.”
>
> In accordance with the principle of relativity the length to be discovered by the operation (a)—we will call it “the length of the rod in the moving system”—must be equal to the length l of the stationary rod.
>
> The length to be discovered by the operation (b) we will call “the length of the (moving) rod in the stationary system.”

> This we shall determine on the basis of our two principles, and we shall find that it differs from l. (Einstein 1905)

That is to say, if the standard measuring equipment defining a physical quantity X^K is, for example, at rest in K and, therefore, moving in K', then the observer in K' does not define the corresponding $X^{K'}$ as the physical quantity obtainable by means of the original standard equipment—being at rest in K and moving in K'—but rather as the one obtainable by means of the same standard equipment *in another state of motion*, namely when it is at rest in K' and moving in K.

3. Let us return to the first problem posed at the end of Point **1**. Now we can specify those different physical phenomena the description of which must have the same form in all inertial frame. For, what we told about the measuring equipments, also holds for the physical systems to be measured. That is to say, the principle says that the description of the behavior of a system when it is co-moving with inertial frame K takes the same form as the description of the same system when it is co-moving with inertial frame K'.

4. Putting all these details together, now we are ready to give a more accurate formulation of the relativity principle:

Relativity Principle *The laws of physics describing the behavior of a system co-moving as a whole with inertial frame K, expressed in terms of the results of measurements obtainable by means of measuring-rods, clocks, etc., co-moving with K takes the same form as the laws of physics describing the similar behavior of the same system when it is co-moving with inertial frame K′, expressed in terms of the measurements with the same equipments when they are co-moving with K′.*

Whether or not the relativity principle holds is—it must be clear—a contingent fact of nature. If the laws of physics known *in any one inertial frame of reference*, say K, account for all physical phenomena then these laws unambiguously predetermine whether the principle holds or not. The reason is that these laws also describe the behavior of moving (relative to K) physical systems including both the measuring equipments co-moving with another inertial frame K' and the system to be measured co-moving with K'.

Nevertheless, there are still vague points here. But before entering in the discussion of these further problems, let us recall how the relativity principle implies Galilean/Lorentz covariance.

Galilean covariance

5. Consider two inertial frames of reference K and K'. Assume that K' is moving at constant velocity v relative to K along the axis of x. Assume that laws of physics are known and empirically confirmed in inertial frame K, including the laws describing the behavior of physical objects in motion relative to K. Denote $x(A), y(A), z(A), t(A)$ the space and time tags of an event A, obtainable by means of measuring-rods and clocks at rest relative to K, and denote $x'(A), y'(A), z'(A), t'(A)$ the similar data of the same event, obtainable by means of measuring-rods and clocks co-moving with K'. Végrehajtva egy és ugyanazon esemény tér és idő koordinátáinak mérését az egyik illetve a másik vonatkoztatási rendszerben nyugvó mérőeszközökkel, a következő összefüggést találjuk:

$$t'(A) = t(A) \tag{1}$$
$$x'(A) = x(A) - v\,t(A) \tag{2}$$
$$y'(A) = y(A) \tag{3}$$
$$z'(A) = z(A) \tag{4}$$

Ez az ún. Galilei-transzformáció. A Galilei-transzformáció tehát a világ egy kontingens ténye. Következménye annak, hogy az órák és a méterrudak hogyan viselkednek akkor, ha mozgásba hozzuk őket. Nevezetesen, az órák ugyanolyan sebességgel járnak és a méterrudak megőrzik a hosszukat—legalábbis abban a közelítésben, amelyben a klasszikus fizika jó pontossággal teljesül.

Since physical quantities are defined by the same operational procedure in all inertial frames, the transformation rules of the space and time coordinates (usually) predetermine the transformations rules of the other physical variables. So, depending on the context, we will mean by Galilean transformation not only the transformation of the space and time tags, but also the corresponding transformation of the other variables in question.

6. In classical physics, the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames can be connected through the Galilean transformation. Consequently, the laws of physics must preserve their forms with respect of the Galilean transformation. Thus, it must be emphasized, the Galilean covariance is a consequence not only of the fact that the laws of physics satisfy the relativity principle but also of the other physical fact that the space and time tags in different inertial frames are connected through the Galilean/Lorentz transformation.

7. Let us now try to unpack the verbal formulations of the relativity principle in a more mathematical way. Let $\mathcal{E}$ be a set of differential equations describing the behavior of the system in question. Let us denote by ψ a typical set of (usually initial) conditions determining a unique solution of $\mathcal{E}$. Let us denote this solution by $[\psi]$. Denote $\mathcal{E}'$ and ψ' the equations and conditions obtained from $\mathcal{E}$ and ψ by substituting every x_i with x_i', and t with t', etc. Denote $G_v(\mathcal{E}), G_v(\psi)$ the set of equations and conditions expressed in the primed variables applying the Galilean transformations (including, of course, the Galilean transformations of all other variables different from the space and time coordinates). Finally, in order to give a strict mathematical formulation of relativity principle, we have to fix two further concepts, the meaning of which are vague: Let a solution $[\psi_0]$ be stipulated to describe the behavior of the system when it is, as a whole, at rest relative to K. Denote ψ_v the set of conditions and $[\psi_v]$ the corresponding solution of $\mathcal{E}$ that are stipulated to describe the similar behavior of the system as $[\psi_0]$ but, in addition, when the system was previously set, as a whole, into a collective translation at velocity v.

Now, what relativity principle in classical phyics states is *equivalent* to the following:

$$G_v(\mathcal{E}) = \mathcal{E}' \quad (5)$$
$$G_v(\psi_v) = \psi_0' \quad (6)$$

8. Although, in conjunction with the Galilean transformation rules, relativity principle implies Galilean covariance, the relativity principle, as we can see, *is not equivalent* to the Galilean covariance (5) in itself. It is equivalent to the satisfaction of (5) in conjunction with condition (6) (The importance of the initial/boundary conditions in symmetry principles is not a novel idea. See Houtappel, Van Dam, and Wigner 1965.)

9. Note, that $\mathcal{E}$, ψ_0, and ψ_v as well as the transformations G_v are given by contingent facts of nature. It is therefore a contingent fact of nature whether a certain law of physics is Galilean covariant, and, *independently*, whether it satisfies the principle of relativity. The relativity principle and its consequence the principle of Galilean and Lorentz covariance (amiről majd később lesz szó) are certainly normative principles in contemporary physics, providing a heuristic tool for constructing new theories. Like Earman (2004), we must emphasize however that these normative principles, as any other fundamental law of physics, are based on empirical facts; they are based on the observation that the behavior of any moving physical object satisfies the principle of relativity.

Does Relativity Principle Hold in Classical Physics?

10. In Point **4** we gave the exact formulation of the relativity principle. It must be noted that there still exists some vagueness in the principle. Namely, the vagueness of the concepts like "a system co-moving as a whole with an inertial frame" and "the similar behavior of the same system when it is co-moving with a given inertial frame". In other words,

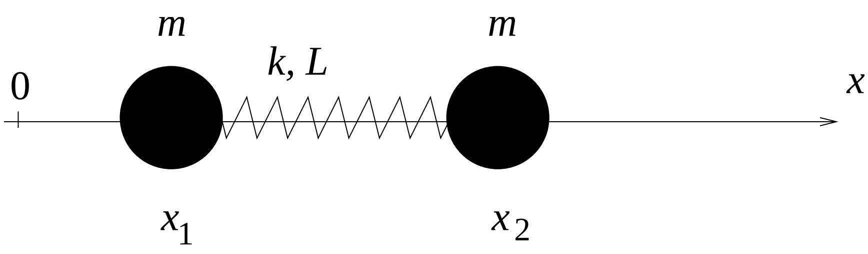


Figure 1: *Two point masses are connected with a spring of equilibrium length L and of spring constant k*

the vagueness of the definitions of conditions ψ_0 and ψ_v (Point **7**). In principle any $[\psi_0]$ can be considered as a "solution describing the system's behavior when it is, as a whole, at rest relative to K". Given any one fixed ψ_0, it is far from obvious, however, what is the corresponding ψ_v. When can we say that $[\psi_v]$ describes the similar behavior of the same system when it was previously set into a collectives motion at velocity v? There is an unambiguous answer to this question in the Galileo covariant classical physics. (But, as we will see, ψ_v is vaguely defined in relativity theory.)

We assume that the relevant equations describing the system are Galilean covariant, that is (5) are satisfied respectively. As it follows from the covariance of the corresponding equations, $G_v^{-1}(\psi_0')$ condition determines a new solution of $\mathcal{E}$. The question is whether this new solution $\left[G_v^{-1}(\psi_0')\right]$ is identical with $[\psi_v]$—the one determined by ψ_v. If so then the relativity principle is satisfied.

The relativity principle in classical mechanics

11. Let us start with an example illustrating how the relativity principle works in classical mechanics. Consider a system consisting of two point masses connected with a spring (Fig. 1). The equations of motion in K,

$$m\frac{d^2x_1(t)}{dt^2} = k(x_2(t) - x_1(t) - L) \qquad (7)$$

$$m\frac{d^2x_2(t)}{dt^2} = -k(x_2(t) - x_1(t) - L) \qquad (8)$$

are indeed covariant with respect to the Galilean transformation, that is, expressing (7)–(8) in terms of variables x', t' they have exactly the same form as before:

$$
\begin{aligned}
m\frac{d^2x_1'(t')}{dt'^2} &= k\left(x_2'(t') - x_1'(t') - L\right) && (9)\\
m\frac{d^2x_2'(t')}{dt'^2} &= -k\left(x_2'(t') - x_1'(t') - L\right) && (10)
\end{aligned}
$$

Consider the solution of the (7)–(8) belonging to an arbitrary initial condition ψ_0:

$$
\begin{aligned}
x_1(t=0) &= x_{10}\\
x_2(t=0) &= x_{20}\\
\left.\frac{dx_1}{dt}\right|_{t=0} &= v_{10}\\
\left.\frac{dx_2}{dt}\right|_{t=0} &= v_{20}
\end{aligned}
\qquad (11)
$$

The corresponding "primed" initial condition ψ_0' is

$$
\begin{aligned}
x_1'(t'=0) &= x_{10}\\
x_2'(t'=0) &= x_{20}\\
\left.\frac{dx_1'}{dt'}\right|_{t'=0} &= v_{10}\\
\left.\frac{dx_2'}{dt'}\right|_{t'=0} &= v_{20}
\end{aligned}
\qquad (12)
$$

Applying the inverse Galilean transformation we obtain a set of conditions $G_v^{-1}(\psi_0')$ determining a new solution of the original equations:

$$
\begin{aligned}
x_1(t=0) &= x_{10}\\
x_2(t=0) &= x_{20}\\
\left.\frac{dx_1}{dt}\right|_{t=0} &= v_{10}+v\\
\left.\frac{dx_2}{dt}\right|_{t=0} &= v_{20}+v
\end{aligned}
\qquad (13)
$$

One can recognize that (13) is nothing but ψ_v. It is the set of the original initial conditions in superposition with a uniform translation at velocity v. That is to say, the corresponding solution describes the behavior of the same system when it was (at $t = 0$) set into a collective translation at velocity v, in superposition with the original initial conditions.

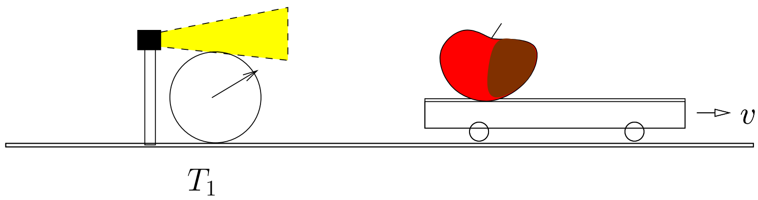
12. In classical mechanics, as we have seen from this example, the equations of motion not only satisfy the Galilean covariance, but also satisfy the condition (6). The principle of relativity holds for *all details of the dynamics* of the system. There is no exception to this rule. In other words, if the world were governed by classical mechanics, relativity principle would be a universally valid principle. With respect to later questions, it is worth noting that the Galilean principle of relativity therefore also holds for the equilibrium characteristics of the system, if the system has dissipations. Imagine for example that the spring has dissipations during its distortion. Then the system has a stable equilibrium state in which the equilibrium distance between the particles is L. When we initiate the system in collective motion corresponding to (13), the system relaxes to another equilibrium state in which the distance between the particles is the same L.

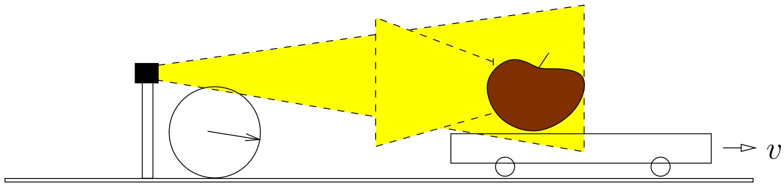
A relativisztikus fizikai valóság

Folyamatok lelassulása

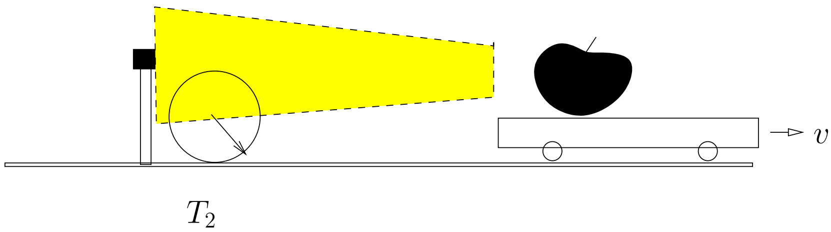
$c = 3 \cdot 10^8 \, \frac{m}{s}$
pl. $v = 10 \, \frac{m}{s}$
$T - T' \approx 10^{-14} s$ másodpercenként

Hogyan lehet a mozgó folyamat egy fázisának időpontját mérni az álló órával?

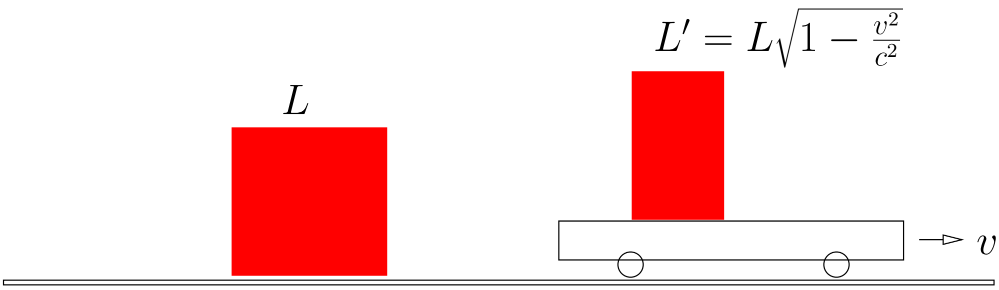




Megrohadás időpontja: $T' = \frac{T_1+T_2}{2}$



Testek kontrakciója



A mozgás irányába eső kiterjedése összehúzódik! (A rá merőleges méret nem változik.)

Hogyan mérjük meg a mozgó test hosszát?

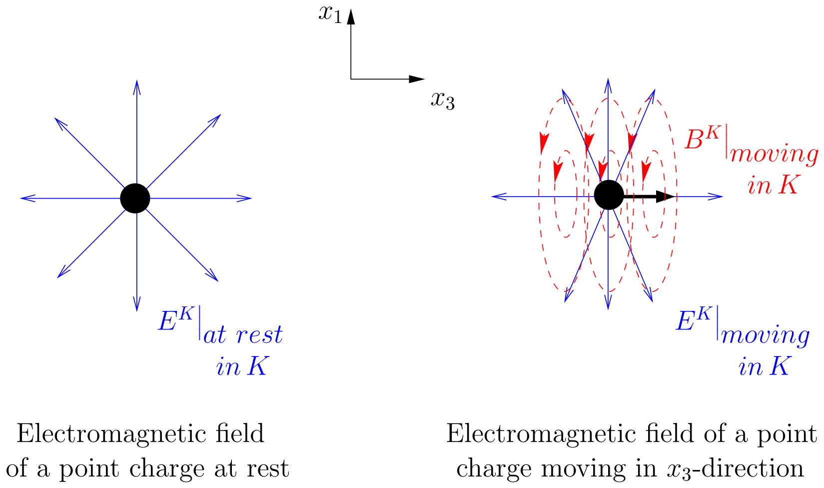
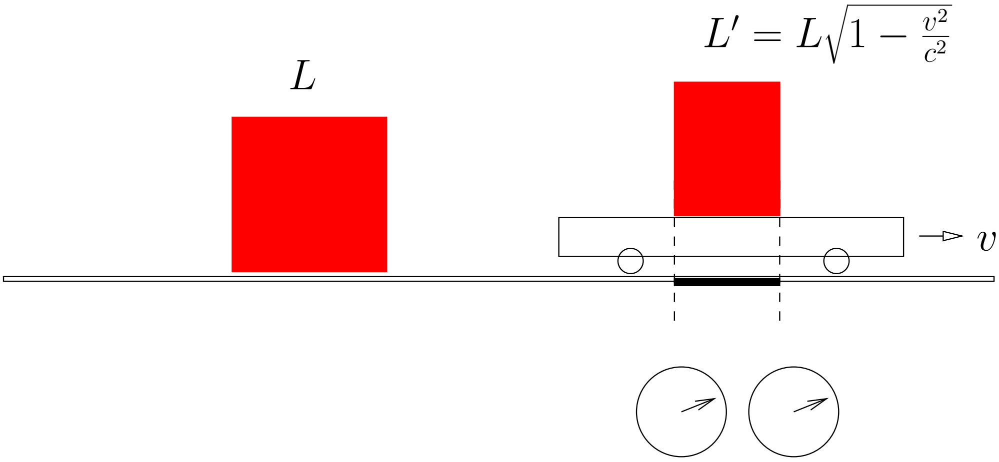


Figure 2: *The change of the electromagnetic field of a point charge*



Nem lepődünk meg azon, hogy egy fizikai tárgynak valamely tulajdonsága megváltozik, ha mozgásba hozzuk:

A többi hasonló testtel együtt, a mérésre használt óra és méterrúd is ugyanígy deformálódik. Ebből azonnal következik valami: Az ugyanazon eseménynek a K vonatkoztatási rendszerben, az ott nyugvó méterrúddal és órával megállapított (x, y, z, t), illetve a K' vonatkoztatási rendszerben, az ott nyugvó – azaz a K'-vel együtt mozgó – méterrúddal és órával megállapított (x', y', z', t') adatai között **nem a Galilei-transzformáció írja le a kapcsolatot!** (Legalábbis ha olyan pontossággal akarjuk leírni ezt a kapcsolatot, amikor az említett relativisztikus effektusok már nem hanyagolhatók el.)

Hanem akkor mi? Hogy erre a kérdésre válaszoljunk, az eddigieknél pontosabban meg kell adnunk, hogyan vannak a tér és idő koordináták empirikusan értelmezve.

A tér és idő koordináták empirikus definíciója I.

13. The empirical definition of a physical quantity requires an *etalon* measuring equipment and a precise description of the operation how the quantity to be defined is measured. For example, assume we choose, as the *etalon* measuring-rod, the meter stick that is lying in the International Bureau of Weights and Measures (BIPM) in Paris. Also assume—this is another convention—that “time” is defined as a physical quantity measured by the standard clock also sitting in the BIPM. When I use the word “convention” here, I mean the semantical freedom we have in the use of the uncommitted signs “distance” and “time”—a freedom what Grünbaum (1974, p. 27) calls “trivial semantical conventionalism”.

14. Now we are going to describe the usual empirical definitions of the space and time tags of an arbitrary event A, relative to the reference frame K in which the the *etalons* are at rest, and to another reference fame K' which is moving (at constant velocity v) relative to K. I call them space and time “tags” rather than space and time “coordinates”. By this terminology I would like to distinguish a particular kind of space and time coordinates which are provided with *direct* empirical meaning from space and time coordinates in general, the empirical meaning of which can be *deduced* from the empirical meaning of the space and time tags. Once we have space and time tags defined, we can introduce arbitrary other *coordinates* on the manifold of space-time tags. The physical/empirical meaning of a *point* of the manifold is however determined by the space-time tags of physical/empirical meaning. Only in this way we can confirm or falsify, empirically, a spatiotemporal physical statement.

In this section, I only reconstruct the definitions as they are understood in classical physics, without any explanation or criticism. I shall say more about the intuition behind the definitions in Points **??**–**??**. Whether these definitions—both classical and relativistic—are tenable will be discussed in Chapter **??**.

For the sake of simplicity consider only one space dimension and assume that the origin of both K and K' is at the BIPM at the initial moment of time.

(D1) Time tag in K according to classical physics

Küldünk az *etalon* órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. The time tag $t^K(A)$ is

$$t^K(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$

(D2) Space tag in K according to classical physics

The space tag $x^K(A)$ of event A is is the distance from the origin of K of the locus of A along the x-axis (straight line is usually defined by a light beam) measured by superposing the standard measuring-rod, being always at rest relative to K.

Hogyan definiáljuk a mozgó K' vonatkoztatási rendszerben a távolságot és az időt?

A következő gondolatmenet az, amit majd a „klasszikus szemléletmódnak” fogunk hívni:

Mivel tudjuk, hogy a mozgásba hozott óra lelassul és a mozgásba hozott méterrúd megrövidül, a távolság nem az lesz, amit a mozgó méterrúddal mérünk, és az idő nem az lesz, amit a mozgó órával mérünk, hanem a mérés eredményét **korrigálni kell**, figyelembe véve a mozgó óra lelassulását és a mozgó méterrúd megrövidülését.

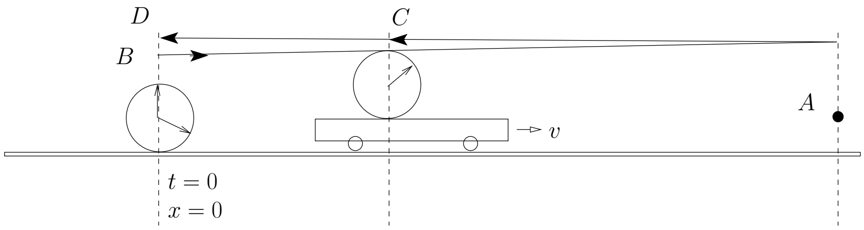
Hogy mennyivel kell korrigálni, azt akkor tudjuk megválaszolni, ha kiszámítjuk, mi lesz a mérés eredménye, ha a lelassult órával és a deformált méterrúddal mérjük meg egy esemény koordinátáit, szó szerint alkalmazva a (D1) és (D2) definíciókat a mozgó rendszerben, úgy *mintha minden az álló vonatkoztatási rendszerben lenne*. Mivel a deformációk miatt ezek nem a helyes tér és idő adatok lesznek, illik őket valahogyan másképpen neveznünk. Hívjuk a mozgó és ezért deformált műszerekkel mért adatokat $\widetilde{\text{tér}}$ és $\widetilde{\text{idő}}$ koordinátáknak.

Hogyan is történik a pontos mérés a mozgó órával és méterrúddal?

(D7) $\widetilde{\textbf{Time}}$ tag in K'

Vesszük az *etalon* óra egy tökéletes példányát, és óvatosan – hogy ne menjen tönkre! – átvisszük a K' rendszerbe. (Tehát óvatosan gyorsítjuk/lassítjuk.) Majd pontosan azt az operációt hajtjuk végre, mint (D1)-ben: Küldünk az órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. The time tag $\tilde{t}^{K'}(A)$ is

$$\tilde{t}^{K'}(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$



3. ábra. *A két óra egybe esik a $t = 0$ pillanatban, az $x = 0$ helyen. A fényjel kibocsájtásának eseménye B. A jel az A esemény bekövetkezésekor verődik vissza. Először a mozgó órához érkezik (C esemény), majd a álló etalon órához (D esemény)*

(D8) $\widetilde{\textbf{Space}}$ tag in K'

The space tag $\widetilde{x}^{K'}(A)$ of event A is the distance from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest.

És most kiszámítjuk, hogy mi lesz a mozgó műszerekkel történő mérés eredménye.

15. Taking the immediate consequences of the facts that the measuring-rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ and the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K', one can directly calculate the $\widetilde{\text{space}}$ tag $\widetilde{x}^{K'}(A)$ and the $\widetilde{\text{time}}$ tag $\widetilde{t}^{K'}(A)$, following the descriptions of operations in (D7) and (D8).

Imagine that a light signal is emitted (event B) when the moving observer meets the standard clock (Fig. 3). Let $t^K(B) = 0$. Event A is marked with the reflection of the signal at time $t^K(A)$. The reflected signal first arrives at the moving observer (event C) and then at the *etalon* clock (event D). By definition, $t^K(A) = \frac{t^K(D)}{2}$. We know that

$$vt^K(C) = x^K(A) - c\left(t^K(C) - t^K(A)\right)$$

and $x^K(A) = ct^K(A)$. Therefore,

$$t^K(C) = \frac{2ct^K(A)}{c+v}$$

Taking into account that the moving observer's "clock"-reading at C is

$$reading(C) = t^K(C)\sqrt{1-\frac{v^2}{c^2}}$$

Therefore, the "time" and "space" coordinates (s)he obtains is

$$\begin{aligned}
\tilde{t}^{K'}(A) &= \frac{reading(C)}{2} = \frac{ct^K(A)}{c+v}\sqrt{1-\frac{v^2}{c^2}} \\
&= \frac{ct^K(A)(c-v)}{(c+v)(c-v)}\sqrt{1-\frac{v^2}{c^2}} \\
&= \frac{t^K(A) - \frac{v}{c^2}x^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (14)
\end{aligned}$$

Now, taking into account that the length of the co-moving meter stick is only $\sqrt{1-\frac{v^2}{c^2}}$, the distance of event A from the origin of K is the following:

$$x^K(A) = t^K(A)v + \tilde{x}^{K'}(A)\sqrt{1-\frac{v^2}{c^2}} \qquad (15)$$

and thus

$$\tilde{x}^{K'}(A) = \frac{x^K(A) - v\,t^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (16)$$

A (14) és (16) formulákat *Lorentz-transzformációnak* nevezzük.

Mindezek alapján tehát meg tudjuk mondani, hogy a mozgó, ezért lelassult és elállítódott óráról leolvasott óraállás és a mozgó, ezért deformálódott méterrúddal történő mérés eredményéből hogyan rekonstruálhatjuk az igazi tér és idő adatokat:

According to the compensatory view, if we measure a distance and the result is X, then the "real distance" is $X\sqrt{1-\frac{v^2}{c^2}}$. Similarly, taking into account the phase shift suffered by a moving clock, we know from (**??**) that if the reading of the clock is T then the "real time" is

$$\frac{T + X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Ennek megfelelően a klasszikus fizika „kompenzációs" szemléletmódja szerint a mozgó vonatkoztatási rendszerben egy esemény tér és idő koordinátája a következőképpen lesz definiálva:

(D6) Space tag of an event in K' according to classical physics

Let X be the "distance" from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest. The space tag $x^{K'}(A)$ of event A is

$$x^{K'}(A) := X\sqrt{1-\frac{v^2}{c^2}} \tag{17}$$

(D5) Time tag of an event in K' according to classical physics

Vesszük az *etalon* óra egy tökéletes példányát és óvatosan átvisszük a K' rendszerbe. Majd pontosan azt az operációt hajtjuk végre, mint (D1)-ben: Küldünk az órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. Legyen

$$T(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$

The time tag $t^{K'}(A)$ is

$$t^{K'}(A) := \frac{T + X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{18}$$

Vegyük észre, hogy mivel a leolvasott X és T értékek nem mások mint $\widetilde{x}^{K'}(A)$ és $\widetilde{t}^{K'}(A)$, az előbb levezetett (14) és (16) felhasználásával [Házi feladat: tessék behelyettesíteni!] azt kapjuk, hogy

$$\begin{aligned} x^{K'}(A) &= x^{K}(A) - vt^{K}(A) \\ t^{K'}(A) &= t^{K}(A) \end{aligned}$$

ami éppen a jól ismert Galilei-transzformáció! Vagyis ez azt jelenti, hogy ha a klasszikus szemléletmódot követjük, és figyelembe vesszük a mozgó órák és méterrudak deformációját, és ennek a deformációnak megfelelően kompenzáljuk a leolvasott értékeket, akkor olyan tér és idő koordinátáink vannak a különböző, egymáshoz képest mozgó vonatkoztatási rendszerekben, — a relativisztikus fizikában is! — melyek teljesítik a Galilei-transzformációt.

Einstein javaslata

Einstein: Ne törődjünk azzal, hogy az órák és méterrudak deformálódnak amikor mozgásba hozzuk őket, és egyszerűen hívjuk a K' mozgó rendszerben „térnek” és „időnek” azokat az adatokat, melyeket a K'-vel együtt mozgó, ezért deformálódott méterrúddal és órával mérünk.

Hívjuk!

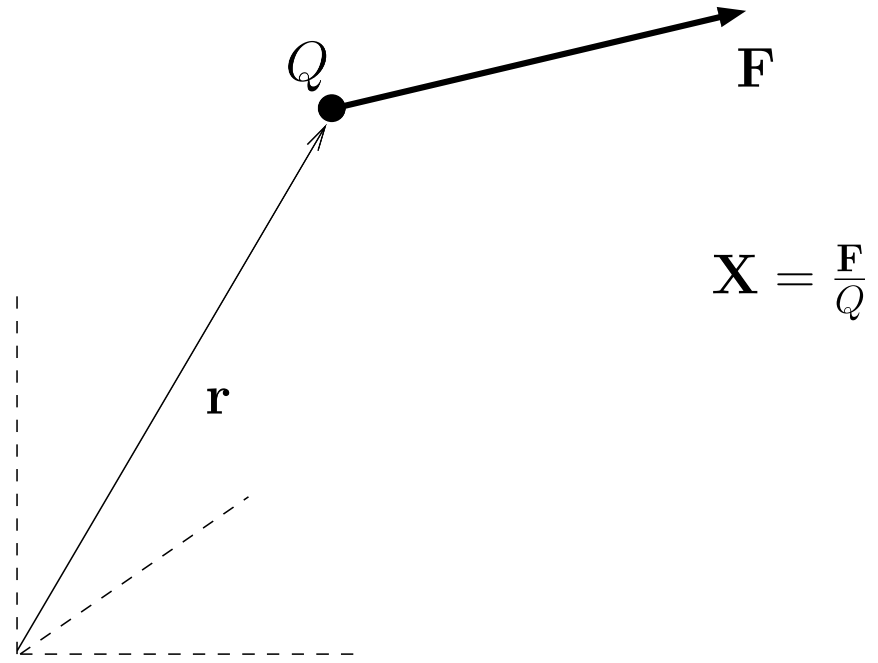


Figure 4: **X** *is defined as the force felt by the unit test charge*

(Elektrodinamikai) tanmese

(az „i" épület lakói másik tanmesét kaptak)

Hívjuk. De tisztában kell lennünk, hogy azzal, hogy valamit *átkeresztelünk*, még sem a fizikánkat, sem a metafizikánkat nem változtatjuk meg!

16. Consider the following definitions of electrodynamical quantities:

$\mathbf{X}(\mathbf{r})$ Locate a test charge Q at point $\mathbf{r}$ and measure the force $\mathbf{F}$ felt by the charge. $\mathbf{X}(\mathbf{r}) = \frac{\mathbf{F}}{Q}$ (Fig. 4).

$\mathbf{Y}(\mathbf{r})$ Locate two contacting metal plates of area A at point $\mathbf{r}$. Separate them and measure the influence charge Q on one of the plates. $Y(\mathbf{r}) = \frac{Q}{A}$. The direction of $\mathbf{Y}(\mathbf{r})$ is determined by the normal vector of the plates, when the charge separation is maximal (Fig. 5).

It is a well known empirical fact that these quantities are not independent of each other. For the sake of simplicity, assume the simplest ma-

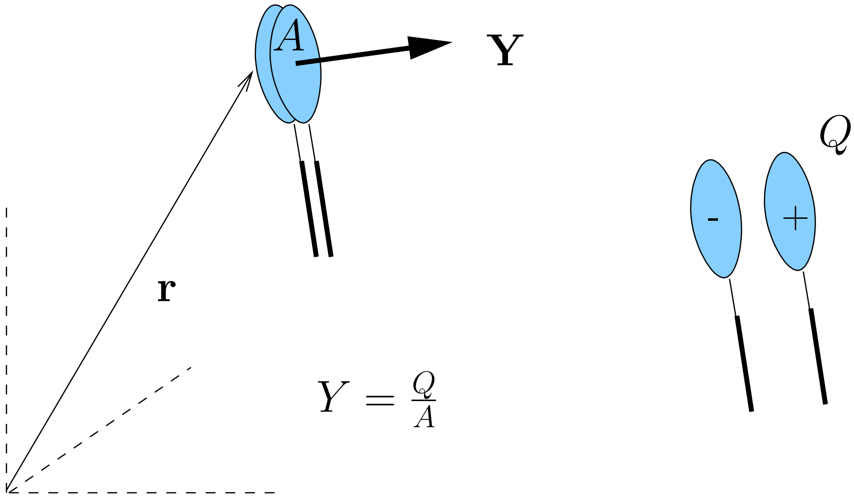


Figure 5: **Y** *is defined by means of the influence charge divided by the surface*

terial equation

$$\mathbf{Y} = \varepsilon \mathbf{X} \tag{19}$$

where ε, called dielectric constant, is a scalar field characterizing the medium.

Traditionally, in phenomenological electrodynamics, physical quantity **X** is called 'electric field strength' and denoted by **E**, and **Y** is called 'electric displacement' and denoted by **D**. Due to the material equation (19) one can eliminate one of the field variables.

17. Imagine a text book (I shall refer to it as the "old" one), which only uses **E**. The equations of electrostatics are written as follows:

$$\text{div}\, {''}\mathbf{E} = \rho \tag{20}$$

$$\text{rot}\, \mathbf{E} = 0 \tag{21}$$

For example, the book contains the following exercise and solution:

> **Exercise** Consider the static electric field around a point charge q located at the border of two materials of dielectric constant ε_1 and ε_2. Is the electric field strength spherically symmetric, or not?
>
> **Solution** (see Fig. 6)
>
> $$\mathbf{E}_1 = \frac{1}{2\pi\left(\varepsilon_1 + \varepsilon_2\right)} \frac{q}{r^3} \mathbf{r} \tag{22}$$

$$\mathbf{E}_2 = \frac{1}{2\pi\left(\varepsilon_1+\varepsilon_2\right)}\frac{q}{r^3}\mathbf{r} \tag{23}$$

Consequently,

(S1) The electric field strength is spherically symmetric.

18. Now, imagine a new electrodynamics text book which is non-traditional in the following sense: it uses only field variable **Y** (traditionally called 'electric displacement' and denoted by **D**), but it systematically calls **Y** 'electric field strength' and denotes it by **E**. Accordingly, the equations of electrostatics are written as follows:

$$\operatorname{div}\mathbf{E} = \rho \tag{24}$$
$$\operatorname{rot}\frac{\mathbf{E}}{\text{„}} = 0 \tag{25}$$

This new book also contains the above-mentioned exercise, but with the following solution:

Solution (see Fig. 7)

$$\mathbf{E}_1 = \frac{\varepsilon_1}{2\pi\left(\varepsilon_1+\varepsilon_2\right)}\frac{q}{r^3}\mathbf{r} \tag{26}$$
$$\mathbf{E}_2 = \frac{\varepsilon_2}{2\pi\left(\varepsilon_1+\varepsilon_2\right)}\frac{q}{r^3}\mathbf{r} \tag{27}$$

Consequently,

(S2) The electric field strength is not spherically symmetric.

Now, does sentence (S2) of the new book contradict to sentence (S1) of the old book? Is it true that the theory presented in the new book is a new theory of electromagnetism? *Does it tell us anything new about electromagnetic field?* Of course, not. Seemingly the two sentences contradict to each other, *on the surface of the words*. However, in order to clarify

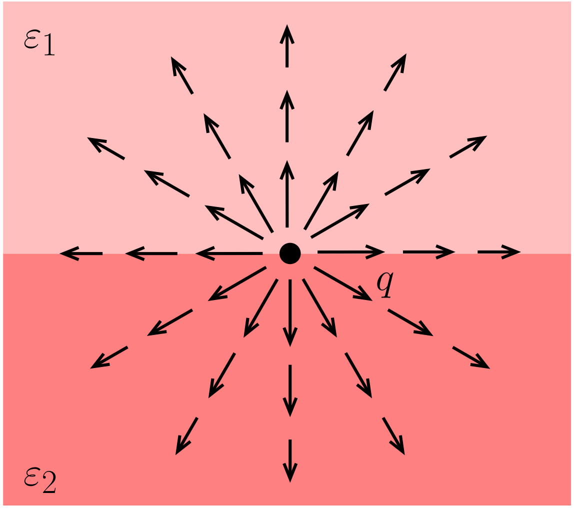


Figure 6: *The 'electric field strength' of the static electric field around a point charge q located at the border of two materials of dielectric constant ε_1 and ε_2*

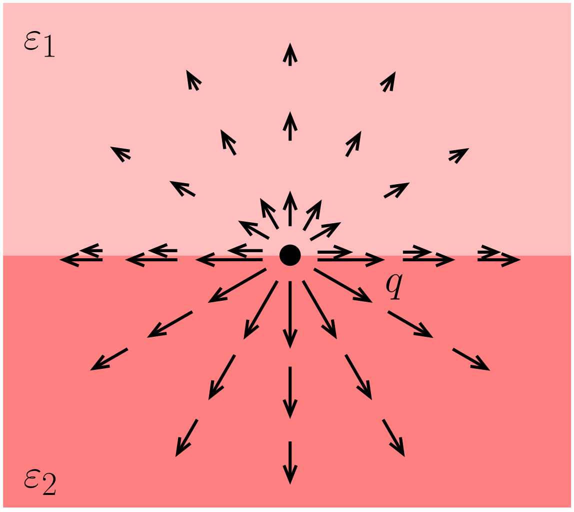


Figure 7: *The 'electric field strength' of the static electric field around a point charge q* located *at the border of two materials of dielectric constant ε_1 and ε_2*

the meaning of sentence (S1) and (S2), one has to go back to the first pages of the corresponding book and to clarify the *empirical definition* of the physical quantity called ‘electric field strength’. And it will be clear that the term ‘electric field strength’ stands for two *different* physical quantities in the two books. Moreover, both text books provide complete descriptions of electromagnetic phenomena. Therefore, although the theory in the old book does not use the field variable **Y**, it is capable to account for the physical phenomena by which physical quantity **Y** is empirically defined. It is capable to determine the influence charge on the separated plates (by calculating εEA). In other words, it is capable to determine the value of **Y**, that is, the value of what the new book calls ‘electric field strength’. And vice versa, on the basis of the theory presented in the new book one can calculate the force felt by a unit test charge (by calculating $\frac{\mathbf{E}}{\varepsilon}$), that is, one can predict the value of what the old book calls ‘electric field strength’. And both, the theory in the old book and the theory in the new book have the same predictions for both, **X** and **Y**. That is to say, although they use *different terminology*, the two text books contain the *same electrodynamics*, they provide the same description of physical reality.

19. Mindebből az derül ki, hogy súlyos konfúzióhoz vezet, ha empirikusan másképpen definiált fizikai mennyiségeket elkezdünk ugyanúgy hívni.

Megoldás: megkülönböztetésül $\widehat{\text{space}}$ and $\widehat{\text{time}}$ címkékről és $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ címkékről fogunk beszélni.

Mindent egybevetve, a következő definícióink vannak:

(D1) $\widehat{\textbf{Time}}$ tag in K

Küldünk az *etalon* órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. The $\widehat{\text{time}}$ tag is

$$\hat{t}^K(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$

(D2) $\widehat{\textbf{Space}}$ tag in K

The $\widehat{\text{space}}$ tag $\hat{x}^K(A)$ of event A is is the distance from the origin of K of the locus of A along the x-axis (straight line is usually defined by a light beam) measured by superposing the standard measuring-rod, being always at rest relative to K.

(D3) $\widetilde{\textbf{Time}}$ tag in K

Küldünk az *etalon* órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. The time tag is

$$\tilde{t}^K(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$

(D4) $\widetilde{\textbf{Space}}$ tag in K

The space tag $\tilde{x}^K(A)$ of event A is is the distance from the origin of K of the locus of A along the x-axis (straight line is usually defined by a light beam) measured by superposing the standard measuring-rod, being always at rest relative to K.

(D5) $\widehat{\text{Time}}$ tag in K'

Vesszük az *etalon* óra egy tökéletes példányát és óvatosan átvisszük a K' rendszerbe. Majd pontosan azt az operációt hajtjuk végre, mint (D1)-ben: Küldünk az órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. Legyen

$$T(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$

The time tag $\hat{t}^{K'}(A)$ is

$$\hat{t}^{K'}(A) := \frac{T + X\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (28)$$

(D6) $\widehat{\text{Space}}$ tag in K'

Let X be the „distance" from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest. The space tag $\hat{x}^{K'}(A)$ of event A is

$$\hat{x}^{K'}(A) := X\sqrt{1 - \frac{v^2}{c^2}} \qquad (29)$$

(D7) $\widetilde{\text{Time}}$ tag in K'

Vesszük az *etalon* óra egy tökéletes példányát, és óvatosan – hogy ne menjen tönkre! – átvisszük a K' rendszerbe. (Tehát óvatosan gyorsítjuk/lassítjuk.) Majd pontosan azt az operációt hajtjuk végre, mint (D1)-ben: Küldünk az órától egy fényjelet t_1 óraállásnál az A esemény helyére, úgy, hogy a fényjel éppen az A bekövetkezése pillanatában reflektálódjon. Legyen a fényjel visszaérkezésének pillanatában az óraállás t_2. The time tag $\tilde{t}^{K'}(A)$ is

$$\tilde{t}^{K'}(A) := t_1 + \frac{1}{2}(t_2 - t_1)$$

(D8) $\widetilde{\text{Space}}$ tag in K'

The space tag $\widetilde{x}^{K'}(A)$ of event A is the distance from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest.

(D9) Velocities in the different cases

Velocity is a quantity derived from the above defined space and time tags:

$$\begin{aligned}
\hat{v}^K &= \frac{\Delta \hat{x}^K}{\Delta \hat{t}^K} \\
\widetilde{v}^K &= \frac{\Delta \widetilde{x}^K}{\Delta \widetilde{t}^K} \\
\hat{v}^{K'} &= \frac{\Delta \hat{x}^{K'}}{\Delta \hat{t}^{K'}} \\
\widetilde{v}^{K'} &= \frac{\Delta \widetilde{x}^{K'}}{\Delta \widetilde{t}^{K'}}
\end{aligned}$$

20. With empirical definitions (D1)–(D8), in every inertial frame we define four different quantities for each event; such that:

$$\hat{x}^K(A) \equiv \widetilde{x}^K(A) \tag{30}$$

$$\hat{t}^K(A) \equiv \widetilde{t}^K(A) \tag{31}$$

for the reference frame of the *etalons*, and

$$\hat{x}^{K'}(A) \not\equiv \widetilde{x}^{K'}(A) \tag{32}$$

$$\hat{t}^{K'}(A) \not\equiv \widetilde{t}^{K'}(A) \tag{33}$$

for any other inertial frame of reference. ($\equiv$ denotes the identical empirical definition.)

In spite of the different empirical definitions, it could be a *contingent* fact of nature that $\hat{x}^{K'}(A) = \widetilde{x}^{K'}(A)$ and/or $\hat{t}^{K'}(A) = \widetilde{t}^{K'}(A)$ for every event A. Let me illustrate this with an example. Inertial mass m_i and gravitational mass m_g are two quantities having different experimental definitions. But, it is a contingent fact of nature (experimentally proved by Eötvös around 1900) that, for any object, the two masses are equal, $m_i = m_g$. However, that is not the case here. $\widetilde{x}^{K}(A), \widetilde{t}^{K}(A)$ are related with $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ through the Lorentz transformation, while $\hat{x}^{K}(A), \hat{t}^{K}(A)$ are related with $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ through the corresponding Galilean transformation, therefore, taking into account identities (30)–(31), $\hat{x}^{K'}(A) \neq \widetilde{x}^{K'}(A)$ and $\hat{t}^{K'}(A) \neq \widetilde{t}^{K'}(A)$, if $v \neq 0$.

Thus, our first partial conclusion is that *different physical quantities are called "space" tag, and similarly, different physical quantities are called "time" tag in special relativity* (Einstein) *and in classical physics* (for example, Lorentz). (This was first recognized by Bridgeman (1927, p. 12), although he did not investigate the further consequences of this fact.) In order to avoid further confusion, from now on $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags will mean the physical quantities defined in (D1), (D2), (D5), and (D6)—according to the usage of the terms in classical physics—, and "space" and "time" in the sense of the relativistic definitions (D3), (D4), (D7) and (D8) will be called $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$.

Special relativity theory makes *different* assertions about somethings which are *different* from $\widehat{\text{space}}$ and $\widehat{\text{time}}$. Symbolically, classical physics claims $G_1\left(\hat{M}\right)$ about $\hat{M}$ and relativity theory claims $G_2\left(\widetilde{M}\right)$ about some other features of reality, $\widetilde{M}$.

The question is: What do special relativity and classical physics say when they are making assertions about the same things?

21. Mielőtt erre a kérdésre válaszolunk, legfőbb ideje, hogy pontosítsuk, mit is értünk „Lorentz-elmélet” és „speciális relativitáselmélet” alatt! Ezt rövidesen pontosabban meg fogjuk mondani. Egyelőre mondjuk azt, hogy

A Lorentz-elmélet – α verzió

(L1) the dimension parallel to $\mathbf{v}$ of a solid measuring rod suffers $\widehat{\text{contraction}}$ by factor $\sqrt{1-\frac{v^2}{c^2}}$ and

(L2) a clock-like physical process $\widehat{\text{slows down}}$ by factor $\sqrt{1-\frac{v^2}{c^2}}$

when they are gently accelerated from the reference frame of the *etalons*, K, to the frame K' moving at velocity $\mathbf{v}$ relative to K.

Speciális relativitáselmélet – α verzió

(R0) The $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\tilde{x}^{K'}(A), \tilde{y}^{K'}(A), \tilde{z}^{K'}(A), \tilde{t}^{K'}(A)$ in K'—defined in (D7)–(D8)—and the similar $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\tilde{x}^{K''}(A), \tilde{y}^{K''}(A), \tilde{z}^{K''}(A), \tilde{t}^{K''}(A)$ in some other K'' are connected through the corresponding Lorentz transformation.

Állítás:

$$\text{(L1)} + \text{(L2)} \Longleftrightarrow \text{(R0)}$$

Más szóval a két elmélet ekvivalens, pontosabban azonos.

Mit is jelent ez? Let A be an arbitrary event and let $K^\star$ be an arbitrary inertial frame of reference. Denote $\left[\hat{x}^{K^\star}(A)\right]_{relativity}$ and $\left[\hat{t}^{K^\star}(A)\right]_{relativity}$ the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags of A predicted by relativity theory and denote $\left[\hat{x}^{K^\star}(A)\right]_{LT}$ and $\left[\hat{t}^{K^\star}(A)\right]_{LT}$ the similar tags predicted by classical physics. Similarly, denote $\left[\tilde{x}^{K^\star}(A)\right]_{relativity}$ and $\left[\tilde{t}^{K^\star}(A)\right]_{relativity}$ the $\widetilde{\text{space}}$

and time tags of A predicted by relativity theory and denote $\left[\tilde{x}^{K^\star}(A)\right]_{LT}$ and $\left[\tilde{t}^{K^\star}(A)\right]_{LT}$ the similar tags predicted by classical physics.

Now, relativity theory would tell us something new if it accounted for physical quantities $\tilde{x}$, $\tilde{t}$, $\hat{x}$ and $\hat{t}$ differently. If there were any event A and any inertial frame of reference $K^\star$ such that $\left[\hat{x}^{K^\star}(A)\right]_{relativity}$ were different from $\left[\hat{x}^{K^\star}(A)\right]_{LT}$ or $\left[\hat{t}^{K^\star}(A)\right]_{relativity}$ from $\left[\hat{t}^{K^\star}(A)\right]_{LT}$ or $\left[\tilde{x}^{K^\star}(A)\right]_{relativity}$ from $\left[\tilde{x}^{K^\star}(A)\right]_{LT}$ or $\left[\tilde{t}^{K^\star}(A)\right]_{relativity}$ from $\left[\tilde{t}^{K^\star}(A)\right]_{LT}$. If, for example, there were any two events simultaneous in relativity theory which were not simultaneous according to classical physics, or vice versa—to touch on a sore point. This is however not the case.

Az egyik irányt, vagyis, hogy

$$\left[\widetilde{t}^{K'}(A)\right]_{LT} = \frac{\hat{t}^K(A) - \frac{v\,\hat{x}^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \left[\widetilde{t}^{K'}(A)\right]_{relativity}$$

illetve

$$\left[\widetilde{x}^{K'}(A)\right]_{LT} = \frac{\hat{x}^K(A) - v\,\hat{t}^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} = \left[\widetilde{x}^{K'}(A)\right]_{relativity}$$

már bebizonyítottuk – lásd a (14) és (16) formulákat. Ebből azonnal következik a fordított irányú állítás is, hiszen a (D5) és (D6) definíciókban szereplő T és X nem más mint $\widetilde{t}^{K'}(A)$ és $\widetilde{x}^{K'}(A)$, melyek értéke mindkét elmélet szerint azonos, tehát

$$\left[\hat{x}^{K'}(A)\right]_{relativity} = \hat{x}^K(A) - \hat{v}^K(K')\hat{t}^K(A) = \left[\hat{x}^{K'}(A)\right]_{LT} \qquad (34)$$

and

$$\left[\hat{t}^{K'}(A)\right]_{relativity} = \hat{t}^K(A) = \left[\hat{t}^{K'}(A)\right]_{LT} \qquad (35)$$

A két elmélet tehát tökéletesen azonos fizikai elmélete a térnek és az időnek!

Fontos észrevétel

1. .Ha egy álló rúd hosszára az álló méterrúd l-szer fér rá, akkor a mozgó és ezért $l\sqrt{1-\frac{v^2}{c^2}}$ hosszúságúra zsugorodott rúdra a mozgó, ezért $\sqrt{1-\frac{v^2}{c^2}}$ hosszúságú méterrúd ugyanúgy l-szer fér rá.

Hasonlóan, az álló rendszerben lezajló folyamat időtartama az álló órával mérve ugyanaz, mint a mozgó rendszerben lezajló, $\sqrt{1-\frac{v^2}{c^2}}$ faktorral lelassult folyamat időtartama a mozgó és ezért $\sqrt{1-\frac{v^2}{c^2}}$ faktorral lassabban járó órával mérve.

Mindezen megfigyelések alapján:

Hipotézis

(L3) For any inertial frame of reference K', the laws of physics in K are such that the laws of physics empirically ascertained by an observer in K', describing the behavior of physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, have the same forms as the similar empirically ascertained laws of physics in in K, describing the similar physical objects co-moving with K, expressed in variables $\widetilde{x}_1^{K}, \widetilde{x}_2^{K}, \widetilde{x}_3^{K}, \widetilde{t}^{K}$, if the observer in K' performs the same measurement operations as the observer in K with the same measuring equipments transferred from K to K', ignoring the fact that the equipments undergo deformations during the transmission.

Ha ez igaz, akkor azonnal több is igaz:

(R1) For any two inertial frames of reference K' and K'', the laws of physics in K' are such that the laws of physics empirically ascertained by an observer in K'', describing the behavior of physical objects co-moving with K'', expressed in variables $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in in K', describing the similar physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transferred from K' to K'', ignoring the fact that the equipments undergo deformations during the transmission.

A hipotézis tehát azt feltételezi, hogy a relativitási elv teljesül a relativisztikus fizikában. Ebben kivétel nélkül hitt Lorentz, Poincaré és Einstein. (Hogy igazuk volt-e, erre még visszatérünk.)

2. .Mutassuk meg, hogy a fény sebessége tetszőleges vonatkoztatási rendszerben ugyanaz, vagyis c.

Megoldás: Tegyük fel, hogy egy fényjelet indítunk (A esemény) a $\hat{t}^K(A) = 0$ pillanatban az $\hat{x}^K(A) = 0$ pontból indul. Nyilvánvaló, hogy $\tilde{t}^{K'}(A) = 0$ és $\tilde{x}^{K'}(A) = 0$. A fényjel valahova beérkezik (B esemény). A fényjel sebessége K-ban c, akkor $\hat{x}^K(B) = c\hat{t}^K(B)$. Legyen ez a B esemény. Számoljuk ki a $\tilde{t}^{K'}(B)$ és $\tilde{x}^{K'}(B)$ koordinátákat! A Lorentz-transzformáció formuláit felhasználva:

$$\tilde{t}^{K'}(B) = \frac{\hat{t}^K(B) - \frac{vc\hat{t}(B)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\tilde{x}^{K'}(B) = \frac{c\hat{t}^K(B) - v\,\hat{t}^K(B)}{\sqrt{1-\frac{v^2}{c^2}}}$$

A fényjel sebessége K'-ben:

$$\tilde{v}^{K'}(light) = \frac{\tilde{x}^{K'}(B)}{\tilde{t}^{K'}(B)} = \frac{\frac{c\hat{t}^K(B)-v\,\hat{t}^K(B)}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{\hat{t}^K(B)-\frac{vc\hat{t}(B)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} = c$$

K' természetesen tetszőleges lehet, tehát azt kaptuk, hogy

(R2) The velocity of a light signal is the same in all inertial reference frames.

Most pontosan megmondjuk, mi a Lorentz elmélet és a speciális relativitáselmélet.

A Lorentz-elmélet – release

(L1) the dimension parallel to $\mathbf{v}$ of a solid measuring rod suffers contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ and

(L2) a clock-like physical process slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$

when they are gently accelerated from the reference frame of the *etalons*, K, to the frame K' moving at velocity $\mathbf{v}$ relative to K.

(L3) For any inertial frame of reference K', the laws of physics in K are such that the laws of physics empirically ascertained by an observer in K', describing the behavior of physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, have the same forms as the similar empirically ascertained laws of physics in in K, describing the similar physical objects co-moving with K, expressed in variables $\widetilde{x}_1^{K}, \widetilde{x}_2^{K}, \widetilde{x}_3^{K}, \widetilde{t}^{K}$, if the observer in K' performs the same measurement operations as the observer in K with the same measuring equipments transferred from K to K', ignoring the fact that the equipments undergo deformations during the transmission.

Speciális relativitáselmélet – release

(R1) For any two inertial frames of reference K' and K'', the laws of physics in K' are such that the laws of physics empirically ascertained by an observer in K'', describing the behavior of physical objects co-moving with K'', expressed in variables $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in in K', describing the similar physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transferred from K' to K'', ignoring the fact that the equipments undergo deformations during the transmission.

(R2) The $\widetilde{\text{velocity}}$ of a light signal is the same in all inertial reference frames.

Einstein az 1905-ös cikkében megmutatta (egyszerű, tisztán matematikai levezetés, amely bármely tankönyvben megtalálható), hogy (R1) + (R2) $\Longleftrightarrow$ (R0). Ezt felhasználva, továbbá figyelembe véve, hogy (L3) $\Longleftrightarrow$ (R1), azt kapjuk, hogy

$$\text{(L1)} + \text{(L2)} + \text{(L3)} \Longleftrightarrow \text{(R1)} + \text{(R2)}$$

> **Vagyis a Lorentz-elmélet és a speciális relativitáselmélet azonos! *Azonos* abban az értelemben, hogy a fizikai világ ugyanazon tulajdonságairól ugyanazt állítják.**

To sum up symbolically, the Lorentz theory and and special relativity theory have identical assertions about both $\hat{M}$ and $\widetilde{M}$: they unanimously claim that $G_1\left(\hat{M}\right) \& G_2\left(\widetilde{M}\right)$.

Tények/Tévedések

Tény: $\widehat{\text{velocity}}$ of light is not the same in all inertial frames of reference, while $\widetilde{\text{velocity}}$ of light is the same in all inertial frames of reference

> ***Tévedés:*** *Korábban azt hittük, hogy egy és ugyanazon fényjel sebessége a különböző, egymáshoz képest mozgó vonatkoztatási rendszerekben különböző, mígnem – Einstein zseniális felismerése révén – rájöttünk, hogy ez nem így van: a fény sebessége minden vonatkoztatási rendszerben ugyanaz.*

Tény: $\widehat{\text{Time}}$ and $\widehat{\text{distance}}$ are invariant, the reference frame independent concepts, $\widetilde{\text{time}}$ and $\widetilde{\text{distance}}$ are not.

> ***Tévedés:*** *Korábban azt hittük, hogy az idő és a távolság vonatkoztatási rendszertől független, invariáns/abszolút mennyiségek, mígnem – Einstein zseniális felismerése révén – rájöttünk, hogy ez nem így van: az idő és a távolság relatív, vonatkoztatási rendszertől függő fogalmak.*

Tény: $\hat{t}$-simultaneity is an invariant, frame-independent concept, while $\tilde{t}$-simultaneity is not.

> ***Tévedés:*** *Korábban azt hittük, hogy az egyidejűség vonatkoztatási rendszertől független, invariáns/abszolút fogalom, mígnem – Einstein zseniális felismerése révén – rájöttünk, hogy ez nem így van: az egyidejűség relatív, vonatkoztatási rendszertől függő fogalom.*

Tény: for arbitrary K' and K'', $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ can be expressed by $\hat{x}^{K''}(A), \hat{t}^{K''}(A)$ through a suitable Galilean transformation; while $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ can be expressed by $\widetilde{x}^{K''}(A), \widetilde{t}^{K''}(A)$ through a suitable Lorentz transformation.

Tévedés: *Korábban azt hittük, hogy egy esemény tér és idő koordinátái a Galilei-transzformáció szerint transzformálódnak, mígnem – Einstein zseniális felismerése révén – rájöttünk, hogy ez nem így van: a tér és idő koordináták helyes transzformációja a Lorentz-transzformáció.*

Tény: $\widetilde{\text{Velocity}}$—which is called "velocity" by relativity theory—is not an additive quantity,

$$\widetilde{v}^{K'}(K''') = \frac{\widetilde{v}^{K'}(K'') + \widetilde{v}^{K''}(K''')}{1 + \frac{\widetilde{v}^{K'}(K'')\widetilde{v}^{K''}(K''')}{c^2}}$$

while $\widehat{\text{velocity}}$—that is, what we traditionally call "velocity"—is an additive quantity,

$$\hat{v}^{K'}(K''') = \hat{v}^{K'}(K'') + \hat{v}^{K''}(K''')$$

where K', K'', K''' are arbitrary three frames.

Tévedés: *Korábban azt hittük, hogy a sebességek egyszerűen összeadódnak (ha a vonat megy $v_1 = 100\frac{km}{h}$ sebességgel, és én a vonaton futok $v_2 = 20\frac{km}{h}$ sebességgel, akkor az én sebességem a földhöz képest $v = v_1 + v_2 = 120\frac{km}{h}$), mígnem – Einstein zseniális felismerése révén – rájöttünk, hogy ez nem így van: a sebességek az alábbi bonyolult formula szerint adódnak össze:*

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} < 120\frac{km}{h}$$

STB, STB...

22. Finally, note that in an arbitrary inertial frame K' for every event A the tags $\hat{x}_1^{K'}(A)$, $\hat{x}_2^{K'}(A)$, $\hat{x}_3^{K'}(A)$, $\hat{t}^{K'}(A)$ can be expressed in terms of $\widetilde{x}_1^{K'}(A)$, $\widetilde{x}_2^{K'}(A)$, $\widetilde{x}_3^{K'}(A)$, $\widetilde{t}^{K'}(A)$ and vice versa. Consequently, we can express the laws of physics—as is done in special relativity—equally well in terms of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ instead of the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$. On the other hand, we should emphasize that the one-to-one correspondence between $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ and $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ also entails that the laws of physics (so called "relativistic" laws included) can be equally well expressed in terms of the traditional $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ instead of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$. In brief, physics could manage equally well with the classical Galileo-invariant conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$.

Are relativistic deformations real physical changes?

23. Many believe that it is an essential difference between the two theories that relativistic deformations like the Lorentz–FitzGerald contraction and the time dilatation are real physical changes in the Lorentz theory, but there are no similar physical effects in special relativity. Let us examine two typical argumentations.

According to the first argument the Lorentz contraction/dilatation of a rod cannot be an objective physical deformation in relativity theory, because it is a frame-dependent fact whether the rod is shrinking or expanding. Consider a rod accelerated from the sate of rest in reference frame K' to the state of rest in reference frame K''. According to relativity theory, "the rod shrinks in frame K' and, at the same time, expands in frame K''. But this is a contradiction, the argument says, if the deformation was a real physical change. In contrast, the argument says, the Lorentz theory claims that the length of a rod is a frame-independent concept. Consequently, in the Lorentz theory, "the contraction/dilatation of a rod" can indeed be an objective physical change.

However, we have already clarified, that the terms "distance" and "time" have different meanings in relativity theory and the Lorentz theory. We must differentiate $\widehat{\text{dilatation}}$ from $\widetilde{\text{dilatation}}$, $\widehat{\text{contraction}}$ from $\widetilde{\text{contraction}}$, and so on. For example, consider the reference frame of the *etalons*, K, and another frame K' moving relative to K, and a rod accelerated from the sate of rest in reference frame K—$state_1$—to the state of rest in reference frame K'—$state_2$. Denote $\hat{l}^K(state_1)$ the $\widehat{\text{length}}$ of the rod in $state_1$ relative to K, $\tilde{l}^K(state_1)$ the $\widetilde{\text{length}}$ of the rod in $state_1$ relative to K, etc. Now, the following statements are true about the rod :

$$\hat{l}^K(state_1) > \hat{l}^K(state_2) \qquad \widehat{\text{contraction}} \text{ in } K \tag{36}$$
$$\hat{l}^{K'}(state_1) > \hat{l}^{K'}(state_2) \qquad \widehat{\text{contraction}} \text{ in } K' \tag{37}$$
$$\tilde{l}^K(state_1) > \tilde{l}^K(state_2) \qquad \widetilde{\text{contraction}} \text{ in } K \tag{38}$$
$$\tilde{l}^{K'}(state_1) < \tilde{l}^{K'}(state_2) \qquad \widetilde{\text{dilatation}} \text{ in } K' \tag{39}$$

And there is no difference between relativity theory and the Lorentz theory: *all* of the four statements (36)–(39) are true *in both theories*. If, in the Lorentz theory, facts (36)–(37) provide enough reason to say that there is a real physical change, then the same facts provide enough reason to say the same thing in relativity theory. And vice versa, if (38)–(39) contradicted to the existence of real physical change of the rod in relativity theory, then the same holds for the Lorentz theory.

24. It should be mentioned, however, that there is no contradiction between (38)–(39) and the existence of real physical change of the rod. Relativity theory and the Lorentz theory unanimously claim that $\widetilde{\text{length}}$ is a relative physical quantity. It is entirely possible that one and the same objective physical change is traced in the increase of the value of a relative quantity relative to one reference frame, while it is traced in the decrease of the same quantity relative to another reference frame (see the example in Fig. 8). (What is more, both, the value relative to one

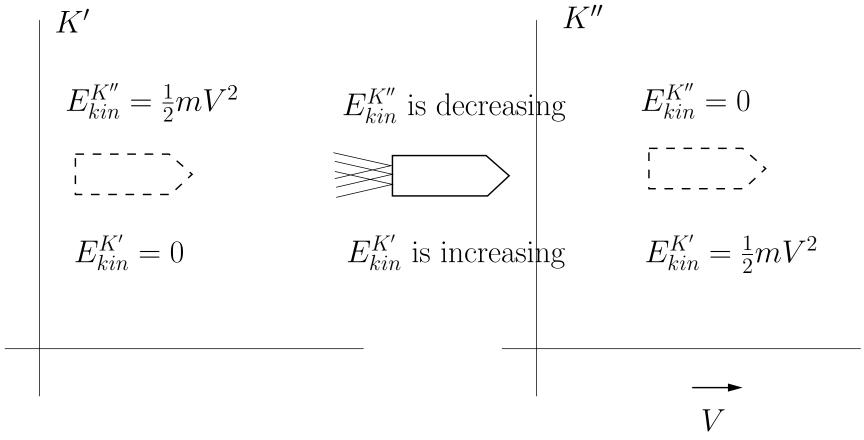


Figure 8: *One and the same objective physical process is traced in the increase of kinetic energy of the spaceship relative to frame K', while it is traced in the decrease of kinetic energy relative to frame K''*

frame and the value relative to the other frame, reflect objective features of the objective physical process in question.)

25. According to the other widespread argument, the relativistic deformations cannot be real physical effects since they can be observed by an observer also if the object is at rest but the observer is in motion at constant velocity. And these relativistic deformations cannot be explained as real physical deformations of the object being continuously at rest.

However, there is a triple misunderstanding behind such an argument:

1. Of course, no real distortion is suffered by an object which is continuously at rest relative to a reference frame K', and, consequently, which is continuously in motion at a constant velocity relative to another frame K''. Contrary to the argument, none of the inertial observers can observe such a distortion. For example,

$$\widetilde{l}^{K'}\begin{pmatrix}\text{distortion free}\\ \text{rod at }\widetilde{t}_1\end{pmatrix} = \widetilde{l}^{K'}\begin{pmatrix}\text{distortion free}\\ \text{rod at }\widetilde{t}_2\end{pmatrix}$$
$$\widetilde{l}^{K''}\begin{pmatrix}\text{distortion free}\\ \text{rod at }\widetilde{t}_1\end{pmatrix} = \widetilde{l}^{K''}\begin{pmatrix}\text{distortion free}\\ \text{rod at }\widetilde{t}_2\end{pmatrix}$$

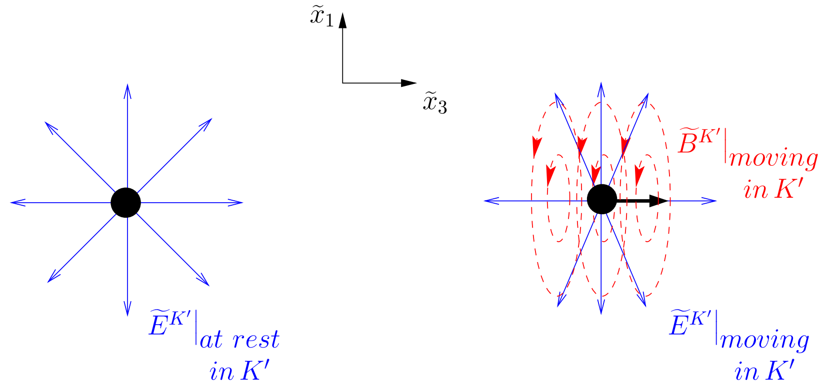


Figure 9: *The change of the electromagnetic field of a point charge*

2. It is surely true for any $\tilde{t}$ that

$$\tilde{l}^{K'}\begin{pmatrix}\text{distortion}\\ \text{free rod at } \tilde{t}\end{pmatrix} \neq \tilde{l}^{K''}\begin{pmatrix}\text{distortion}\\ \text{free rod at } \tilde{t}\end{pmatrix} \tag{40}$$

   This fact, however, does not express a contraction of the rod—neither a real nor an apparent contraction.

3. On the other hand, inequality (40) is a *consequence* of the real physical distortions suffered by the measuring equipments—with which the space and time tags are empirically defined—when they are transfered from the BIPM to the other reference frame in question. (For further details of what a moving observer can observe by means of his or her distorted measuring equipments, see Bell 1983, pp. 75–76.)

26. Finally, let me give an example for a well known physical phenomenon which is of exactly the same kind as the relativistic deformations, but nobody would question that it is a real physical change. Consider the electromagnetic field of a point charge q. One can easily solve the Maxwell equations when the particle is at rest in a given K'. The result is the familiar spherically symmetric Coulomb field. (Fig. 9)

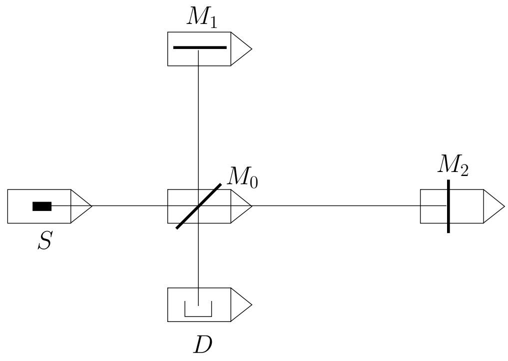
How does this field change if we set the charge in motion with constant $\widetilde{\text{velocity}}$ $\widetilde{v}$ along the $\widetilde{x}_3$ axis? Maxwell's equations can also answer this question. The electromagnetic field of the charge *change*s: t*here appears* a magnetic field (turning the magnetic needle, for example) and the electric field *flattened* in the direction of motion (Fig. 9). No physicist would say that this is not a real physical change in the electromagnetic field of the charge, only because we can express the new electromagnetic field of the moving charge in terms of the variables relative to the co-moving reference frame K'', and it has the same form as the old electromagnetic field, when the charge was at rest in K', expressed in the terms of the variables relative to K'.

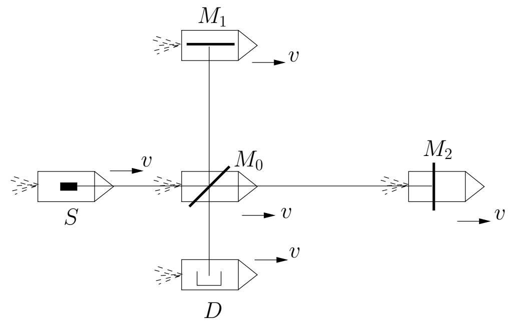
27. Thus, relativistic deformations are real physical deformations also in special relativity theory. One has to emphasize this fact because it is an important part of the physical content of relativity theory. It must be clear, however, that this conclusion is independent of our main concern. What is important is the following: The Lorentz theory and special relativity have identical assertions about $\widehat{\text{length}}$ and $\widetilde{\text{length}}$, $\widehat{\text{duration}}$ and $\widetilde{\text{duration}}$, $\widehat{\text{shrinking}}$ and $\widetilde{\text{shrinking}}$, etc. Consequently, whether or not these facts provide enough reason to say that relativistic deformations are real physical changes, the conclusion is common to both theories.

On the null result of the Michelson–Morley experiment

A Michelson–Morley kísérletről magyarázat és demó:
http://phil.elte.hu/leszabo/terido/Michelson-Morley-demo.html

Megjegyzés: Képzeljünk el egy Michelson-Morley kísérletet úgy, hogy a fényforrás (S), a féligáteresztő tükör (M_0), a két tükör (M_1 és M_2), valamint a fénydetektor (D) egy-egy rakétán van elhelyezve. A rakéták tömege azonos és azonos erejű a hajtóművük. Első helyzetben a rakéták állnak egy adott inerciarendszerben. A detektorban egy adott interferencia-képet figyelünk meg. Most egyszerre beindítjuk a hajtóműveket, majd egyszerre kikapcsoljuk. Így a rakétákat ugyanarra a v sebességre gyorsítjuk fel. Tapasztalunk-e eltolódást az interferencia-képben?





A válasz az, hogy igen! Az eredeti kísérletben ugyanis fontos az, hogy a tükrök egy merev kőlapra vannak erősítve, amelyik Lorentz-kontrakciót szenved a mozgás irányában. Vagyis a null-effektushoz az kell, hogy az M_0 és M_2 tükrök távolsága a $\sqrt{1-\frac{v^2}{c^2}}$ faktorral csökkenjen; a rakéták közötti távolság azonban változatlan marad.

28. Mit konfirmál tehát a Michelson–Morley-kísérlet? Consider the following passage from Einstein:

> A ray of light requires a perfectly definite time T to pass from one mirror to the other and back again, if the whole system be at rest with respect to the aether. It is found by calculation, however, that a slightly different time T^1 is required for this process, if the body, together with the mirrors, be moving relatively to the aether. And yet another point: it is shown by calculation that for a given velocity v with reference to the aether, this time T^1 is different when the body is moving

> perpendicularly to the planes of the mirrors from that resulting when the motion is parallel to these planes. Although the estimated difference between these two times is exceedingly small, Michelson and Morley performed an experiment involving interference in which this difference should have been clearly detectable. But the experiment gave a negative result — a fact very perplexing to physicists. (Einstein 1920, p. 49)

The "calculation" that Einstein refers to is based on the Galilean "kinematics", that is, on the invariance of "time" and "simultaneity", on the invariance of "distance", on the classical addition rule of "velocities", etc. That is to say, "distance", "time", and "velocity" in the above passage mean the classical $\widehat{\text{distance}}$, $\widehat{\text{time}}$, and $\widehat{\text{velocity}}$ defined in (D1), (D2), (D5), and (D6). The negative result was "very perplexing to physicists" because their expectations were based on traditional the concepts of $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and they could not imagine other that if the $\widehat{\text{speed}}$ of light is c relative to one inertial frame then the $\widehat{\text{speed}}$ of the same light signal cannot be the same c relative to another reference frame.

29. On the other hand, Einstein continues this passage in the following way:

> Lorentz and FitzGerald rescued the theory from this difficulty by assuming that the motion of the body relative to the aether produces a contraction of the body in the direction of motion, the amount of contraction being just sufficient to compensate for the difference in time mentioned above. Comparison with the discussion in Section 11 shows that also from the standpoint of the theory of relativity this solution of the difficulty was the right one. But on the basis of the theory of relativity the method of interpretation is incomparably more satisfactory. According to this theory there is no such thing as a "specially favoured" (unique) co-ordinate system

> to occasion the introduction of the aether-idea, and hence there can be no aether-drift, nor any experiment with which to demonstrate it. Here the contraction of moving bodies follows from the two fundamental principles of the theory, without the introduction of particular hypotheses; and as the prime factor involved in this contraction we find, not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point. Thus for a co-ordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a co-ordinate system which is at rest relatively to the sun. (Einstein 1920, p. 49)

What "rescued" means here is that Lorentz and FitzGerald proved, within the framework of the classical space-time theory and Galilean kinematics, that if the assumed deformations of moving bodies exist then the expected result of the Michelson–Morley experiment is the null effect. On the other hand, we have already clarified, what Einstein also confirms in the above quoted passage, that these deformations also derive from the two basic postulates of special relativity. Putting all these facts together (see Schema 1), we must say that the null result of the Michelson–Morley experiment simultaneously confirms *both*, the classical rules of Galilean kinematics for $\hat{x}$ and $\hat{t}$, and the violation of these rules (Lorentzian kinematics) for the space and time tags $\widetilde{x}, \widetilde{t}$. It confirms the classical addition rule of velocities, on the one hand, and, on the other hand, it also confirms that velocity of light is the same in all frames of reference.

This actually holds for all other experimental confirmations of special relativity. That is why the only difference Einstein can mention in the quoted passage is that special relativity does not refers to the aether. (As a historical fact, this difference is true. Although, as we will see in Points **36–37** and **38–40**, the concept of aether can be entirely removed

$$\left.\begin{array}{c}[\text{deformations}]\\ \& \\ \underbrace{\left[\begin{array}{c}\text{Galilean } \widehat{\text{kinematics}}\\ \text{for } \hat{x},\hat{t} \text{ (the } \widehat{\text{speed}}\\ \text{of light is NOT}\\ \text{the same in all}\\ \text{inertial frame)}\end{array}\right]}\end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{the result of the}\\ \text{Michelson-Morley}\\ \text{experiment must}\\ \text{be the null effect}\end{array}\right]$$

$$\Updownarrow$$

$$\left.\begin{array}{c}\overbrace{\left[\begin{array}{c}\text{Lorentz } \widetilde{\text{kinematics}}\\ \text{for } \tilde{x},\tilde{t} \text{ (the } \widetilde{\text{speed}}\\ \text{of light IS the same}\\ \text{in all inertial frame)}\end{array}\right]}\\ \Downarrow \\ [\text{deformations}]\end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{the result of the}\\ \text{Michelson-Morley}\\ \text{experiment must}\\ \text{be the null effect}\end{array}\right]$$

Schema 1: *The null result of the Michelson–Morley experiment simultaneously confirms both, the classical rules of Galilean* $\widehat{\textit{kinematics}}$ *for* $\hat{x}$ *and* $\hat{t}$*, and the violation of these rules (Lorentzian* $\widetilde{\textit{kinematics}}$*) for the* $\widetilde{\textit{space}}$ *and* $\widetilde{\textit{time}}$ *tags* $\widetilde{x},\widetilde{t}$.

from the recent logical reconstruction of the Lorentz theory.)

30. Finally, it is no surprise that the deformations can be "derived" from the Lorentz kinematics. The *physical* information about the deformations suffered by objects accelerated from one state of motion to another, say from the state of rest relative to K' to the state of rest relative to K'', is inbuilt into the relationship between the tags $\tilde{x}^{K'}(A), \tilde{t}^{K'}(A)$ and $\tilde{x}^{K''}(A), \tilde{t}^{K''}(A)$. For these relations are determined by the *physical behavior* of measuring rods and clocks during the acceleration and relaxation process, as Einstein warns us (see the quotation in Point **47**).

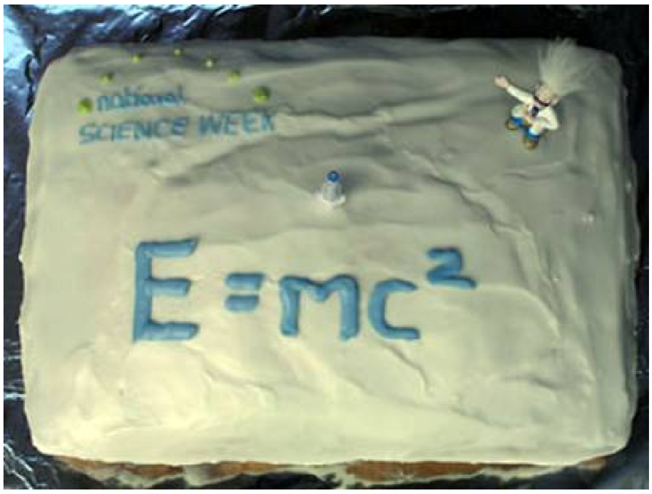
A relativisztikus fizika harmadik fizikai jelensége, amelyről még nem volt szó: a tömegnövekedés

Empirikusan megfigyelhető tény: tekintsünk egy részecskét, amelyik áll a K vonatkoztatási rendszerben. Ilyenkor megállapított tömege $m(0)$. Ha a részecskét mozgásba hozzuk, és v sebességgel mozog K-hoz képest, akkor a tömege megváltozik:

$$m(v) = \frac{m(0)}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{41}$$

Töltött részecskék mozgásának megfigyelése alapján: Thomson 1881-ben már feltételezte, hogy a tömeg változik a sebességgel. ... Ezt a formulát Lorentz írta fel először, különféle elvi megfontolásokból. Empirikusan csak 1940-ben sikerült megfelelő pontossággal igazolni.

... Celebrating Einstein's anniversary A Café Scientifique took place on 15th March at Bar So in the Royal Exeter Hotel in Bournemouth: Dr Jim Smith led with a brief talk on Einstein's "E=mc-squared; the equation that shook the world", followed by a buffet, celebration of Einstein's life and work and pitting their wits in the BA Science Week pub quiz.



http://www.ceh.ac.uk/news/sciweek05.html

Szorozzuk be a (41) egyenletet c^2-tel, majd használjuk fel a matematikából ismert $\frac{1}{\sqrt{1-x}} \approx 1 + \frac{1}{2}x$ közelítő formulát:

$$m(v)c^2 = \frac{m(0)c^2}{\sqrt{1-\frac{v^2}{c^2}}} \approx m(0)c^2 + \underbrace{\frac{1}{2}m(0)v^2}_{\text{Ez a mozgási energia!}}$$

Vagyis az egész $m(v)c^2$ olyan, mint a mozgási energiának és valami „nyugalmi energiának" az összege. Hipotézis: az összes energia $E = m(v)c^2$. (És valóban, ez a mennyiség megmarad a különböző folyamatokban.)

Minkowski-geometria

1907-ben Minkowski rájött, hogy az $\tilde{x}^{K*}, \tilde{y}^{K*}, \tilde{z}^{K*}, \tilde{t}^{K*}$ mennyiségekre vonatkozó szabályokat – melyeket már ismerünk! – elegáns formában le lehet írni egy 4-dimenziós nem euklideszi geometriával.

Szótár

Euklideszi geometria

n-dimenziós euklideszi térnek nevezzük az a következő párost: $(\mathbb{R}^n, s)$, ahol $\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R}$, azaz a valós számokból álló $(x_1, x_2, \ldots x_n)$ nesek halmaza; s pedig a tér „pontjai” közötti távolság-függvény, amely az euklideszi térben úgy van definiálva, hogy

$$s^2(X,Y) := (x_1 - y_1)^2 + (x_2 - y_2)^2 + \ldots + (x_n - y_n)^2 \tag{42}$$

Az 1-dimenziós euklideszi tér tehát maga a számegyenes a szokásos távolsággal, a 2-dimenziós euklideszi tér a sík a pontok közötti szokásos távolsággal, a 3-dimenziós euklideszi tér (ez volna a hagyományos értelemben vett euklideszi geometria standard modellje) a három dimenziós tér a szokásos távolsággal, stb.

Nem-euklideszi geometria

Valamilyen $(\mathbb{R}^n, s)$, ahol az s távolság-függvény az (42)-tól eltérő valami.

Például:

Minkowski-geometria

$(\mathbb{R}^4, s)$, ahol $s^2(X,Y) := (x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 - (x_4 - y_4)^2$. Tehát a távolság-négyzet lehet pozitív, nulla, vagy negatív. Meg ilyesmik. Bármi, ... hiszen – ne felejtsük el – csak matematikáról van szó!

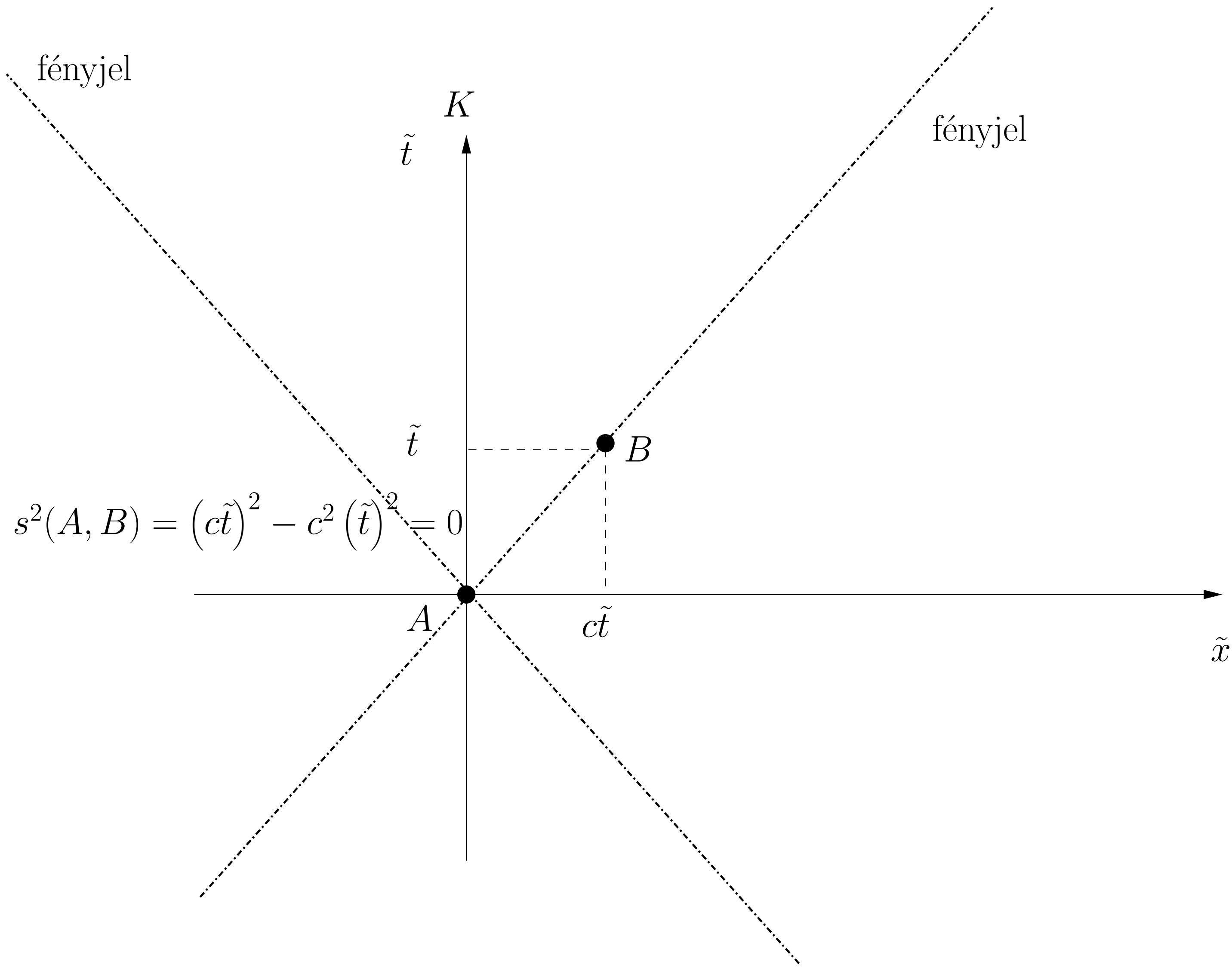
Téridő

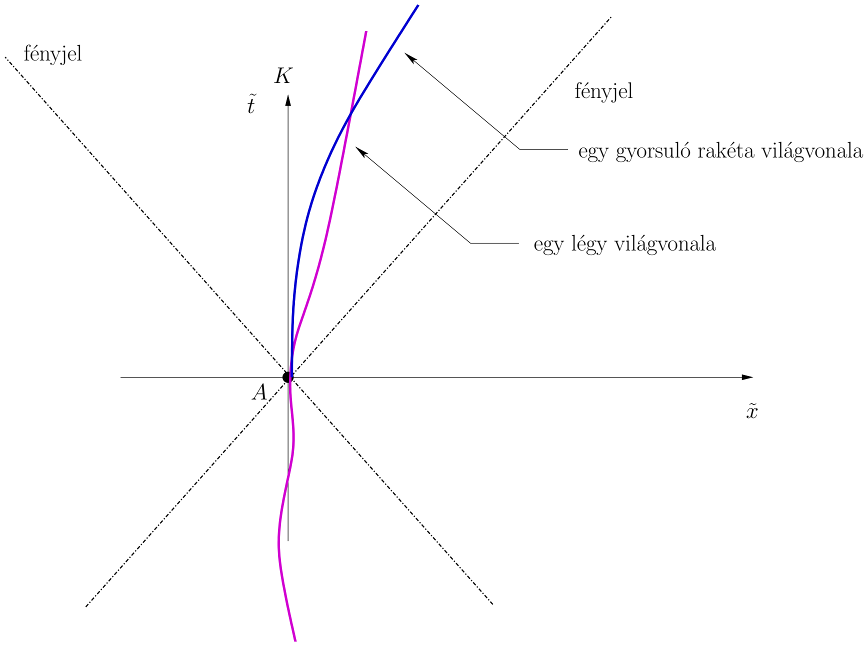
Megtehetjük, hogy az eseményeket a „tér” és „idő” címkéik alapján egy négy dimenziós térben reprezentáljuk. Ezt hívjuk „téridőnek”.

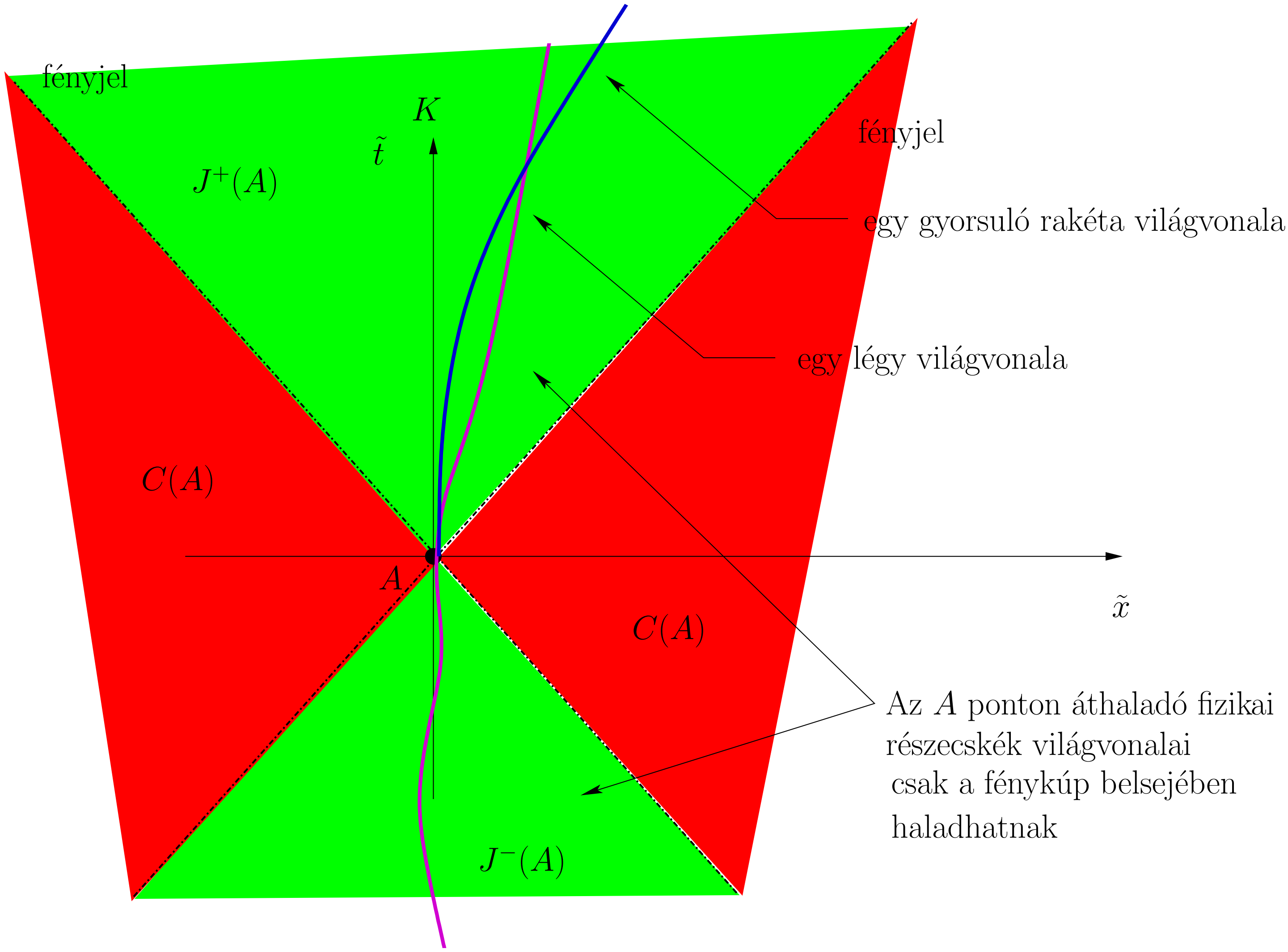
Minkowski-$\widetilde{\text{téridő}}$

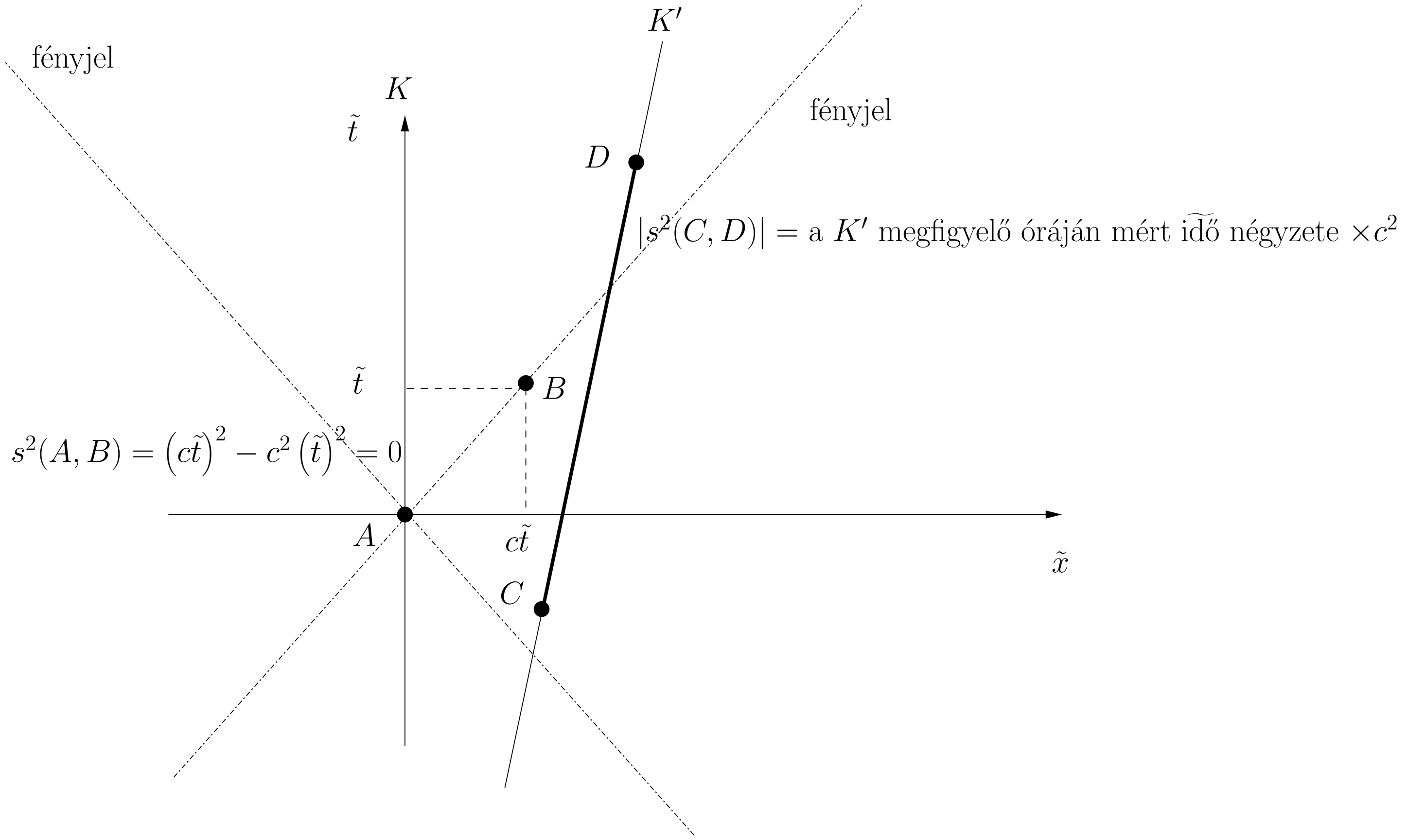
Egy 4-dimenziós tér, amelyben a „távolság” egy nem-euklideszi metrikával van definiálva. Az egyszerűség kedvéért csak egy tér- és az idő-dimenziót feltételezve, tehát 2 dimenzióban:

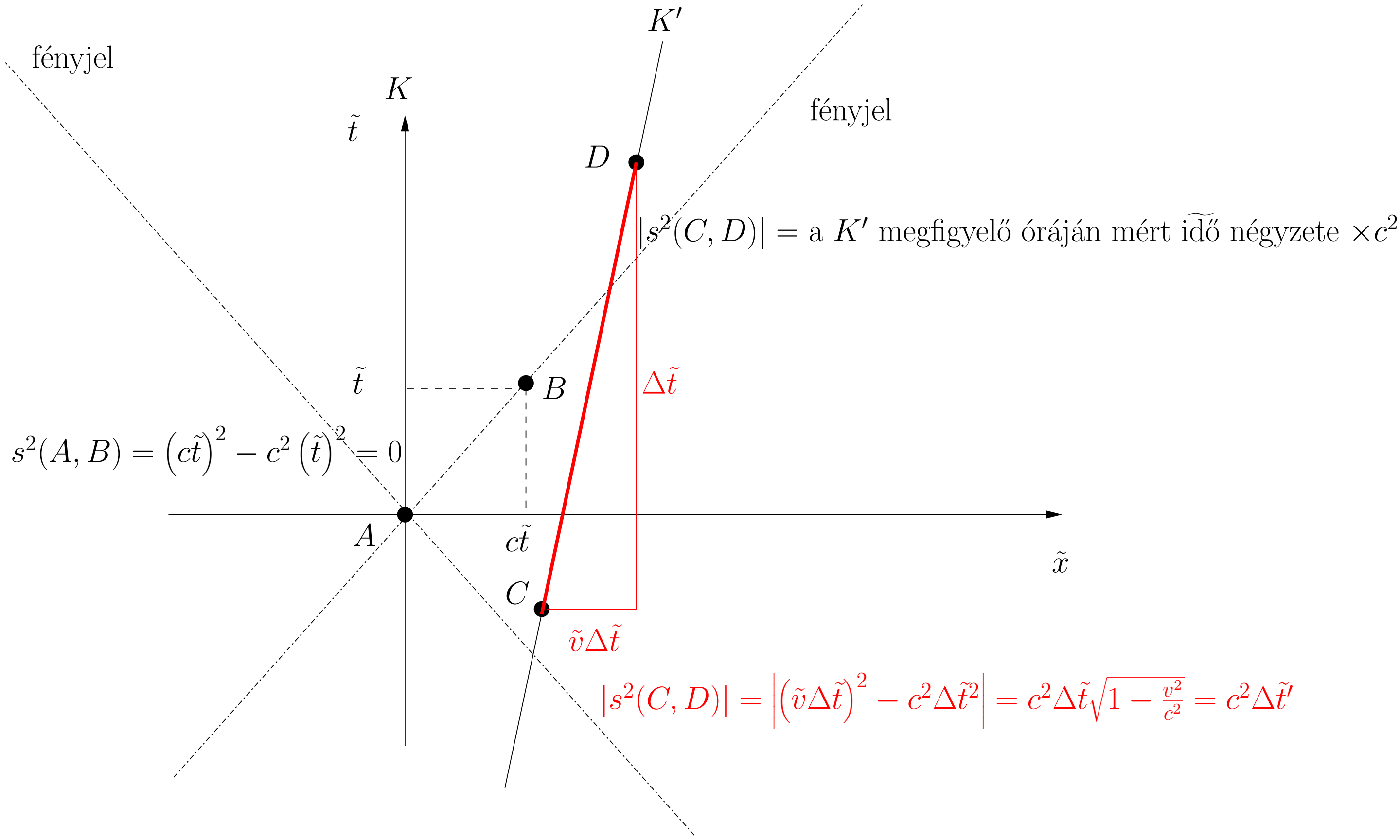
$$s^2(A,B) = \left(\tilde{x}^{K*}(B) - \tilde{x}^{K*}(A)\right)^2 - c^2\left(\tilde{t}^{K*}(B) - \tilde{t}^{K*}(A)\right)^2$$


fényjel
K
$\tilde{t}$
fényjel
$\tilde{t}$
B
$s^2(A,B) = \left(c\tilde{t}\right)^2 - c^2\left(\tilde{t}\right)^2 = 0$
A
$c\tilde{t}$
$\tilde{x}$


fényjel
K
$\tilde{t}$
fényjel
egy gyorsuló rakéta világvonala
egy légy világvonala
A
$\tilde{x}$

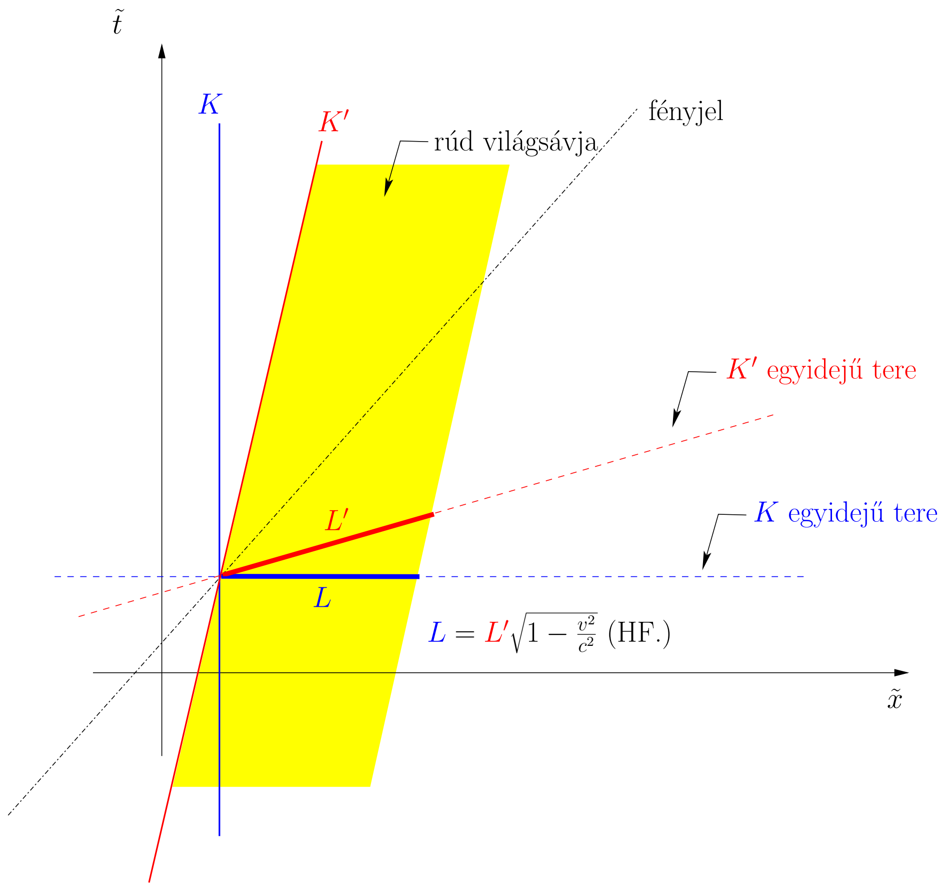

fényjel
K
$\tilde{t}$
$J^+(A)$
fényjel
egy gyorsuló rakéta világvonala
egy légy világvonala
$C(A)$
A
$\tilde{x}$
$C(A)$
Az A ponton áthaladó fizikai
részecskék világvonalai
csak a fénykúp belsejében
haladhatnak
$J^-(A)$

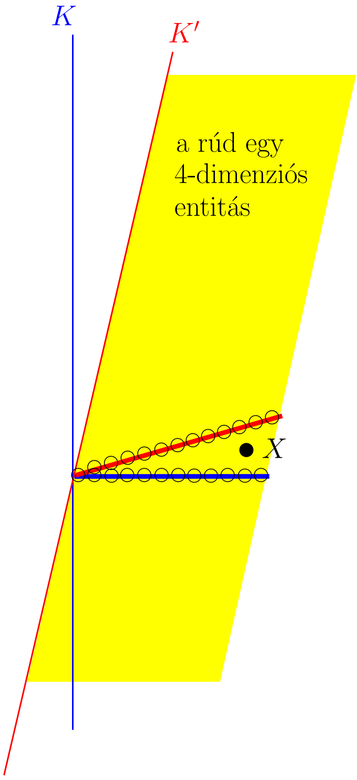

fényjel
K'
K
$\tilde{t}$
fényjel
D
$|s^2(C,D)| =$ a K' megfigyelő óráján mért $\widetilde{\text{idő}}$ négyzete $\times c^2$
$\tilde{t}$
B
$s^2(A,B) = \left(c\tilde{t}\right)^2 - c^2\left(\tilde{t}\right)^2 = 0$
A
$c\tilde{t}$
$\tilde{x}$
C



fényjel
K'
K
fényjel
$\tilde{t}$
D
$|s^2(C,D)| =$ a K' megfigyelő óráján mért $\widetilde{\text{idő}}$ négyzete $\times c^2$
$\tilde{t}$
B
$\Delta\tilde{t}$
$s^2(A,B) = \left(c\tilde{t}\right)^2 - c^2\left(\tilde{t}\right)^2 = 0$
A
$c\tilde{t}$
$\tilde{x}$
C
$\tilde{v}\Delta\tilde{t}$
$|s^2(C,D)| = \left|\left(\tilde{v}\Delta\tilde{t}\right)^2 - c^2\Delta\tilde{t}^2\right| = c^2\Delta\tilde{t}\sqrt{1-\frac{v^2}{c^2}} = c^2\Delta\tilde{t}'$

Például a Lorentz-kontrakció Minkowski-téridőben így néz ki:

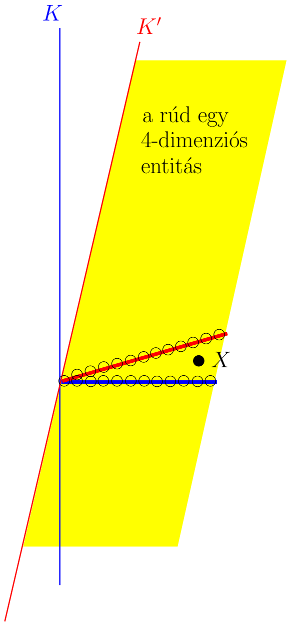




Filozófiai szempontból az einsteini-téridős-tildés relativitáselmélet üzenete:

– Találkozásuk pillanatában a két megfigyelő számára a rúd nem ugyanazon események összessége.

– A fizikai tárgyak 4-dimenziós entitások: Csak az a valóság, ami 4-dimenzióban van. Ezzel szemben a 3-dimenziós tér és az idő csupán csupán a 4-dimenziós világ vetületei.



Egyes filozófusok ebből arra következtetnek, hogy

– nincs objektív bekövetkezés; az események ontológiai státusza nem változik, pontosabban nem különbözik. Pl. X az egyik megfigyelő rendszerében (találkozásuk pillanatában) még „nem bekövetkezett”, még „nem determinált”, a másik szerint már „megmásíthatatlanul megtörtént”. Sőt, egy harmadik szerint, „éppen most történő, valóságos esemény”. Következésképpen: (a) a prezentizmus téves (b) az endurantizmus téves (c) objektív indeterminizmus lehetetlen.

– Mindezeket ráadásul a világról szerzett *empirikus* ismereteinkből olvastuk ki. Tehát tévesek Kant transzcendentális filozófiájának azon alapelvei, melyek szerint a világ geometriája és alapvető kinematikai törvényei *a priori* euklidesziek és newtoniak – merthogy empirikusak, nem euklidesziek és nem newtoniak.

Fontos azonban, hogy ezeket a filozófiai következményeket a helyi-értékükön kezeljük! A téridő itt $\widetilde{\text{téridő}}$-t, a tér $\widetilde{\text{tér}}$-t és az idő $\widetilde{\text{idő}}$-t jelent – olyan mennyiségeket, amelyek a mozgatáskor deformálódott órákkal és méterrudakkal vannak értelmezve. Miközben megmutattuk, hogy a relativisztikus fizika tökéletesen elmondható a klasszikus $\widehat{\text{tér}}$ és $\widehat{\text{idő}}$ nyelvén is. Kérdés, hogy a filozófiának melyik mennyiségeket kell metafizikailag relevánsnak tartania?

Megtehetjük, hogy a relativisztikus fizika leírásában a klasszikus fizika $\widehat{\text{tér}}$ és $\widehat{\text{idő}}$ fogalmait használjuk Elégséges-e ez ahhoz, hogy rehabilitáljuk Kant transzcendentális esztétikáját? Valamint juthatunk-e ezzel arra a metafizikai konklúzióra, hogy van objektív bekövetkezés, hogy (a) a prezentizmus igaz és az eternalizmus téves, hogy (b) az endurancia-elmélet igaz és a perdurancia-elmélet nem, hogy (c) lehetséges objektív indeterminizmus? Ezekre a kérdésekre még visszatérünk!

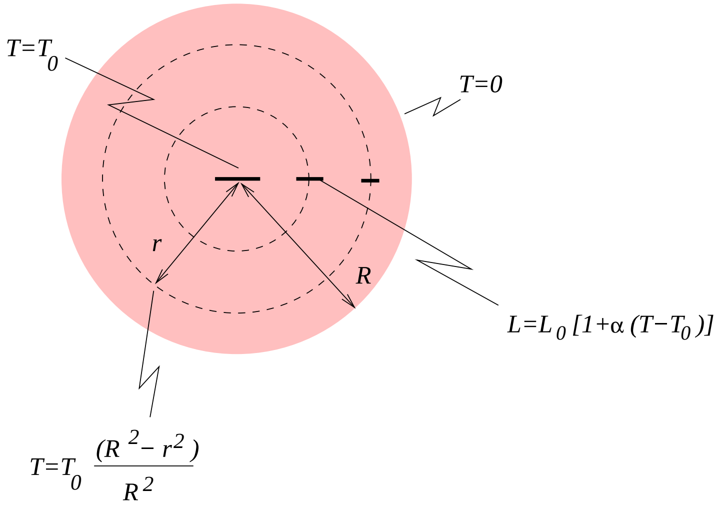


Figure 10: Megmutatható, hogy ha a körlap lakói úgy veszik, hogy méterrúdjaik hossza mindenütt egyforma, akkor arra a konklúzióra jutnak, hogy egy végtelen kiterjedésű, konstans negatív görbületű Bolyai–Lobacsevkszki-felületen élnek.

The conventionalist approach

31. According to the conventionalist thesis, the Lorentz theory and Einstein's special relativity are two alternative scientific theories which are equivalent on empirical level (see Friedman 1983, p. 293; Einstein 1983, p. 35). Due to this empirical underdeterminacy, the choice between these alternative theories is based on external aspects. (Cf. Zahar 1973; Grünbaum 1974; Friedman 1983; Brush 1999; Janssen 2002.)

Tekintsük először Poincaré ismert példáját (Fig. 10). Képzeljünk el olyan kétdimenziós lényeket, akik egész életüket egy euklideszi körlap belsejében élik le. Vannak méterrúdjaik, melyekkel távolságot tudnak mérni. A körlap hőmérsékleteloszlása nem homogén. A középponttól kifelé haladva a $T(r) = T_0 \frac{(R^2 - r^2)}{R^2}$ formula szerint csökken. Tegyük fel az egyszerűség kedvéért, hogy méterrúdjaik a hőmérséklettel arányosan változtatják hosszukat, tehát a kör széléhez közelítve a méterrudak hossza nullához tart. Megmutatható, hogy ha e körlap lakói úgy veszik, hogy méterrúdjaik hossza mindenütt egyforma, akkor arra a konklúzióra jutnak, hogy egy végtelen kiterjedésű, konstans negatív görbületű Bolyai–Lobacsevkszki-felületen élnek. A geometria és a fizikai elmélet együtt kell hogy megfeleljen mindannak, amit e kétdimenziós lények műszereikkel és érzékszerveikkel a világból tapasztalnak:

$$\left(\begin{array}{c}\textit{Bolyai–}\\ \textit{Lobacsevszki-}\\ \textit{geometria}\end{array}\right) + \left(\begin{array}{c}\textit{a világ hőmérséklete}\\ \textit{állandó}\end{array}\right) = \left(\begin{array}{c}\textit{empirikus}\\ \textit{tények}\end{array}\right)$$

$$\left(\begin{array}{c}\textit{euklideszi}\\ \textit{geometria}\end{array}\right) + \left(\begin{array}{c}\textit{a világ hőmérséklete:}\\ T(r) = T_0\frac{\left(R^2-r^2\right)}{R^2}\end{array}\right) = \left(\begin{array}{c}\textit{empirikus}\\ \textit{tények}\end{array}\right)$$

Following Poincaré's argument about the relationship between geometry, physics, and the empirical facts, the conventionalist thesis asserts the following relationship between the Lorentz theory and special relativity:

$$\left[\begin{array}{c}\text{classical}\\ \text{space-time}\\ \mathbb{E}^3\times\mathbb{E}^1\end{array}\right] + \left[\begin{array}{c}\text{physical}\\ \text{content of}\\ \text{Lorentz}\\ \text{theory}\end{array}\right] = \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right]$$

$$\left[\begin{array}{c}\text{relativistic}\\ \text{space-time}\\ \mathbb{M}^4\end{array}\right] + \left[\begin{array}{c}\text{special}\\ \text{relativistic}\\ \text{physics}\end{array}\right] = \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right]$$

Continuing the symbolic notations we used in Points **??**–**??**, denote Z those objective features of physical reality that are described by the alternative physical theories P_1 and P_2 in question. With these notations, the logical schema of the conventionalist thesis can be described in the following way: We cannot distinguish by means of the available experiments whether $G_1(M)\,\&P_1(Z)$ is true about the objective features of physical reality $M\cup Z$, or $G_2(M)\,\&P_2(Z)$ is true about the *same* objective features $M \cup Z$. Schematically,

$$[G_1(M)] + [P_1(Z)] = \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix}$$

$$[G_2(M)] + [P_2(Z)] = \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix}$$

32. However, it is clear from the previous sections that the terms "space" and "time" have different meanings in the two theories. The Lorentz theory claims $G_1(\hat{M})$ about $\hat{M}$ and relativity theory claims $G_2\left(\widetilde{M}\right)$ about some other features of reality $\widetilde{M}$. Of course, this terminological confusion also appears in the physical assertions. Let us symbolize with $\hat{Z}$ the objective features of physical reality, such as the $\widehat{\text{length}}$ of a rod, etc., described by physical theory P_1. And let $\widetilde{Z}$ denote some (partly) different features of reality described by P_2, such as the $\widetilde{\text{length}}$ of a rod, etc. Now, as we have seen, both theories actually claim that $G_1(\hat{M})\,\&G_2\left(\widetilde{M}\right)$. It is also clear that, for example, within the Lorentz theory, we can legitimately query the $\widetilde{\text{length}}$ of a rod. For the Lorentz theory has complete description of the behavior of a moving rigid rod, as well as the behavior of a moving clock and measuring-rod. Therefore, it is no problem to predict, in the Lorentz theory, the result of a measurement of the "length" of the rod, if the measurement is performed with a co-moving measuring equipments, according to empirical definition (D8). This prediction will be exactly the same as the prediction of special relativity. And vice versa, special relativity would have the same prediction for the $\widehat{\text{length}}$ of the rod as the prediction of the Lorentz theory. That is to say, the physical contents of the Lorentz theory and special relativity also are identical: both claim that

$P_1\left(\hat{Z}\right) \& P_2\left(\widetilde{Z}\right)$. So we have the following:

$$\left[G_1\left(\hat{M}\right) \& G_2\left(\widetilde{M}\right)\right] + \left[P_1\left(\hat{Z}\right) \& P_2\left(\widetilde{Z}\right)\right] = \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix}$$

$$\left[G_1\left(\hat{M}\right) \& G_2\left(\widetilde{M}\right)\right] + \left[P_1\left(\hat{Z}\right) \& P_2\left(\widetilde{Z}\right)\right] = \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix}$$

In other words, there are no different theories; consequently there is *no choice*, based neither on internal nor on external aspects.

33.. .Fenti konklúziónk arra figyelmeztet, hogy Poincaré konvencionalista felfogását is újra átgondoljuk. Abban a szabadságban, hogy a téridő geometriáját megválaszthatjuk, a konvencionalitásnak két esete keveredik egyszerre:

1. *Nem triviális konvencionalizmus*: az elméletek empirikusan aluldetermináltak.

2. *Szemantikai konvencionalizmus*: az elnevezés szabadsága, vagyis az a szabadságunk, hogy az empirikusan értelmezett (fizikai) mennyiségek közül mit minek nevezünk, például melyiket nevezzük tér- illetve időkoordinátának.

Az empirikus aluldetermináltságra példa a téridő topológiájának konvencionális jellege. Ebben az esetben tisztán látjuk érvényesülni a Poincaré által tételezett választás szabadságát a téridő lehetséges struktúrái és a hozzájuk tartozó fizikai elméletek között. Ez egyben példa az empíria „elmélet-terhességére” is. Gondoljunk Wheeler fizikushallgatójára: a laboratóriumban látott egyszerű jelenség a tér nem triviális topológiájának éppúgy lehet az evidenciája, mint a töltés létezésének, s ez csupán az elméleti előfeltevésektől függ.

Ugyanakkor Poincaré korongos példájában szó sincs az empíriát megelőző elméleti feltevésekről, és arról a fajta konvecionalitásról, melynek illusztrálására a példa született. Egyszerű szemantikai konvencióról van szó! Túl könnyen, és túl gyakran szokás az empíriát megelőző elméleti feltevésekről beszélni! Olyan egyszerű mérési operáció, mint a távolság mérése – állítja Reichenbach[1] – nem végezhető el, pontosabban nem értelmezhető anélkül az elméleti előfeltevés nélkül, hogy a merev méterrúd egy szakasz mentén egymás után lehelyezve nem változtatja a hosszát, pontosabban, hogy nincsen olyan univerzális deformáció, melyet a méterrudak is és azok az objektumok is, melyek hosszát mérni kívánjuk, elszenvednek. A koronglakók példájára lefordítva, a távolságmérés során más eredményre „jutnak”, és ezzel a világuk geometriáját másnak fogják „tapasztalni”, ha azzal az elméleti előfeltevéssel élnek, hogy a világuk hőmérséklete állandó, illetve, ha változik – állítja Poincaré. Jobban meggondolva azonban – hasonlóan a relativitáselmélet kontra Lorentz-elmélet esetéhez –, ez nem igaz! Egyszerűen kétféle fizikai mennyiséget vezethetünk be:

$$\begin{array}{lll} \tilde{x}: & \begin{array}{c}\text{a méterrúd } n\text{-szer} \\ \text{fér rá a szakaszra}\end{array} & \Rightarrow \tilde{x} = n \\ \hat{x}: & \begin{array}{c}\text{a méterrúd } n\text{-szer} \\ \text{fér rá a szakaszra}\end{array} & \Rightarrow \hat{x} = \sum_{i=1}^{n} \frac{\left(\hat{R}^2 - \hat{r}_i^2\right)}{\hat{R}^2} \end{array} \tag{43}$$

ahol $\widetilde{r}_i$ az i-edik helyen álló méterrúd $\widetilde{x}$-távolsága a középponttól, azaz a második esetben minden egyes „métert” egy $\frac{\left(\widetilde{R}^2 - \widetilde{r}_i^2\right)}{\widetilde{R}^2}$ faktorral súlyozunk (azaz „figyelembe vesszük a hőmérsékletváltozás miatti korrekciót”). Valójában mindegy, hogy milyen teoretikus előfeltevések alapján definiáljuk így az egyik mennyiséget illetve a másikat, a lényeg az, hogy empirikus értelemben két különböző mennyiséget értelmeztünk. Hasonlóan két további fizikai mennyiséget értelmezhetünk (valahol a :

$$\begin{array}{lll} \tilde{T}: & \begin{array}{c}\text{a méterrúd } n\text{-szer fér rá} \\ \text{a hőmérő higanyszálára}\end{array} & \Rightarrow \tilde{T} = n \\ \hat{T}: & \begin{array}{c}\text{a méterrúd } n\text{-szer fér rá} \\ \text{a hőmérő higanyszálára}\end{array} & \Rightarrow \hat{T} = n\frac{\left(\hat{R}^2 - \hat{r}^2\right)}{\hat{R}^2} \end{array} \tag{44}$$

(A hőmérőt kicsinek tekintjük, tehát elhanyagoltuk az egyszerűség kedvéért azt, hogy a hőmérő higanyszálának hosszán belül is változik a

[1] Reichenbach 1951, 131. o.

méterúd hossza.)

Most, hogy $\hat{x}$ vagy $\widetilde{x}$ mennyiséget *nevezzük* „távolságnak”, nem igazán jelentős kérdés, illetve az sem különösebben fontos, hogy $\hat{T}$-t vagy $\widetilde{T}$-ot kereszteljük el „hőmérsékletnek”. Ez az a szabadság, melyet szemantikai konvencionalizmusnak nevezünk. Bárhogyan is döntöttünk az elnevezések ügyében, a fizika törvényeit és a tér geometriáját a mérési eredmények egyértelműen meghatározzák: Fizikai törvény lesz például, hogy $\tilde{T}(\tilde{r}) = $ konstans, illetve az is, hogy $\hat{T}(\hat{r}) = T_0 \frac{\left(\hat{R}^2 - \hat{r}^2\right)}{\hat{R}^2}$. Hasonlóan, a mérési eredmények egyértelműen determinálják, hogy az $\tilde{x}$-geometria nem euklideszi, az $\hat{x}$-geometria pedig euklideszi. Miután tehát eldöntöttük, hogy mit jelent a „távolság” szó, az $\hat{x}$, vagy az $\widetilde{x}$ mennyiséget, a mérések egyértelműen meghatározzák a tér geometriáját. Hasonlóan az, hogy a hőmérséklet a mérések szerint állandó, vagy változik, csupán attól függ, hogy az állandó $\widetilde{T}$, vagy a radiálisan csökkenő $\hat{T}$ mennyiséget kereszteljük el „hőmérsékletnek”. Nincs tehát szó két fizikai elméletről és két térgeometriáról, csupán a szavak más használatáról!

Methodological remarks

34. It is to be noted that my argument is based on the following very weak “operationalist” premise: physical terms, assigned to measurable physical quantities, have different meanings if they have different empirical definitions. This premise is one of the fundamental preassumptions of Einstein’s 1905 paper and is widely accepted among physicists. Without clear empirical definition of the measurable physical quantities a physical theory cannot be empirically confirmable or disconfirmable. In itself, this premise is not yet equivalent to operationalism or verificationalism. It does not generally imply that a statement is necessarily meaningless if it is neither analytic nor empirically verifiable. However, when the physicist assigns time and space tags to an event, relative to a reference frame, (s)he is already after all kinds of metaphysical considerations about “What is space and what is time?”

and means definite physical quantities with already settled empirical meanings.

35. In saying that the meanings of the words “space” and “time” are different in relativity theory and in classical physics, it is necessary to be careful of a possible misunderstanding. I am talking about something entirely different from the incommensurability thesis of the relativist philosophy of science (Kuhn 1970, Chapter X; Feyerabend 1970). How is it that relativity makes any assertion about classical $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and vice versa, how can the Lorentz theory make assertions about quantities which are not even defined in the theory? As we have seen, each of the two theories is sufficiently complete account of physical reality to make predictions about those features of reality that correspond—according to the empirical definitions—to the variables used by the other theory, and we can *compare* these predictions. For example, within the Lorentz theory, we can legitimately query the reading of a clock slowly transported in K' from one place to another. That exactly is what we calculated in Point **??**. Similarly, in relativity theory, we can legitimately query the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags of an event in the reference frame of the *etalons* and then apply formulas (**??**)–(**??**). This is a fair calculation, in spite of the fact that the result thus obtained is not explicitly mentioned and named in the theory. This is what we actually did. And the conclusion was that not only are the two theories commensurable, but they provide completely identical accounts of the same physical reality.

Privileged reference frame

36. Due to the popular/textbook literature on relativity theory, there is a widespread aversion to a privileged reference frame. However, like it or not, there is a privileged reference frame in both special relativity and classical physics. It is the frame of reference in which the *etalons* are at rest. This privileged reference frame, however, has nothing to do with the concepts of “absolute rest” or the aether; it is not privileged by nature, but it is privileged by the trivial semantical convention providing meanings for the terms "distance" and "time", by the fact that of all possible measuring-rod-like and clock-like objects floating in the universe, we have chosen the ones floating together with the International Bureau of Weights and Measures in Paris. In Bridgman's words:

> It cannot be too strongly emphasised that there is no getting away from preferred operations and unique standpoint in physics; the unique physical operations in terms of which interval has its meaning afford one example, and there are many others also. (Bridgman 1936, p. 83)

37. Many believe that one can avoid a reference to the *etalons* sitting in a privileged reference frame by defining, for example, the unit of $\widetilde{\text{time}}$ for an arbitrary (moving) frame of reference K' through a cesium clock, or the like, co-moving with K'. In this way, one needs not to refer to a standard clock accelerated from the reference frame of the *etalons* into reference frame K'. But, as it follows from our considerations in Point **??**, such a definition has several difficulties. For if this operation is regarded as a convenient way of *measuring* $\widetilde{\text{time}}$, then we still have $\widetilde{\text{time}}$ in the theory, together with the privileged reference frame of the *etalons*. If, however, this operation is regarded as the empirical *definition* of a physical quantity, then it must be clear that this quantity is not $\widetilde{\text{time}}$ but a new physical quantity, say $\widetilde{\widetilde{\text{time}}}$. In order to establish any relationship between $\widetilde{\widetilde{\text{time}}}$ tags belonging to different reference frames, it is a must to use an "*etalon* cesium clock" as well as to refer to its behavior when accelerated from one inertial frame into the other, or, in some other way, to describe the others' behaviors in term of the physical quantity defined with the *etalon*.

The aether

38. Many of those, like Einstein himself (see Point **29**), who admit the "empirical equivalence" of the Lorentz theory and special relativity argue that the latter is "incomparably more satisfactory" (Einstein) because it has no reference to the aether. As it is obvious from the previous sections, we did not make any reference to the aether in the logical reconstruction of the Lorentz theory. It is however a historic fact that, for example, Lorentz did. In this section, I want to clarify that Lorentz's aether hypothesis is logically independent from the actual physical content of the Lorentz theory. In other words, the concept of aether is merely a verbal decoration in Lorentz's theory, which can be interesting for the historians, but negligible from the point of view of the recent logical reconstructions. (Actually the same holds for the "denial of aether" by Einstein's special relativity.)

Consider, for example, Lorentz's aether-theoretic formulation of the relativity principle—to touch on a sore point.

39. Let us introduce the following notation:

$A\left(K',K''\right) :=$ The laws of physics in K' are such that the laws of physics empirically ascertained by an observer in K'', describing the behavior of physical objects co-moving with K'', expressed in variables $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in in K', describing the similar physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transfered from K' to K'', ignoring the fact that the equipments undergo deformations during the transmission.

Taking this statement, the usual Einsteinian formulation of the relativity principle is the following:

$$\begin{bmatrix} \text{Einstein's} \\ \text{Relativity} \\ \text{Principle} \end{bmatrix} = (\forall K')\,(\forall K'')\;A\,(K', K'')$$

Many believe that this version of relativity principle is essentially different from the similar principle of Lorentz, since Lorentz's principle makes explicit reference to the motion relative to the aether. Using the above introduced notations, it says the following:

$$\begin{bmatrix} \text{Lorentz's} \\ \text{Principle} \end{bmatrix} = (\forall K'')\;A\,(\text{aether}, K'')$$

It must be clearly seen, however, that *Lorentz's principle and Einstein's relativity principle are logically equivalent to each other.* On the one hand, it is trivially true that

$$\begin{aligned} \begin{bmatrix} \text{Einstein's} \\ \text{Relativity} \\ \text{Principle} \end{bmatrix} &= (\forall K')\,(\forall K'')\;A\,(K', K'') \\ &\Rightarrow (\forall K'')\;A\,(\text{aether}, K'') \\ &= \begin{bmatrix} \text{Lorentz's} \\ \text{Principle} \end{bmatrix} \end{aligned}$$

On the other hand, it follows from the *meaning* of $A\,(K', K'')$ that

$$(\exists K')\,(\forall K'')\;A\,(K', K'') \Rightarrow (\forall K')\,(\forall K'')\;A\,(K', K'')$$

The reason is that the laws of physics in K' completely determine the results of the measurements performed by a moving—relative to K'—observer on moving physical objects with moving measuring equipments. Consequently,

$$\begin{aligned} \begin{bmatrix} \text{Lorentz's} \\ \text{Principle} \end{bmatrix} &= (\forall K'')\;A\,(\text{aether}, K'') \\ &\Rightarrow (\exists K')\,(\forall K'')\;A\,(K', K'') \end{aligned}$$

$$\Rightarrow \; (\forall K')\,(\forall K'')\;\; A\,(K',K'')$$
$$= \left[\begin{array}{c}\text{Einstein's}\\ \text{Relativity}\\ \text{Principle}\end{array}\right]$$

Thus, it is Lorentz's principle itself—which refers to the aether—that renders any claim about the aether a logically separated hypothesis outside of the scope of the factual content of both the Lorentz theory and special relativity. It is Lorentz's principle itself—again, which refers to the aether—that implies that the role of the aether could be played by anything else; the aether does not constitute a privileged reference frame.

As the Lorentz theory and special relativity unanimously claim, physical systems undergo deformations when they are transferred from one inertial frame K' to another frame K''. One could say, these deformations are caused by the transmission of the system from K' to K''. You could say they are caused by the "wind of aether". By the same token you could say, however, that they are caused by "the wind of *anything*", since if the physical system is transfered from K' to K'' then its state of motion changes relative to an arbitrary third frame of reference.

40. On the other hand, it must be mentioned that special relativity does not exclude the existence of the aether. (Not to mention that already in 1920 Einstein himself argues for the existence of some kind of aether. See Reignier 2000.) Neither does the Michelson–Morley experiment. If special relativity/Lorentz theory is true then there must be no indication of the motion of the interferometer relative to the aether. Consequently, the fact that we do not observe indication of this motion is not a challenge for the aether theorist. Thus, the hypothesis about the existence of aether is logically independent of both the Lorentz theory and special relativity.

Heuristic and explanatory values

41. The Lorentz theory and special relativity, as completely identical theories, offer the same symmetry principles and heuristic power. As we have seen, both theories claim that quantities $\widetilde{x}^{K'}, \widetilde{t}^{K'}$ in an arbitrary K' and the similar quantities $\widetilde{x}^{K''}, \widetilde{t}^{K''}$ in another arbitrary K'' are related through a suitable Lorentz transformation. This fact in conjunction with the relativity principle (within the scope of validity of the principle) implies that laws of physics are to be described by Lorentz covariant equations, if they are expressed in terms of variables $\widetilde{x}$ and $\widetilde{t}$, that is, in terms of the results of measurements obtainable by means of the corresponding co-moving equipments—which are distorted relative to the *etalons*. There is no difference between the two theories that this space-time symmetry provides a valuable heuristic aid in the search for new laws of nature.

42. Finally, let us return to the question of the alleged difference, that special relativity is a theory of principle whereas the Lorentz theory is a constructive theory (see Point **??**). As we have seen in Point **??**, the two sets of basic principles $\{(L1) + (L2) + (L3)\}$ and $\{(R1) + (R2)\}$ are logically equivalent; the two theories are identical. Therefore, the statements of "both theories" can be derived either from $\{(L1) + (L2) + (L3)\}$ or from $\{(R1) + (R2)\}$. So, if the fact that the statements of special relativity can be derived form $\{(R1) + (R2)\}$ provides enough reason to say that special relativity is a principle theory, then the same fact provides enough reason to say the same thing about the Lorentz theory. And vice versa, if the fact that the statements of the Lorentz theory can be derived form basic assumptions $\{(L1) + (L2) + (L3)\}$ provides enough reason to say that it is a constructive theory, then the same fact provides enough reason to say the same thing about special relativity.

Though, it is a historic fact that Lorentz, FitzGerald, and Larmor, in contrast to Einstein, made an attempt to understand how these laws actually come about from the molecular forces. These are perfectly legitimate *additional* questions. Moreover, as we have seen in Chapter , a careful consideration of these details reveals that the principle of relativity is not a universal principle; it does not hold for the whole range of validity of the Lorentz covariant laws of relativistic physics, but only for the equilibrium quantities characterizing the equilibrium states of dissipative systems. So, the situation is more complex: both the Lorentz theory and special relativity are principle theories, but the main "principle" on which they are based is not a general principle. Yet, it is suitable for some derivations; for example, for the derivation of the transformation rules of $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags. It is because $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags are obtained by means of measuring-rods and clocks co-moving with different inertial reference frames, and the principle of relativity holds for such equilibrium quantities as the length of a measuring-rod and the characteristic period of a clock.

43. With these comments I have completed the argumentation for the claim that special relativity and the Lorentz theory are completely identical. Again, the historical questions are not important from the point of view of our analysis. What is important is the logical possibility of a Lorentz-type theory: the classical Galileo-invariant spatiotemporal conceptions + deformations of moving objects, governed by the relativity principle. And, what we proved is that such a theory is completely identical to special relativity in both senses, as theories about space and time and as theories about the behavior of moving physical objects. They are not only "empirically equivalent", as sometimes claimed, but they are identical in all sense; they are identical physical theories.

Consequently, in comparison with the classical Galileo-invariant conceptions, special relativity theory tells us nothing new about the spatiotemporal features of the physical world. As we have seen, the longstanding belief that it does is the result of a simple but subversive terminological confusion.

Violation of relativity principle in relativistic physics

Example

Consider a rod at rest in K. The length of the rod is l. At a given moment of time t_0 we take a record about the positions and velocities of all particles of the rod:

$$r_i(t = t_0) = R_i \tag{45}$$

$$\left.\frac{dr_i(t)}{dt}\right|_{t=t_0} = w_i \tag{46}$$

Then, forget this system, and imagine another one which is initiated at moment $t = t_0$ with the initial condition (45)–(46). No doubt, the new system will be identical with a rod of length l, that continues to be at rest in K.

Now, imagine that the new system is initiated at $t = t_0$ with the initial condition

$$r_i(t = t_0) = R_i \tag{47}$$

$$\left.\frac{dr_i(t)}{dt}\right|_{t=t_0} = w_i + v \tag{48}$$

instead of (45)–(46). No doubt, in a very short interval of time $(t_0, t_0 + \Delta t)$ this system is a rod of length l, moving at velocity v; the motion of each particle is a superposition of its original motion, according to (45)–(46), and the collective translation at velocity v. In other words, it is a rod co-moving with the reference frame K'. Still, its length is l, contrary to the principle of relativity, according to which the rod should be of length $l\sqrt{1 - \frac{v^2}{c^2}}$—as a consequence of $l' = l$.

The restricted relativity principle as a principle of thermodynamics

44. The resolution of this "contradiction" is that the system initiated in state (47)–(48) at time t_0 finds itself in a non-equilibrium state and then, due to certain dissipations, it *relaxes* to the *new* equilibrium state. What such a new equilibrium state is like, depends on the details of the dissipation/relaxation process. It is, in fact, a *thermodynamical* question. The concept of "gentle acceleration" not only means that the system does not go irreversibly far apart from its equilibrium state, but, more essentially, it incorporates the assumption that there is such a dissipation/relaxation phenomenon.

Without entering into the quantum mechanics of solid state systems, a good way to picture it is imagine that the system is radiating during the relaxation period.

45. Thus, we must draw the conclusion that, in spite of the Lorentz covariance of the equations, whether or not the solution determined by the condition $\Lambda_v^{-1}(\psi_0')$ is identical with the solution belonging to condition ψ_v, in other words, whether or not the relativity principle holds, depends on the details of the dissipation/relaxation process in question, *given that 1) there is dissipation in the system at all and, 2) the physical quantities in question, to which the relativity principle applies, are equilibrium quantities characterizing the equilibrium properties of the system.* For instance, in our example, the relativity principle does not hold for all dynamical details of all particles of the rod. The reason is that many of these details are sensitive to the initial conditions. The principle holds only for some macroscopic equilibrium properties of the system, like the length of the rod. It is a typical feature of a dissipative system that it unlearns the initial conditions; some of the properties of the system in equilibrium state, after the relaxation, are independent from the initial conditions. The equilibrium length of a solid rod is a good example. These equilibrium properties are completely determined by the equations themselves *independently of the initial conditions*. If so, the Lorentz covariance of the equations in itself guarantees the satisfaction of the principle of relativity *with respect to these properties*: Let X be the value of such a physical quantity—characterizing the equilibrium state of the system in question, fully determined by the equations independently of the initial conditions—ascertained by the measuring devices at rest in K. Let X' be the value of the same quantity of the same system when it is in equilibrium and at rest relative to the moving reference frame K', ascertained by the measuring devices co-moving with K'. If the equations are Lorentz covariant, then $X = X'$. We must recognize that whenever in relativistic physics we derive correct results by applying the principle of relativity, we apply it for such particular equilibrium quantities. *But the relativity principle, in general, does not hold for the whole dynamics of the systems in relativity theory*, in contrast to classical mechanics.

46. When claiming that relativity principle, in general, does not hold for the whole dynamics of the system, a lot depends on what we mean by the system set into uniform motion. One has to admit that this concept is still vague. As we pointed out, it was not clearly defined in Einstein's formulation of the principle either. By leaving this concept vague, Einstein tacitly assumes that these details are irrelevant. However, they can be irrelevant only if the system has dissipations and the principle is meant to be valid only for some equilibrium properties with respect to which the system unlearns the initial conditions. So the best thing we can do is to keep the classical definition of ψ_v: Consider a system of particles the motion of which satisfies the following "initial" conditions:

$$\begin{aligned} \mathbf{r}_i(t = t_0) &= \mathbf{R}_{i0} \\ \left.\frac{d\mathbf{r}_1}{dt}\right|_{t=t_0} &= \mathbf{V}_{i0} \end{aligned} \tag{49}$$

(A condition like (49) does not necessarily mean either that $t_0 = 0$ nor that the solution in question describes the motion only for $t \geq t_0$, it just fixes a particular solution by prescribing the state of the particles at a given moment of time.) The system is set in collective motion at velocity $\mathbf{v}$ at the moment of time t_0 if its motion satisfies

$$\begin{aligned} \mathbf{r}_i(t = t_0) &= \mathbf{R}_{i0} \\ \left.\frac{d\mathbf{r}_1}{dt}\right|_{t=t_0} &= \mathbf{V}_{i0} + \mathbf{v} \end{aligned} \tag{50}$$

I have basically two arguments for such a choice:

(a) The first is a methodological one. The usual Einsteinian derivation of the Lorentz transformation, simultaneity in K', etc., starts with the declaration of the relativity principle. In order to formulate the principle, we need the concept of a physical system in uniform motion relative to K. This concept, therefore, must logically precede relativity theory.

(b) The second support comes from what Bell calls "Lorentzian pedagogy".

> Its special merit is to drive home the lesson that the laws of physics in any one reference frame account for all physical phenomena, including the observations of moving observers. And it is often simpler to work in a single frame, rather than to hurry after each moving objects in turn. (Bell 1987, p. 77.)

In reference frame K, the concept of setting a system of state (49) in collective motion at velocity $\mathbf{v}$ in turn means nothing but setting it in state (50).

47. Thus, we have seen that in classical mechanics the principle of relativity is, indeed, a universal principle. It holds, without any restriction, for *all* dynamical details of *all* possible systems described by classical mechanics. In contrast, in relativistic physics this is not the case:

1. The principle of relativity is not a universal principle. It does not hold for the whole range of validity of the Lorentz covariant laws

of relativistic physics, but only for the equilibrium quantities characterizing the equilibrium states of dissipative systems. Since dissipation, relaxation, and equilibrium are thermodynamical conceptions *par excellence*, the special relativistic principle of relativity is actually a thermodynamical principle, rather than a general principle satisfied by all dynamical laws of physics describing all physical processes in details. One has to recognize that the special relativistic principle of relativity is experimentally confirmed only in such restricted sense.

2. The satisfaction of the principle of relativity in such restricted sense is indeed guaranteed by the Lorentz covariance of those physical equations that determine, independently of the initial conditions, the equilibrium quantities for which the principle of relativity holds. In general, however, Lorentz covariance of the laws of physics does not guarantee the satisfaction of the relativity principle.

3. It is an experimentally confirmed fact of nature that some laws of physics are *ab ovo* Lorentz covariant. However, since relativity principle is not a universal principle, it does not entitle us to infer that Lorentz covariance is a fundamental symmetry of physics.

4. The fact that the space and time tags obtained by means of measuring-rods and clocks co-moving with different inertial reference frames can be connected through the Lorentz transformation is compatible with our general observation that the principle of relativity only holds for such equilibrium quantities as the length of a solid rod or the characteristic periods of a clock-like system.

The fact that relativity principle is not a universal principle throws new light upon the discussion of how far the Einsteinian special relativity can be regarded as a principle theory relative to the other (constructive) approaches (cf. Einstein 1969a, p. 57; Bell 1992; Brown and Pooley 2001; Brown 2001; 2003). (See Point **42**.) It can also be interesting from the point of view of other reflections on possible violations of Lorentz covariance (see, for example, Kostelecký and Samuel 1989).

It must be emphasized that the physical explanation of this more complex picture is rooted in the physical deformations of moving measuring-rods and moving clocks by which the space and time tags are defined in moving reference frames. In Einstein's words:

> A Priori it is quite clear that we must be able to learn something about the physical behavior of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. (Einstein 1920, p. 35)

Since therefore the Lorentz transformation itself is not merely a mathematical concept without contingent physical content, we must not forget the real physical content of Lorentz covariance and relativity principle.

Empirical Foundation of the Absolute Theory of Space and Time

48. Faithfully reflecting how "space" and "time" tags are understood in classical physics and relativity theory, definitions (D1)–(D8) in Point **14** answered the purpose of demonstrating that Einstein's special relativity has exactly the same claims about space and time as classical physics and the Lorentz theory. However, neither the classical nor the relativistic definitions are trouble free; they are actually sloppy and circular. As we will see, they are based on several pre-assumptions about contingent facts of nature which cannot be known or even formulated *prior* to the definitions of space and time tags.

The central issue of relativistic physics is the relationship between the space and time tags defined in different inertial frames moving relative to each other. In this chapter we will focus on the difficulties encountered with respect to the empirical definitions of space and time tags in one single inertial frame of reference. In other words, we will focus on what is common to both the classical and relativistic approaches, definitions (D1)–(D4).

49. The first difficulty is caused by the fact that distance is defined by means of measuring rod. The problem I mean is different from the one proposed by Reichenbach (1958), namely that the length of the rod may be altered by some universal forces when the rod is transported from one place to the another. This—known, or unknown—deformation of the *etalon* is, in fact, no problem from logical/operational point of view, as long as the operational procedure provides an unambiguous definition. For example, we are completely aware of the Lorentz contraction of the measuring rod. But this is no problem; procedure (D8) in Point **14** provides an unambiguous definition of $\widetilde{\text{space}}$ tags $\widetilde{x}^{K'}(A)$. As in his final conclusion Reichenbach points out, the very posing of the problem presupposes the Newtonian concept of "absolute length" of the rod; something that is independent of any empirical definition of "distance" whatsoever (see Point **67**). This concept is, however, meaningless or at least is outside of the scope of physics. If space tags are defined, for example, according to (D2) then the length of the measuring rod is—by definition—constant, no matter what is our metaphysical pre-assumption about the "length" of the rod *ansich.*

There are, however, real circularities in definitions (D1)–(D4) that appear at the very operational level. The operations described in (D2) and (D4) rest on the concept of a measuring rod at rest relative to a given reference frame. But, we encounter the following difficulties:

(a) We have seen in Point **??** that the concept of a rod "at rest" relative to a reference frame is problematic in itself.

(b) One might think that this is no problem if the measuring rod is always in equilibrium state when we are measuring with it. It must be clear however that the equilibrium state of a rod cannot be ascertained prior to the definition of its length (and other extensive properties), that is, prior to the definition of distance.

(c) The concept of rest relative to a reference frame is problematic not only for the measuring rod as a whole but even for one single particle of the rod. The reason is that we are missing a prior definition of velocity relative to a given reference frame.

(d) Throughout definitions (D1)–(D9) we nonchalantly used the term "reference frame". Of course, it is no problem to give the usual meaning to this term *after* having defined space and time tags of events; when we already have the concepts of simultaneity, the distance of simultaneous events, dimensions, straight lines, etc. But the term "reference frame" has no meaning prior to the space and time tags. We encounter this wrong circularity in definitions (D2) and (D4): we ought to superpose the measuring-rod along a straight line, such that the rod is always at rest relative to the reference frame.

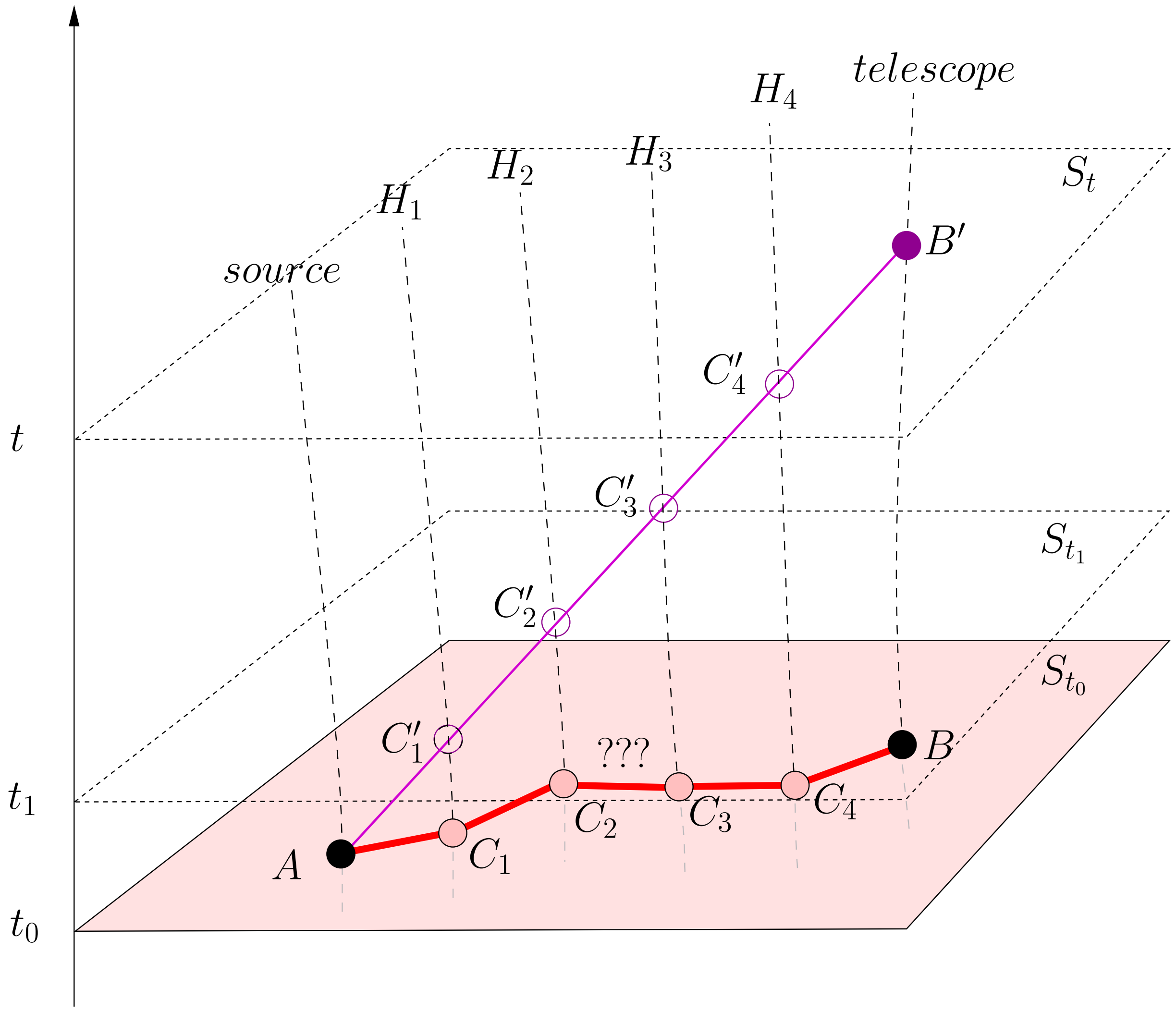


Figure 11: *Operational definition of straight line by means of light signal*

(e) We also used the term "inertial" frame of reference. This is another term that has no meaning without a previous definition of space and time tags (see Point **69**).

50. Some of the circularity problems are independent of the relativistic effects and they are already there in classical physics. Let me illustrate this with a typical example.

Assume, the time tags of events are somehow defined. So we have the concept of "space" S_t, that is the set of simultaneous events at a given time t. I shall discuss the widespread view that the concept of straight line in a given S_{t_0} can be defined by light signal. There has been a long discussion about the conventionality versus objectivity of the spatiotemporal concepts defined by empirical operations of this kind;

whether or not there exist "universal forces" deviating the path of the light from the "real straight line", etc. But nobody contested that the operational definition in itself is meaningful. This is, however, not the case.

The definition says: A light signal is emitted from a point-like source. It passes through little optical holes and arrives at a telescope. The path from the source, through the holes, to the telescope is a straight line of the physical space, by definition.

But, let us examine this operation in more details (Fig. 11). The light is emitted from the source—event $A \in S_{t_0}$. Then it passes through the first hole H_1—event C'_1. Event C'_1 is, however, already not in S_{t_0} but in S_{t_1}. The next similar event C'_2 is in S_{t_2}, an so on; and the observation of the signal in the telescope, event B', is in S_t. Now, even if we accept that the path $A, C'_1, C'_2, \ldots B'$ is a straight line (geodesic) of spacetime (in some objective sense or/and by convention—it does not matter now), it cannot define the concept of "straight line" in one given space S_{t_0} of simultaneous events. In fact, it cannot designate a path in S_{t_0}, whatsoever, without a previous definition of "rest", without the previous concept of "persistent space locus", that is, a previous concept of identity of two locuses of space at two different times.

It worth noting that the same problem occurs if, instead of a light signal, we want to define straight line by means of a "standard inertial/free motion".

51. The upshot of these considerations is that, in order to avoid the circularities mentioned above and to minimize the conventional elements in the empirical foundation of our physical theory of space and time, we must avoid using standard measuring rod in the definition of distance. We must also abstain from relying on the concept of rigid body, reference frame, and inertial motion. Instead, we will use one standard clock and light signals. A light signal should not be understood as a "light ray" or a "light beam", that is, we should not assume—in advance—that the light signal propagates along a "straight line".

Of course, using one standard clock and light signals for coordination of space-time is an old idea; as old as the widespread belief that the task is as trivial as it seems from the two-dimensional textbook examples, and that the resulted spatiotemporal structure is, at least locally, necessarily identical with the standard space-time geometry of special relativity. What will be new in our analysis is the consequent performance of this task without operational circularities. As we will see, the task is not trivial; and the analysis of the spatiotemporal conceptions so obtained will raise some still open—although experimentally testable—questions.

Empirical definition of space and time tags

52. First we chose an *etalon* clock. That is to say, we chose a system (a sequence of phenomena) floating somewhere in the universe. Without loss of generality we may stipulate that this is an equipment having a pointer and the readings are real numbers. (For example, the clock in the U.S. Naval Observatory, used by the GPS.) There is no assumption that this is a clock measuring "proper time". There is no assumption that it "runs uniformly". And there is no assumption that it is "at rest" relative to anything, or that it is of "inertial motion". The reason is that none of these concepts is defined yet.

We will call "marker" an equipment which can be triggered by a physical event and can transmit and receive modulated radio waves containing some information. Assume we have as many markers as we need, with the following functions:

1. There is a distinguished marker floating together with the standard clock and continuously transmitting the actual reading of the standard clock.

2. The others continuously receive the regular time signals from the standard clock.

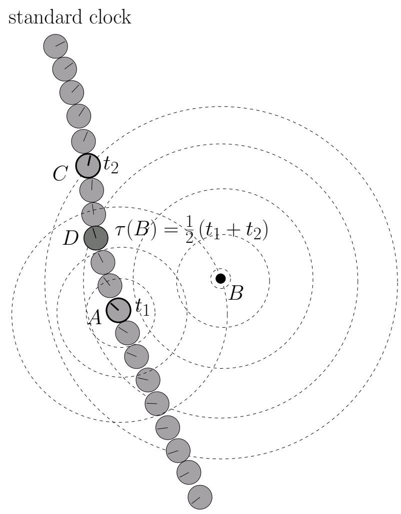


Figure 12: *Operational definition of time tags. (This is just a symbolic sketch, not a real "two dimensional space-time diagram" or the like.)*

3. They can transmit radio signals containing the following information: a) an ID code of the device and information about the standard clock reading, so from the signal they send it always can be known which device was the transmitter and what was the standard clock reading received by the transmitter at the moment of the emission of the signal, b) information about the event on the occasion of which the signal was transmitted.

4. They can receive the signals transmitted by the others.

By the emission of a radio signal the marker marks an event. It is far from obvious, however, what must be regarded as an event in general—prior to the concepts of time and distance. (See Brown 2005, pp. 11-14.) We do not dwell on this problem here. The reader can easily imagine various operational solutions of how to use a marker for marking various physical events/phenomena.

53. Consider the experimental arrangement in Fig. 12. The marker at the standard clock emits a radio signal at clock-reading t_1 (event A). The signal is received by another marker which immediately emits another signal (event B). This "reflected" signal is detected by the marker at the standard clock at t_2 (event C). We assume, as an empirical fact, that the clock we have chosen is such that a given reflected signal is received by the standard clock only once, at reading t_2, and

$$t_2 \geq t_1 \tag{51}$$

by which we have chosen, conventionally, an "arrow of time" (not the arrow of physical processes in time; see Price 1996, p. 16 and 58). (In fact, we made two choices here. One is the choice of the direction of the parametrization of the clock's pointer positions (51). There is however a more important one: by applying the terms "sending" and "receiving" a signal, we previously determined the causal order of events A and C. To what extent this causal order is purely conventional? How can we—without prior spatiotemporal conceptions—distinguish whether an event is a "sending" or a "receiving" of a signal? How is this choice of causal order related to the change of information content of the signal? To what extent this choice is determined by our free will and free action experience at the modulation of the radio waves? Is this freedom an objective openness of future or merely a subjective experience? These are delicate metaphysical questions; into the discussion of which it is not our present purpose to enter.)

Definition (A1) The *absolute time* tag of event B is the following:

$$\tau(B) := t_1 + \varepsilon(t_2 - t_1) \tag{52}$$

where $\varepsilon = \frac{1}{2}$ by convention. (Of course, it could be a contingent fact of nature that $t_2 = t_1$, in which case the choice of the value of ε would not matter.)

It is important to emphasize that the choice of using radio signals in definition (A1) is purely conventional. This choice is by no means justified by the "constancy and isotropy of the (round-trip) velocity of light"; simply because we are prior to any spatiotemporal concepts that would make any statement about the "velocity" of light meaningful.

54. Denote S_τ the set of simultaneous events with time tag τ. One might think that we are ready to define the distance between simultaneous events in the usual way. Surely, we can define the distance between the simultaneous events D and B in Fig. 12 as $\frac{1}{2}(t_2 - t_1)\,c$, where the value of c is taken as a convention. In this way, however, we can define the distance only *from the standard clock*, but there is no way to extend this definition for arbitrary pair of simultaneous events. In order to define the distance between two arbitrary simultaneous evens we need further preparations.

We would like to base the definition of distance to the definition of time: the distance between two points in a given S_τ will be defined through the period of time in which a radio signal runs "from the one point to the other". Therefore, instead of signals sent and received by the marker at the standard clock, we will use radio signals "sent from the one point and received at the other". However, we encounter the following difficulty. We would like to define distance between *simultaneous* events; but the travel of the signal takes some time; the emission of the signal and the receiving of the signal are not simultaneous events. Whose distance is the one measured by the time of travel of the signal—and when? We encounter the same problem as in Point **50**. The distance obtained by means of the time of travel of the signal depends on the concept of "rest"; the concept of "being at the same place at different times" (Fig 13). So, in order to define the distance of simultaneous events we need a previous concept of "rest"; and, moreover, we have to define this concept by the only means of the standard clock and radio signals.

It is necessary to be careful of a possible misunderstanding. Although they are close to each other, the problem we are addressing here is different from the problem of persistence of physical objects (Butterfield 2005). What we would like to define is the identity of two locuses of space at two different times, and not the genidentity of the physical objects occupying these locuses. One might think that some definition of genidentity of physical objects must be prior to our operational definition of space and time tags, at least in the case of the standard clock. This is, however, not necessarily the case. The standard clock is just an ordered (ordered by the clock readings) sequence of physical events,

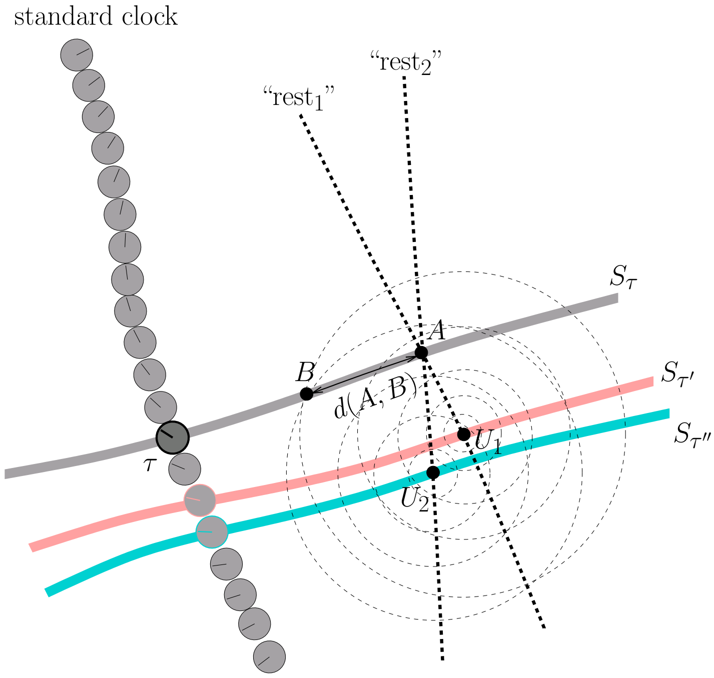


Figure 13: *The distance defined by means of the time of travel of the radio signal depends on the concept of "rest"; the concept of "being at the same place at different times"*. In general, $\tau(B) - \tau(U_1) \neq \tau(B) - \tau(U_2)$

but without the further metaphysical assumption that these events belong to the same physical object. (We definitely do not make such assumption in the case of a "clock-like" sequence of events that we will call a rest time sequence below.)

Definition (A2) A one-parameter family of events $\gamma(\tau)$ is called *time sequence* if $\gamma(\tau) \in S_\tau$ for all τ.

One has to recognize that a time sequence is a "clock-like" sequence of events. For every event, one can define a time-like tag in the same way as (A1): Event A (Fig. 14) is marked with the emission of a radio signal at time $\tau(A)$. The signal is reflected at event B. Event C is the first detection of the reflected signal at time $\tau(C)$. We define the following time-like tag for event B:

$$\tau^\gamma(B) := \tau(A) + \varepsilon\left(\tau(C) - \tau(A)\right)$$

(If there is no detection of the reflected signal at all, then, say, $\tau^\gamma(B) := \infty$.)

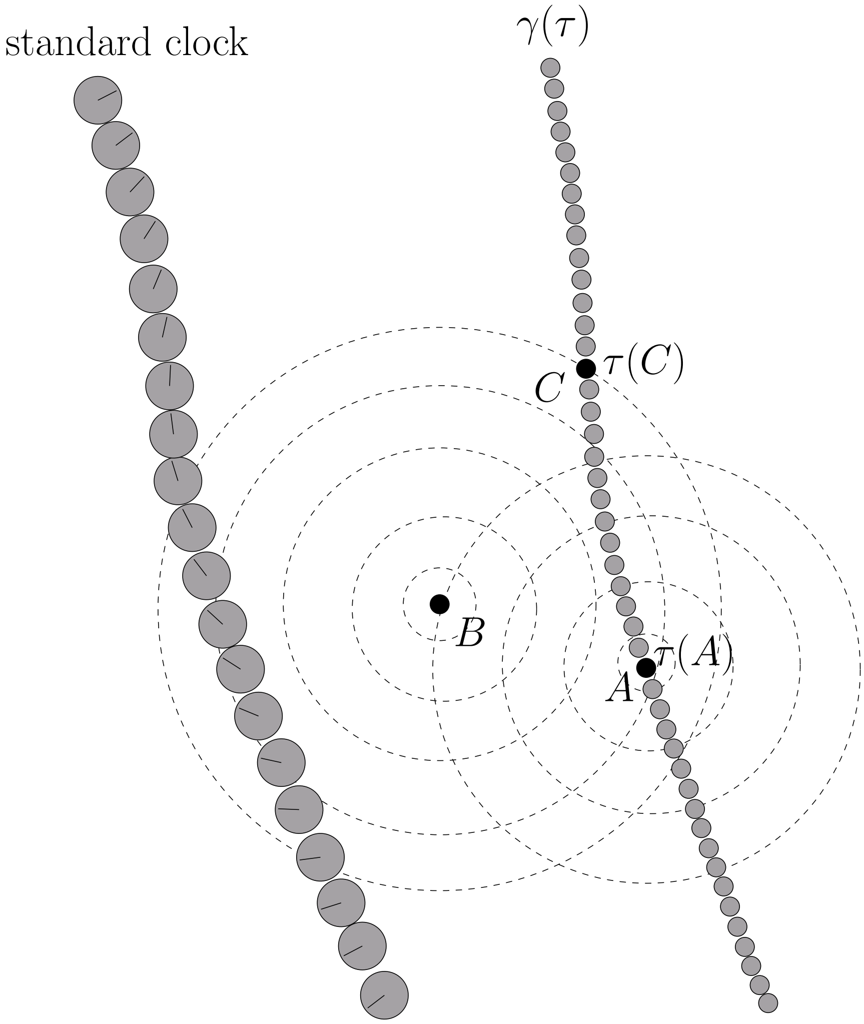


Figure 14: *Clock-like time sequence*

It is an empirical fact that $\tau^\gamma(B) \neq \tau(B)$ in general. It is another empirical observation however that for some particular cases $\tau^\gamma(B) = \tau(B)$.

Definition (A3) A time sequence $\gamma(\tau)$ is a *rest time sequence* if for every event B $\tau^\gamma(B) = \tau(B)$.

Whether or not there exist rest time sequences is an empirical question. We stipulate the following:

Empirical fact (E1) For any event A there exists a unique rest time sequence $\gamma(\tau)$ such that

$$A = \gamma(\tau(A))$$

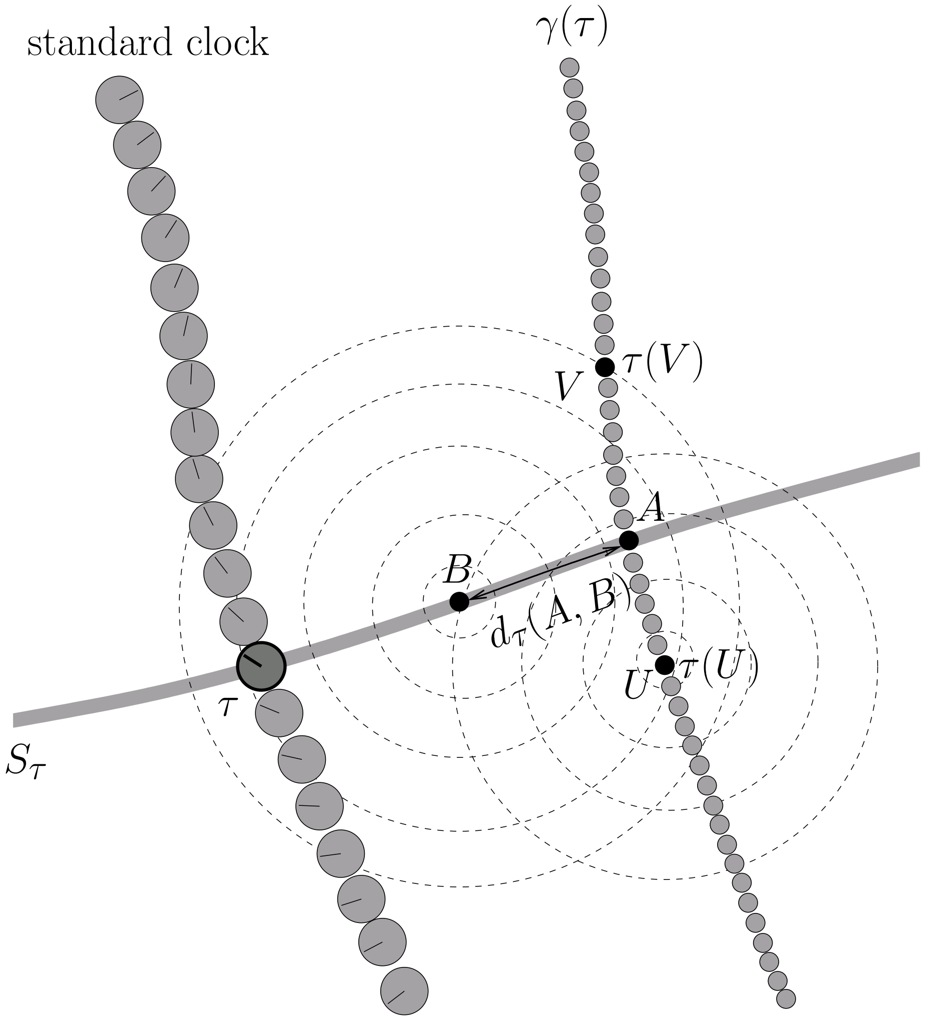


Figure 15: *The distance between two simultaneous events*

Rest time sequence is a concept defined only by means of the standard clock and radio signals. It singles out a "world line" through every event, that will play the role of the "world line of a particle being at rest relative to the standard clock".

55. Now we are ready to define the distance between simultaneous events.

Definition (A4) The *absolute distance* between two simultaneous evens $A, B \in S_\tau$ is operationally defined in the following way. Take a rest time sequence γ such that $A = \gamma(\tau)$. (See Fig. 15) Let $U = \gamma\left(\tau(U)\right)$ be an event marked with the emission of a radio signal at absolute time $\tau(U)$, such that the signal is received and reflected at event B. The detection of the reflected signal marks the event $V = \gamma\left(\tau(V)\right)$ of time tag $\tau(V)$.

The absolute distance is

$$d_\tau(A,B) := \frac{1}{2}\left(\tau(V) - \tau(U)\right) c \tag{53}$$

where $c = 299792458\frac{m}{s}$ by convention.

56. From (51) and from other observations we know the following properties of d_τ:

Empirical fact (E2) For all $A, B, C \in S_\tau$

$$\begin{aligned} d_\tau(A,B) &\geq 0 && (54)\\ d_\tau(A,A) &= 0 && (55)\\ d_\tau(A,B) &= 0 \text{ only if } A = B && (56)\\ d_\tau(A,B) + d_\tau(B,C) &\geq d_\tau(A,C) && (57)\\ d_\tau(A,B) &= d_\tau(B,A) && (58) \end{aligned}$$

One has to recognize that a function $S_\tau \times S_\tau \to \mathbb{R}$ with properties (54)–(57) and (58) is what the mathematician calls metric on S_τ. Thus, we can stipulate that (S_τ, d_τ) is a metric space for every moment of absolute time τ.

57. Having metric, that is distance, defined on S_τ, we introduce the following abbreviations:

$$\begin{aligned} Cong_\tau(A,B,C,D) &\iff d_\tau(A,B) = d_\tau(C,D)\\ Bet_\tau(A,B,C) &\iff d_\tau(A,C) = d_\tau(A,B) + d_\tau(B,C) \end{aligned}$$

In terms of these abbreviations we formulate the following—not necessarily new—empirical facts:

Empirical facts

(E3) $\forall A \forall B \; Cong_\tau(A,B,B,A)$

(E4) $\forall A \forall B \forall C \; Cong_\tau(A,B,C,C) \to A = B$

(E5) $\forall A \forall B \forall C \forall D \forall E \forall F \; Cong_\tau(A,B,C,D)$
$\wedge Cong_\tau(C,D,E,F) \to Cong_\tau(A,B,E,F)$

(E6) $\forall A \forall B \; Bet_\tau(A, B, A) \rightarrow A = B$

(E7) $\forall A \forall B \forall C \forall D \forall E \; Bet_\tau(A, D, C) \wedge Bet_\tau(B, E, C))$
$\rightarrow \exists F \, (Bet_\tau(D, F, B) \wedge Bet_\tau(E, F, A)$

(E8) $\exists E \forall A \forall B \; A \in \alpha \wedge B \in \beta \rightarrow Bet_\tau(E, A, B)$
$\rightarrow \exists F \forall A \forall B \; A \in \alpha \wedge B \in \beta \rightarrow Bet_\tau(A, F, B)$
where α and β are two sets of events in S_τ.

(E9) $\exists A \exists B \exists C \exists D \exists E \; \neg D = E \wedge Cong_\tau(A, D, A, E)$
$\wedge Cong_\tau(B, D, B, E) \wedge Cong_\tau(C, D, C, E)$
$\wedge \neg Bet_\tau(A, B, C) \wedge \neg Bet_\tau(B, C, A) \wedge \neg Bet_\tau(C, A, B)$

(E10) $\forall A \forall B \forall C \forall D \forall E \forall F \; \neg D = E \wedge \neg D = F \wedge \neg E = F$
$\wedge Cong_\tau(A, D, A, E) \wedge Cong_\tau(A, D, A, F)$
$\wedge Cong_\tau(B, D, B, E) \wedge Cong_\tau(B, D, B, F)$
$\wedge Cong_\tau(C, D, C, E) \wedge Cong_\tau(C, D, C, F)$
$\rightarrow Bet_\tau(A, B, C) \vee Bet_\tau(B, C, A) \vee Bet_\tau(C, A, B)$

(E11) $\forall A \forall B \forall C \forall D \forall E \forall F \; Bet_\tau(A, B, F) \wedge Cong_\tau(A, B, B, F)$
$\wedge Bet_\tau(A, D, E) \wedge Cong_\tau(A, D, D, E)$
$\wedge Bet_\tau(B, D, C) \wedge Cong_\tau(B, D, D, C)$
$\rightarrow Cong_\tau(B, C, F, E)$

(E12) $\forall A \forall B \forall C \forall D \forall E \forall F \forall G \forall H \; \neg A = B \wedge Bet_\tau(A, B, C)$
$\wedge Bet_\tau(E, F, G) \wedge Cong_\tau(A, B, E, F)$
$\wedge Cong_\tau(B, C, F, G) \wedge Cong_\tau(A, D, E, H)$
$\wedge Cong_\tau(B, D, F, H) \rightarrow Cong_\tau(C, D, G, H)$

(E13) $\forall A \forall B \forall C \forall D \exists E \; Bet_\tau(D, A, E) \wedge Cong_\tau(A, E, B, C)$

The quantification runs over S_τ. In brief, we stipulate, as an empirical fact, that the two relations $Cong_\tau$ and Bet_τ, determined by the distances of simultaneous events, satisfy the axioms of 3-dimensional Euclidean geometry, namely Tarski's axioms of 3-dimensional Euclidean geometry (Tarski and Givant 1999). It must be emphasized that all the state-

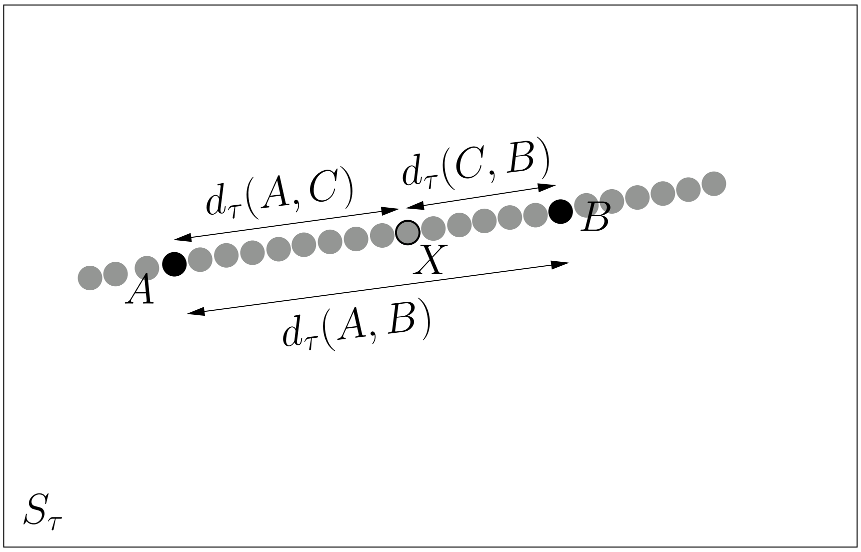


Figure 16: *Straight line*

ments (E3)–(E13) are stipulated, via inductive generalization, merely on the basis of observations about distances of simultaneous events.

58. Within this axiomatic framework, one can define the basic geometrical concepts in the usual way; and one can derive a body of theorems, well known from the textbooks on Euclidean geometry. Below are a few of the typical definitions and theorems we will use in the construction of space tags.

Definition *A* subset $\sigma \subset S_\tau$ is called (straight) *line* if satisfies the following conditions (Fig. 16):

1. for any $A, B, C \in \sigma$ exactly one of the following three relations hold:

$$\begin{aligned} d_\tau(A,C) + d_\tau(C,B) &= d_\tau(A,B) \\ d_\tau(A,B) + d_\tau(B,C) &= d_\tau(A,C) \\ d_\tau(B,A) + d_\tau(A,C) &= d_\tau(B,C) \end{aligned}$$

2. σ is maximal for property 1.

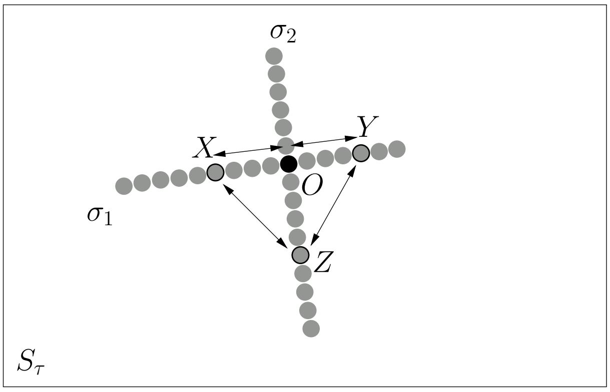


Figure 17: *Orthogonal lines*

Definition Let σ_1 and σ_2 be two lines in S_τ such that $\sigma_1 \cap \sigma_2 = \{O\}$ (see Fig. 17). σ_2 is *orthogonal* to σ_1 if for every $Z \in \sigma_2$ and for every $X, Y \in \sigma_1$

$$d_\tau(X, O) = d_\tau(O, Y) \Leftrightarrow d_\tau(X, Z) = d_\tau(Y, Z)$$

Theorem For every $A, B \in S_\tau$ there exists a unique line containing A and B.

Theorem Let $A \in S_\tau$ be an arbitrary event and let $\sigma_1 \subset S_\tau$ be an arbitrary line. There always exists a line σ_2 orthogonal to σ_1, such that $A \in \sigma_2$.

Definition Using the notations of the above theorem, let $\sigma_1 \cap \sigma_2 = \{O\}$. Event O is called *the orthogonal projection of A to σ_1*. Distance $d_\tau(A, O)$ is called *the distance of A from σ_1*.

Definition Let $\sigma_1 \subset S_\tau$ be a line. A line σ_2 is *parallel* to σ_1 if for all $X \in \sigma_2$ the distance of X from σ_1 is the same.

Theorem Let $\sigma_1 \subset S_\tau$ be a line and let $C \in S_\tau$ be an arbitrary event. There exists exactly one line σ_2 such that $C \in \sigma_2$ and σ_2 is parallel to σ_1.

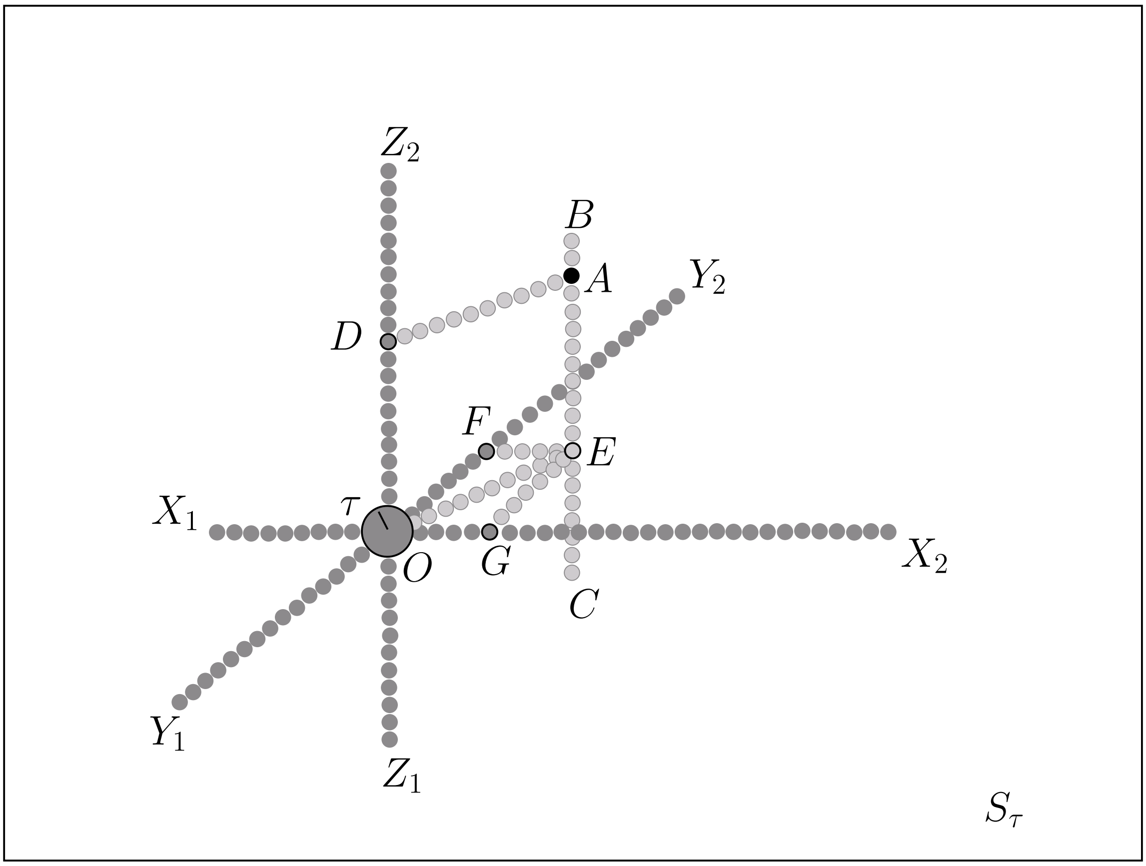


Figure 18: *Cartesian coordinates in S_τ*

Definition Let $A, B \in \sigma$ be two events on line σ. *Line segment* between events $A, B \in S_\tau$ is the following subset of σ:

$$\sigma(A, B) := \{X \in \sigma | \, d_\tau(A, X) + d_\tau(X, B) = d_\tau(A, B)\} \tag{59}$$

These are however only examples. In what follows, the whole usual system of definitions and theorems of Euclidean geometry are supposed to be known.

59. Now we are going to to define the usual Cartesian coordinates in S_τ. First we need a 3-frame.

Definition (A6) A *3-frame* in S_τ consists of three pairwise orthogonal lines, $\sigma\,(X_1, X_2)$, $\sigma\,(Y_1, Y_2)$, $\sigma\,(Z_1, Z_2)$, such that

$$\sigma\,(X_1, X_2) \cap \sigma\,(Y_1, Y_2) \cap \sigma\,(Z_1, Z_2) = \{O\}$$

O is called the origin of the frame (Fig. 18).

The end points play marginal role, but we do not assume that these segments have "infinite" length. The segments are supposed to be long enough for the purposes of the empirical coordination of the physical events in question. The origin of the 3-frame is arbitrary, although it could be a natural choice to take the "τ-event" of the standard clock as origin.

In the following definition we give the operational definition of space tags in one given S_τ. Let us call them τ-space tags.

Definition (A7) Let A be an arbitrary event in S_τ. Take a line segment $\sigma(B,C) \ni A$ parallel to $\sigma(Z_1, Z_2)$. (See Fig. 18.) Take another line segment $\sigma(A,D)$ orthogonal to $\sigma(Z_1, Z_2)$ such that $D \in \sigma(Z_1, Z_2)$. Let $\sigma(O,E)$ be a line segment parallel to $\sigma(A,D)$ such that $E \in \sigma(B,C)$. Finally, take the line segments $\sigma(E,F)$ and $\sigma(E,G)$ such that $\sigma(E,F)$ is parallel to $\sigma(X_1, X_2)$ and $F \in \sigma(Y_1, Y_2)$, and $\sigma(E,G)$ is parallel to $\sigma(Y_1, Y_2)$ and $G \in \sigma(X_1, X_2)$. Now, the space tags are defined as follows:

$$x_\tau(A) := \begin{cases} d_\tau(G,O) & \text{if } G \in \sigma(O, X_2) \\ -d_\tau(G,O) & \text{if } G \in \sigma(O, X_1) \end{cases}$$

$$y_\tau(A) := \begin{cases} d_\tau(F,O) & \text{if } F \in \sigma(O, Y_2) \\ -d_\tau(F,O) & \text{if } F \in \sigma(O, Y_1) \end{cases}$$

$$z_\tau(A) := \begin{cases} d_\tau(D,O) & \text{if } D \in \sigma(O, Z_2) \\ -d_\tau(D,O) & \text{if } D \in \sigma(O, Z_1) \end{cases}$$

60. It must be emphasized that with the above definitions we only defined the space tags in a given set of simultaneous events S_τ. Yet, we have no connection whatsoever between two S_τ and $S_{\tau'}$ if $\tau \neq \tau'$. In principle, there exist "infinitely" many possible bijections between the different S_τ's. This is true, even if we prescribe that the bijection must be an isomorphism preserving distances.

According to some vague intuition, a time sequence $\gamma(\tau)$ satisfying that

$$x_\tau(\gamma(\tau)) = \text{const.} \tag{60}$$

$$
\begin{aligned}
y_\tau(\gamma(\tau)) &= \text{const.} && (61)\\
z_\tau(\gamma(\tau)) &= \text{const.} && (62)
\end{aligned}
$$

corresponds to a localized physical object being at rest. "At rest"—relative to what? The actual behavior described by these equations depends on how the different 3-frames are chosen in the different S_τ's. One might think that an object is at rest if equations (60)–(62) hold in one and the same 3-frame in all S_τ. But, what does it mean that "one and the same 3-frame in all S_τ"? When can we say that a line segment $\sigma(X'_1, X'_2)$ in $S_{\tau'}$ is *the same* 3-frame axis as $\sigma(X_1, X_2)$ in S_τ? When can we say that an event A' is in the same place in $S_{\tau'}$ as event A in S_τ?

61. When we are seeking for a correspondence between S_τ and $S_{\tau'}$, our aim is not simply to find a mathematically "canonical" bijection—whatever it means. What we wish is a one-to-one map

$$\mathbb{T}_\tau^{\tau'} : S_\tau \rightarrow S_{\tau'}$$

of natural physical meaning:

(a) It must be defined by means of physical operations.

(b) For all $A, B \in S_\tau$, we require that $d_{\tau'}\left(\mathbb{T}_\tau^{\tau'}(A), \mathbb{T}_\tau^{\tau'}(B)\right) = d_\tau(A, B)$.

(c) It must reflect our intuition about being "at rest". (For example, in our traditional language, if the standard clock moves along a time-like straight line of the Minkowski space-time, $\mathbb{T}_\tau^{\tau'}$ must be equal to the map $(\tau, x, y, z) \mapsto (\tau', x, y, z)$, in the frame of reference of the standard clock. Of course, this example should be understood only intuitively, in the sense of Points **68–70**.)

62. We have already defined a concept of the unique rest time sequence through every event. So condition (c) basically means that for any rest time sequence γ we require that $\mathbb{T}_\tau^{\tau'}(\gamma(\tau)) = \gamma(\tau')$. In fact, we will base the connection between different time slices on the rest time sequences.

A következőt lehet bebizonyítani:

Lemma Let γ_1 and γ_2 be arbitrary two rest time sequences. For any two moments of absolute time τ and τ'

$$d_\tau\left(\gamma_1\left(\tau\right), \gamma_2\left(\tau\right)\right) = d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right) \tag{63}$$

63. Now, the following isomorphism satisfies the conditions we required in Point **61**.

Definition (A8)

$$\begin{aligned} \mathbb{T}_\tau^{\tau'} : S_\tau &\rightarrow S_{\tau'} \\ A &\mapsto \mathbb{T}_\tau^{\tau'}(A) = \gamma(\tau') \end{aligned}$$

where γ is a rest time sequence such that $A = \gamma(\tau)$. Let us call $\mathbb{T}_\tau^{\tau'}$ the *time shift* between S_τ and $S_{\tau'}$.

It follows from (E1) and Lemma 2 that this definition is sound and $\mathbb{T}_\tau^{\tau'}$ is a distance preserving bijection (Fig. 19).

64. Now we have everything at hand to define the space tags of events.

Definition (A9) Let A be an arbitrary event. The *absolute space tags* of A are defined as follows:

$$\begin{aligned} \xi_1(A) &:= x_0\left(\mathbb{T}_{\tau(A)}^0\left(A\right)\right) \\ \xi_2(A) &:= y_0\left(\mathbb{T}_{\tau(A)}^0\left(A\right)\right) \\ \xi_3(A) &:= z_0\left(\mathbb{T}_{\tau(A)}^0\left(A\right)\right) \end{aligned}$$

Thus, we are given *the absolute space and time tags* for every event: $\xi_1(A), \xi_2(A), \xi_3(A), \tau(A)$.

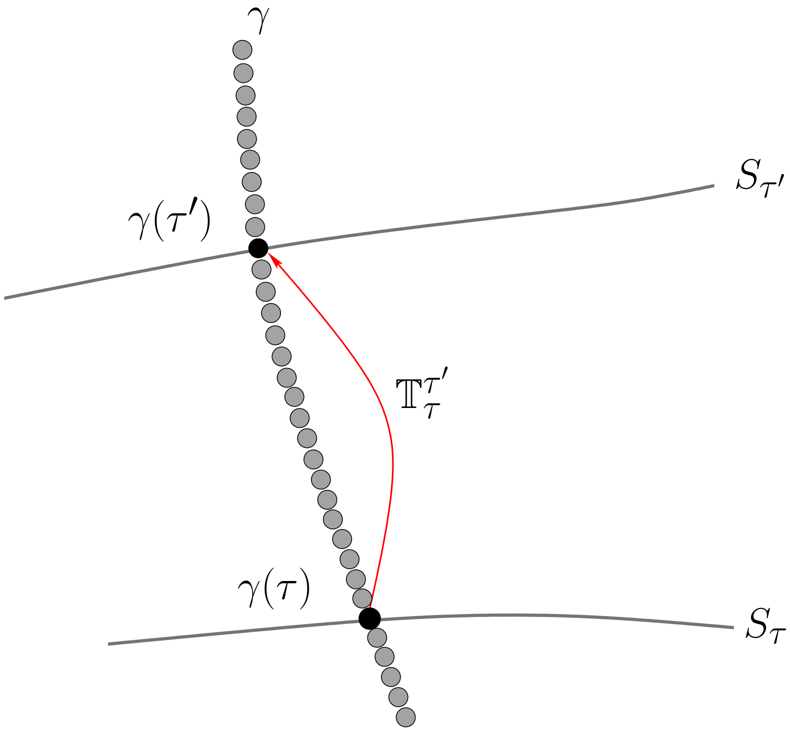


Figure 19: *Due to the existence of rest time sequences one can define a natural distance-preserving map* $\mathbb{T}_\tau^{\tau'}$ *between the different sets of simultaneous events, only by means of physical operations*

65. Having absolute space and time tags for every event, one can define further kinematical concepts. For example, the *absolute velocity* of a time sequence $\gamma(\tau)$ is obviously defined as

$$\mathbf{v}(\tau) := \begin{pmatrix} \frac{d\xi_1(\gamma(\tau))}{d\tau} \\ \frac{d\xi_2(\gamma(\tau))}{d\tau} \\ \frac{d\xi_3(\gamma(\tau))}{d\tau} \end{pmatrix}$$

I omit the further (but straightforward) definitions of quantities reducible to the space and time tags.

66. So we know the space and time *tags* for all events; the distance of simultaneous events; the concept of straight line; the dimension of the space of simultaneous events, etc. We can compact all these empirical facts—about those features of the physical events that are defined by (A1)–(A9)—by representing each physical event A by a corresponding point $p_A \in E^3 \times \mathbb{R}$, where E^3 is a 3-dimensional Euclidean space, such that the usual Cartesian coordinate map φ_C assigns the corresponding space and time tags to each point:

$$\begin{aligned} \varphi_C : E^3 \times \mathbb{R} &\rightarrow \mathbb{R}^4 \\ p_A &\mapsto (\xi_1(A), \xi_2(A), \xi_3(A), \tau(A)) \end{aligned}$$

On the 4-dimensional manifold $E^3 \times \mathbb{R}$ we can introduce many different coordinates:

$$\begin{aligned} \varphi : E^3 \times \mathbb{R} &\rightarrow \mathbb{R}^4 \\ p &\mapsto (u_1(p), u_2(p), u_3(p), u_4(p)) \end{aligned}$$

Normally, these coordinates have no physical/empirical meanings; except what one can deduce from the coordinate transformation map $\varphi_C \circ \varphi^{-1}$, that is, from the physical/empirical meanings of the space and time tags $(\xi_1(A), \xi_2(A), \xi_3(A), \tau(A))$.

67. I call $\tau(A)$ "absolute time" not in the sense of what Newton called "absolute, true and mathematical time", that is independent of any empirical definition, but in the sense of what the 20th century physics calls absolute time; it is "independent of the position and the condition of motion of the system of co-ordinates" (Einstein 1920, p. 51). The space and time tags $\xi_1(A), \xi_2(A), \xi_3(A), \tau(A)$ are *absolute in the sense that they are not relative to a reference frame* but prior to any reference frame. (The concept of "reference frame" is still not defined, and actually will never be defined in our construction.)

Absolute space and time tags are, of course, "relative" to the trivial semantical convention by which we define the meaning of the terms. They are "relative" to the *etalon* clock-like process we have chosen in the universe; and to the particular way in which the space and time tags are defined, including the usage of radio signals, the choice of "$\varepsilon = \frac{1}{2}$", etc. This kind of "relativism" is however common to all physical quantities having empirical meaning. As we pointed out in Points **36–37**, there is no getting away from such preferred operations. (The space tags $\xi_1(A), \xi_2(A), \xi_3(A)$ have some additional conventional element; they also are relative to the chosen 3-frame in S_0. This additional conventionality is, however, of marginal importance; it is nothing more than what we would call in our usual language "the choice of a 3-coordinate basis in a given reference frame".)

To what extend these choices are actually free will be discussed later.

Inertial motion

68. A remark is in order on the empirical facts (E1)–(E13) to which we refer in constructing the space and time tags. One might think that we can know these facts from our ordinary physical theories. The ordinary physical theories are however based on the ordinary, problematic, space and time conceptions, relaying on "reference frames realized by rigid bodies" and the likes, without proper, non-circular, empirical definitions. Thus, especially in the context of defining the two most fundamental physical quantities, distance and time, we must not regard our ordinary physical theories as empirically meaningful and empirically confirmed claims about the world. Whether these statements are true or not is, therefore, an empirical question, and it is far from obvious whether they would be completely confirmed if the corresponding experiments were performed with higher precision, similar to the recent GPS measurements, especially for larger distances. Strangely enough, according to my knowledge, these very fundamental facts have never been tested experimentally; no textbook or monograph on space-time physics refers to such experimental results; actually, with a very few exceptions (for example, Milne 1935 Part I; Bridgman 1965), it is not even attempted to provide a clear, non-circular empirical definition of "time" and "distance" in one single (inertial) frame, as if it would be a problem only in the case of an accelerated observer (cf. Märzke and Wheeler 1964; Pauri and Vallisneri 2000).

So, the best we can do is to *believe* that (E1)–(E13) are true. (E1) is necessary for to define any “distance” whatsoever. The rest (E2)–(E13) are statements about the properties of the distance so obtained. It is, therefore, of utmost importance whether (E1) is true or not. As we will see, this question is, in some intuitive sense, related to the inertiality of the *etalon* clock.

69. Throughout the definition of space and time tags, we avoided the term “inertial”, and because of a good reason. First of all, if “inertial” is regarded as a kinematical notion based on the concept of straight line and constancy of velocity, then it cannot be antecedent to the concept of space-time tags. If, on the other hand, it is understood as a manner of existence of a physical object in the universe, when the object is undergoing a free floating, in other words, when it is “free from forces”, then the concept is even more problematic. The reason is that “force” is a concept defined through the deviation from the trajectory of inertial motion (first circularity), and neither the inertial trajectory nor the measure of deviation from it can be expressed without spatiotemporal concepts; consequently, they cannot be antecedent to the definition of space and time tags (second circularity). So there is *no* precise, non-circular definition of inertial motion. (And this is—in my view—the major difficulty with Märzke and Wheeler’s (1964) “geodesic clock” approach, too.)

Two remarks:

1. It is to be emphasized that this operational/logical circularity is a problem even in a special relativistic/flat/local space-time; and, therefore, it has nothing to do with the problem of conventionality of demarcation between “inertial” or “geodetic” motion versus gravitation as universal force.

2. Point **50** implies that even if we chose, as a convention, a standard “inertial motion” (just by pointing at a physical object in the universe, by saying that “This object is of inertial motion, by definition.”), it would not supersede empirical definitions (A3)–(A9);

and it would remain an empirical question whether the conventionally chosen "inertial motion" has anything to do with the concept of straight line in a space of simultaneous events.

70. According to our believed special relativistic physical theory, space-time is a 4-dimensional Minkowski space and inertial trajectory is a time-like straight line in the Minkowski space. Since we are prior to the empirical definitions of the basic spatiotemporal quantities, we cannot regard this claim as an empirically confirmed physical theory. Nevertheless, let us assume for a moment that our special relativistic theory is the true description of the world "from God's point of view". It is straightforward to check that all the facts (E1)–(E13) are true if 1) the standard clock moves along an inertial world line in the Minkowski space-time and 2) it reads the proper time, that is, it measures the length of its own word line, according to the Minkowski metric. However, we human beings can know neither whether the standard clock (chosen by us) is of inertial motion in God's Minkowskian space-time nor whether it reads the proper time. What if these conditions fail? What does special relativistic kinematics say about (E1)–(E13) if the standard clock is accelerated and/or it does not read the proper time?

In order to answer this question, we have to follow up the operational definitions (A1), (A2),... and *calculate* whether statements (E1), (E2),... are true or not if the standard clock moves along a given world line γ and the "time" it reads is, say, a given function of the Minkowskian coordinate time, $\chi(t)$. Although the task is straightforward, the calculation is too complex to give a general answer by analytic means. But, for the most crucial question of (E1), the problem can be solved by computer. One finds the following—perhaps surprising—results.

For the sake of the contrast, let me first mention that one obtains a very misguiding result if, for the sake of simplicity, the calculation is made in a *2-dimensional* Minkowski space-time:

No matter if the standard clock moves along a non-inertial world line γ, no *matter if it reads a time $\chi(t)$ which is an arbitrary monotonic function of the Minkowskian coordinate time, different from the proper time along its world line, facts (E1) is always true.*

If this 2-dimensional result were the final truth, one would conclude that no spatiotemporal measurement can ascertain whether the standard clock moves inertially or not; the very concept of "inertial" motion would remain a purely conventional one; fact (E1) would always be true, independently of the "objective" fact of how the standard clock moves in God's Minkowski space-time.

In contrast, the real *4-dimensional* calculation leads to the following results:

(RES1) *Fact (E1) is always true if the standard clock moves along an inertial world line, no matter if the clock reads a time $\chi(t)$ which is an arbitrary monotonic function of the Minkowskian coordinate time, different from the proper time along its world line.*

(RES2) *If the standard clock moves along a non-inertial world line γ, fact (E1) is never true, no matter if the clock reads the proper time or not.*

71. There are remarkable consequences of the above results:

1. In accord with our intuition based on the believed physical theories, we can give an objective meaning to "inertial motion" by means of correct—neither logically nor operationally circular—experiments: *the standard clock is of inertial motion if statements (E1) is true.* Assuming that the standard clock is not only inertial but also satisfies conditions (E2)–(E13), one can extend the concept of inertiality for an arbitrary time sequence $\gamma(\tau)$ of events: $\gamma(\tau)$ corresponds to an inertial motion if the absolute space tags $\xi_1(\gamma(\tau)), \xi_2(\gamma(\tau)), \xi_3(\gamma(\tau))$ are linear functions of the absolute time tag τ.

2. On the basis of our believed physical theories, one cannot, however, predict whether (E1) is true or false. It is still an open *empirical* question.

3. Imagine that (E1) are not satisfied. It not only means that the standard clock we have chosen is non-inertial but it also means that the chosen clock is inappropriate for the definition of space-time tags. More exactly, one has to stop at definition (A1). One can define the time tags but cannot define the spatial notions, in particular the distances between simultaneous evens.

4. Consequently, it is meaningless to talk about "non-inertial reference frame", "space-time coordinates (tags) defined/measured by an accelerated observer", and the likes.

5. Az (E1) kondíció mellett természetesen a további kondícióknak is teljesülniük kell ahhoz, hogy a térkoordinátákat értelmezhessük. Ez további megszorításokat jelenthet az *etalon* óra lehetséges választására. A fent vázolt konstrukcióban, az egyszerűség kedvéért feltettük, hogy az euklideszi geometria axiómáival ekvivalens (E2)–(E13) tulajdonságok igazak. Megmutatható, hogy ebben az esetben a lehetséges *etalon* órák által mutatott „idők" egymás lineáris függvényei.

Life in absolute space and time

72. Let us continue to assume that the empirical facts (E1)–(E13) hold, and let us also assume the following:

Empirical fact (E14) The empirically confirmed laws of physics expressed in terms of the absolute space and time tags are exactly the same as the ordinary laws of the believed (special) relativistic physics, expressed in one single space-time coordinates, namely in the ones we called absolute space and time tags (i. e., "in the frame of reference of the standard clock", in the usual formulation).

If so, then, as it can be easily seen, the whole relativistic physics can be reconstructed within the framework of absolute space and time.

Egyszerűen azért, mert – mint azt már sokszor megállapítottuk – a fizika törvényei a jelenségek egészét le tudják írni egyetlen kiválasztott vonatkoztatási rendszerben.

Szorgalmi olvasmány

As an example, consider how a moving observer describes the "space" and "time" coordinates of an event in his/her own "frame of reference". (Note that "frame of reference" is a concept we have not defined; and here I use the term only symbolically. Actually, the concept of reference frame as a rigid system of material points—rigid body of a spacecraft, three orthogonal rigid rods co-moving with the observer, etc.—is a vague and very problematic notion which ought to be expelled from the conceptual vocabulary of physics.) We will assume that the observer moves along a time sequence the absolute velocity of which is smaller than the speed of light. Now, imagine that the observer has a clock-like device and performs exactly the same operational procedure as (A1)–(A9). If (s)he can go through all the steps, then—according to assumption (E14) and Point **70**—(s)he is an inertial observer. (Otherwise it would be meaningless to talk about the "space" and "time" coordinates in his/her "frame of reference".) What can we say about the "space" and "time" tags so obtained?

Of course, we can say nothing in the general case when the observer's clock-like device has nothing to do with the standard clock. Assume, however, that the observer's clock-like device is an identical copy of the standard clock, which was gently accelerated up to the velocity of the observer; therefore, it is almost like a clock, except that it runs slower by the factor $\sqrt{1-\frac{v^2}{c^2}}$, due to assumption (E14). In this case, the observer obtains "space" and "time" tags equal to the ones we would obtain from the absolute space and time tags by applying the Lorentz transformation. Let me illustrate this with a simple two-dimensional calculation. (Most megismételjük a tildés mennyiségek definícióját!)

Imagine that a radio signal is emitted (event B) when the observer

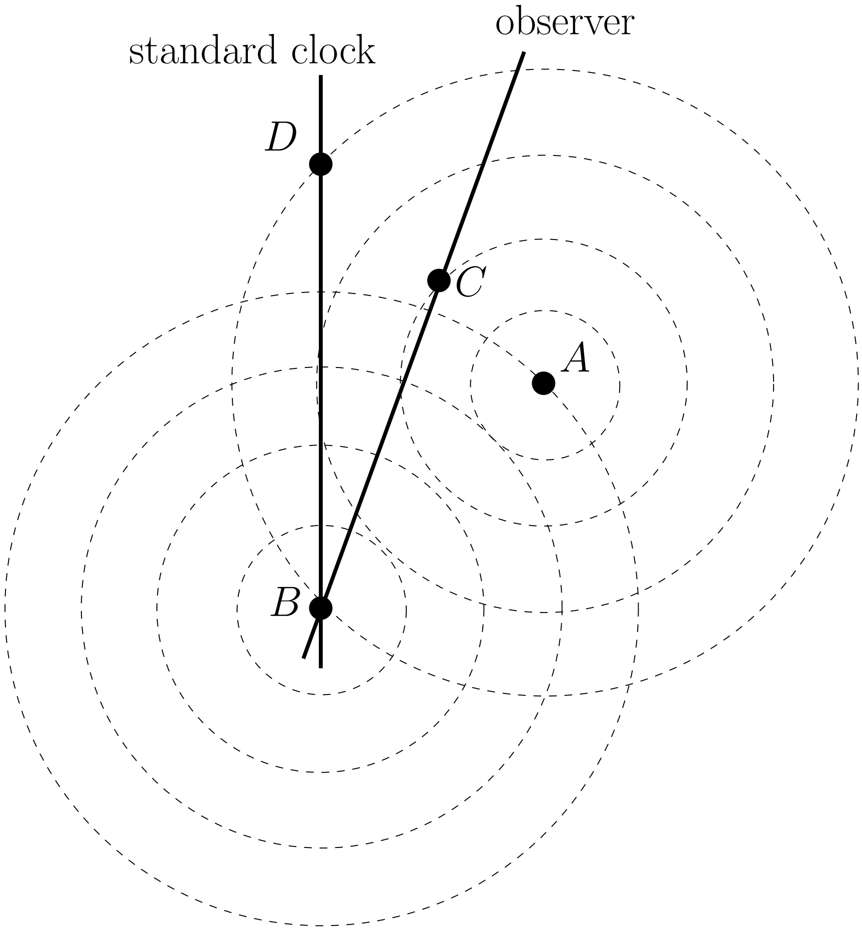


Figure 20: *When the observer meets the standard clock, a radio signal is emitted (event B). Event A is marked with the reflection of the signal. The reflected signal first arrives at the observer (event C) and then at the standard clock (event D)*

meets the standard clock (Fig. 20). Let $\tau(B) = 0$. Event A is marked with the reflection of the signal at time $\tau(A)$. The reflected signal first arrives at the observer (event C) and then at the standard clock (event D). By definition, $\tau(A) = \frac{\tau(D)}{2}$. We know that

$$v\tau(C) = \xi(A) - c\left(\tau(C) - \tau(A)\right)$$

where, by definition, $\xi(A) = c\tau(A)$. Therefore,

$$\tau(C) = \frac{2c\tau(A)}{c+v}$$

Taking into account assumption (E14), the observer's "clock"-reading at C is

$$t(C) \;=\; \tau(C)\sqrt{1-\frac{v^2}{c^2}}$$

Therefore, the "time" and "space" coordinates (s)he obtains is

$$\begin{aligned} t(A) &= \frac{t(C)}{2} = \frac{c\tau(A)}{c+v}\sqrt{1-\frac{v^2}{c^2}} \\ &= \frac{c\tau(A)(c-v)}{(c+v)(c-v)}\sqrt{1-\frac{v^2}{c^2}} \\ &= \frac{\tau(A)-\frac{v}{c^2}\xi(A)}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned} \tag{64}$$

and

$$x(A) = \frac{\xi(A)-v\tau(A)}{\sqrt{1-\frac{v^2}{c^2}}}$$

These are nothing but the corresponding Lorentz transformations.

The "time" and "space" coordinates defined in this way are nothing but the $\widetilde{\text{time}}$ and $\widetilde{\text{space}}$ coordinates $\left(\tilde{t}^{K'}(A), \tilde{x}^{K'}(A)\right)$ in Point **14**.

73. It is also an interesting question: How can a moving observer ascertain the absolute time and space tags of an arbitrary event A (in order, for example, to assign $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\left(\hat{t}^{K'}(A), \hat{x}^{K'}(A)\right)$ to A, by following the definitions in Point **14**)? This is actually very easy. For that we only need to equip the standard clock and the marking devices with functions similar to the ones described in Point **??**. In addition, let the standard clock be continuously writing and broadcasting a "log file", containing all the relevant information: when a signal was transmitted and when it was received back from which marker, etc. By reading off this "log file", the remote moving observer can reconstruct the absolute time and space tags of all events.

74. The practical details are, of course, not interesting. What is important is the in principle possibility that any observer can ascertain the absolute time and space tags of all events, by applying one and the same *etalon* clock and one and the same semantical convention. So, absolute time and space tags not only are not relative to the "reference frame" of an observer but they are not even relative to an observer. For, the only thing the absolute time tag is relative to is the *etalon* clock and the semantical convention by which the term is defined.

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To what extent are spatiotemporal concepts conventional?

75. As any other physical quantities of empirical meaning, absolute space and time tags are relative to the trivial semantical conventions by which they are defined. They are relative to the *etalon* clock-like process we have chosen in the universe; and to the particular way in which the space and time tags have been defined, including the usage of radio signals, the choice of "ε", etc.

But there are two things that do not follow from this kind of conventionality. On the one hand, it does not follow that these physical quantities cannot describe *objective* features of physical reality. On the other hand, it does not follow either that there are no *objective* constraints on the semantical conventions themselves. In this section we will discuss to what extent these objective constraints restrict the various components of the semantical convention defining absolute space and time tags.

76. There has been a long discussion in the literature about the conventionality of simultaneity. (See, for example, Reichenbach 1956; Bridgeman 1965; Winnie 1970; Grünbaum 1974; Salmon 1977; Malament 1977; Friedman 1983; Anderson *et al.* 1998; Minguzzi 2002; Ben-Yami 2006.) As it is obvious from (52), we chose the standard "$\varepsilon = \frac{1}{2}$-

synchronization". This choice was *a part of the trivial semantical convention* defining the term "absolute time tag". It is, therefore, prior to any claim about the one-way or even round-trip speed of electromagnetic signals, because there is no such a concept as "speed" prior to the definition of time and space tags; it is, of course, prior to "the metric of Minkowski space-time", in particular to the "light-cone structure of the Minkowski space-time", because we have no words to tell this structure prior to the space and time tags; and it is prior to the causal order of physical events, because—even if we could know this causal order prior to temporality—we cannot know in advance how causal order is related with temporal order (which we have defined here). It is actually prior to any discourse about two locuses in space, because there is no "space" (S_τ) prior to definition (A1) and there is no concept of a "persistent space locus" prior to definitions (A3) and (A8).

This all seems to support the arguments *for* the conventionality of simultaneity. But, as we will see, the situation is more complex.

77. To what extent is the choice of ε free? Before answering this question, it must be emphasized that changing the value of ε in (A1) changes the *semantical* convention; by choosing a different value of ε a different physical quantity would be called "absolute time". Since the term "absolute time" has been already used for the case of $\varepsilon = \frac{1}{2}$, let us call these other quantities absolute time$_\varepsilon$ (that is, time$\equiv$time$_{\frac{1}{2}}$).

So far, it seems, we are entirely free in the choice of the value of ε, that is in the choice of which objective feature of the physical reality—time$_\varepsilon$—we want to deal with. One might think that starting with some ε, that is with some time$_\varepsilon$ tags $\tau_\varepsilon(A)$ and the corresponding ε-simultaneity slices S_τ^ε, one finally obtains some space$_\varepsilon$ tags $\xi_1^\varepsilon(A)$, $\xi_2^\varepsilon(A)$, and $\xi_3^\varepsilon(A)$, corresponding to the given value of ε. This is true only if we can go through all the operational definitions (A1)–(A13), and all the empirical facts (E1)–(E13) are true for the given $\varepsilon \neq \frac{1}{2}$. This is, however, not necessarily the case. For, imagine we repeat the operations described in (A1), (A2) and (A3) with some $\varepsilon \neq \frac{1}{2}$, and obtain the concept of a (rest time sequence)$_\varepsilon$. Then, we encounter the question of whether empirical fact (E1) is true or not. Normally, in case of $\varepsilon = \frac{1}{2}$, we assumed that there exists a unique rest time sequence through every event. This assumption was based on result (RES1) of Point **70**, derived from our believed physical theories. But, a similar calculation in case of $\varepsilon \neq \frac{1}{2}$ leads to the following result:

(RES3) *Fact (E1) is never true if $\varepsilon \neq \frac{1}{2}$, no matter if the standard clock moves along an inertial world line, and no matter if the clock reads the proper time along its world line.*

Of course, whether or not (E1) is true is still an open *empirical* question in both the $\varepsilon = \frac{1}{2}$ and the $\varepsilon \neq \frac{1}{2}$ cases. Nevertheless, assuming that the future empirical findings will confirm what our present physical theories tell about (E1), there seems no way to build up the *spatial* concepts (rest$_\varepsilon$, distance$_\varepsilon$, space$_\varepsilon$ tags, etc.) operationally, if $\varepsilon \neq \frac{1}{2}$. And, given that our aim is to define not only the temporal but also the spatial concepts, this is a strong experimentally testable argument against the $\varepsilon \neq \frac{1}{2}$-synchronization.

78. Combining (RES1), (RES2), and (RES3), one can arrive at the following conclusions:

- The choice of the standard clock is not entirely conventional. It must be of "inertial trajectory" and also the clock's reading may have some restrictions, otherwise we cannot go through all the definitions (A1)–(A13); whether the standard clock satisfies these conditions is an empirically testable question. Still, one must recognize, that both the clock's trajectory and the clock's reading have some conventional degrees of freedom.

- The choice of Reichenbach's ε in the synchronization is not conventional. Its value is restricted to $\varepsilon = \frac{1}{2}$ by empirically testable conditions.

Az eddigiek összegzése (olvasmány)

79. The aim of our empiricist analysis was to free our accepted (relativistic) physical theories from the vicious circular and/or aprioristic pre-assumptions about space, time, reference frames, relativity principle, inertial motions, etc.; and to provide an empirical/operational foundation of the basic spatiotemporal concepts. These pre-assumptions proved themselves vicious indeed; it turned out that many of them are simply meaningless or false, or, at least, true only in some particular cases:

(I) **The basic postulate of special relativity theory, the relativity principle, does not hold in relativistic physics in general.** In classical mechanics, Galilean covariance and the principle of relativity are completely equivalent and hold for all possible dynamical processes. In contrast, in relativistic physics Lorentz covariance and the principle of relativity are not completely equivalent; Lorentz covariance in itself does not guarantee that the physical laws in question satisfy the relativity principle in general. Actually, the principle of relativity is a kind of thermodynamical principle: it only holds for the equilibrium quantities that characterize the thermodynamical equilibrium of the systems in question. In the light of this fact, there is no reason to regard Lorentz covariance as a fundamental symmetry of the laws of physics.

Besides our empiricist approach, our analysis of the relativity principle was based on what Bell calls "Lorentzian pedagogy". Namely, that the laws of physics in any one reference frame account for all physical phenomena, including the behavior of moving objects and the observations of moving observers. In other words, not only is the question of whether or not the relativity principle holds an empirical matter, not only is the validity of this "second order" law determined by the empirically confirmed "first order" laws of physics, but it is completely determined by the empirically confirmed laws of physics in a single reference frame.

(II) **In comparison with the pre-relativistic theories, special relativity theory tells us nothing new about the spatiotemporal features of the physical world; except the apparent differences on the surface of the words.** It turned out from the empirical definitions of the space and time tags of events that different physical quantities are called "time", and similarly, different physical quantities are called "distance" in special relativity and in classical physics. (We called them $\widetilde{\text{time}}$ and $\widetilde{\text{distance}}$ versus $\widehat{\text{time}}$ and $\widehat{\text{distance}}$.) Special relativity theory makes "novel" claims not about what were originally called "space" and "time" in pre-relativistic physics ($\widehat{\text{space}}$ and $\widehat{\text{time}}$), but about some different properties of physical reality ($\widetilde{\text{space}}$ and $\widetilde{\text{time}}$). On the other hand, the detailed calculations show that both special relativity and the pre-relativistic Lorentz theory have identical assertions about all of the four quantities $\widetilde{\text{time}}$, $\widetilde{\text{distance}}$, $\widehat{\text{time}}$, and $\widehat{\text{distance}}$.

The terminological distinction between $\widehat{\text{space}}$ and $\widehat{\text{time}}$ versus $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ was based on the following very weak "operationalist" premise: physical terms, assigned to measurable physical quantities, have different meanings if they have different empirical definitions.

(III) **Special relativity and the Lorentz theory are completely identical theories.** In terms of $\widehat{\text{space}}$, $\widehat{\text{time}}$, $\widetilde{\text{space}}$, and $\widetilde{\text{time}}$, our analysis shed more light on the relationship between special relativity and pre-relativistic physics. Actually, as we proved in Point **??**, special relativity and the Lorentz theory are completely identical in both senses, as theories about space-time and as theories about the behavior of moving physical objects. They have identical claims about whether or not relativistic deformations are real physical changes; they have identical claims about simultaneity, about the velocity of light; about the addition rules of velocities; about Galilean and Lorentz covariance; the two theories have identical predictions about the validity of the relativity principle; they have identical predictions about the result of the Michelson–Morley experiment; they both are logically independent of any hypothesis about the existence of aether. Contrary to the conventionalist thesis, there is no actual choice between the two theories, because, in the contemporary reconstruction, there are no two different theories.

(IV) The consequent non-circular definition of spatiotemporal concepts is highly non-trivial; it leads to the concepts of absolute space and time; and raises open empirical questions. In the definitions of $\widehat{\text{space}}$, $\widehat{\text{time}}$, $\widetilde{\text{space}}$, and $\widetilde{\text{time}}$ tags we just reconstructed how "space" and "time" tags of events were understood in pre-relativistic physics and special relativity theory. However, neither the classical nor the relativistic definitions are trouble free. They are based on vicious circularities and aprioristic pre-assumptions about reference frames, inertial motions, etc. To avoid these difficulties we tried to define the space and time tags of events by means of a consequent "standard clock + light signals" method. As it turned out, the task is not trivial. (Traditionally speaking, the definition of space and time tags is a non-trivial problem even "in a single inertial frame of reference".) The analysis of the problem led to the following conclusions: 1) Although the space and time tags so obtained are relative to the trivial semantical convention by which we define the meaning of the terms in question, they are absolute in the sense that they are not relative to a reference frame but prior to any reference frame. 2) "Proper" time is what the *etalon* clock reads, by definition. However, conditions (E1)–(E13) imply some restrictions on the possible choice of the standard clock; first of all, it must be inertial. 3) It is therefore meaningless to talk about "non-inertial reference frame", "space-

> time coordinates (tags) defined/measured by an accelerating or rotating observer", and the likes. 4) Whether the standard clock used in the contemporary physical laboratories is appropriate at all for the definition of the space and time tags is still an open empirical question. Finally, 5) we pointed out that the usual arguments, pro and con, about the conventionality of simultaneity are pointless in a real circularity-free operational context; there is no additional conventionality attached to simultaneity over and above the original choice of the value of ε in definition (A1), as a part of the trivial semantical convention; and this freedom merely consists in the choice of a single real number between 0 and 1; but it cannot depend on "space coordinates" and/or "direction", simply because there are no such things as "space coordinates" and "direction" prior to fixing the value of ε. On the other hand, however, we gave an empirically testable argument against the $\varepsilon \neq \frac{1}{2}$-synchronization.

These results of our analysis are astounding, not only in contrast to some long-accepted views held in relativistic physics, but also in contrast to how philosophers and historians of science think about the "revolution" brought about by Einstein's special relativity.

<Olvasmány vége>

80. **From philosophical point of view**, the most important fact is that special relativistic physics could manage equally well with the classical Galileo-invariant conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$. What is more, relativistic physics can be formulated even in terms of *absolute* (inertial-observer-independent) space and time tags; that is, in terms of the *only* spatiotemporal concepts that have consistent non-circular empirical definitions; which are logically/operationally prior to the relative (inertial-observer-dependent) concepts, and without which the relative concepts cannot be even defined.

At first sight, this recognition significantly affects a long series of metaphysical arguments that are based on special relativity theory. Objective becoming and time flow, reality of time, A-theory versus B-theory of time, openness of future versus determinism, endurance versus perdurance, presentism versus eternalism, etc., are typical metaphysical issues in the discussion of which the arguments form special relativity have been especially influential. The arguments in question operate with the non-objectivity (that is, reference frame dependence) of simultaneity; with the non-objectivity of separation between past, present, and future. These ideas find support in Einstein's own utterances. For example, shortly before his own death, Einstein wrote to the family of his friend Michele Besso, who had just died:

> So in quitting this world he has once again preceded me by a little. That doesn't mean anything. For those of us who believe in physics, this separation between past, present, and future is only an illusion, however tenacious. (Wang 1995)

81. Let me illustrate the general structure of these arguments with the example of the problem of objective becoming. In a common sense view, the ontological status of things in the past and future differs from that of present things. However, from a scientific/philosophical point of view, it is far from obvious whether this is truly the case. For the "becoming" of physical events in our temporal awareness does not itself guarantee that becoming has a mind-independent status. Is "becoming" then an objective, mind-independent feature of physical events, as the common-sense view supposes it to be?

Many authors (for example, Rietdijk 1966; 1976; Maxwell 1985; Putnam 1967) argued that relativity theory is incompatible with objective becoming (becoming determined, becoming existing, etc.). I do not want to recall here the various formulations of this argument. Instead, I will try to formulate the general schema of the argument in an abstract way, in order to avoid the previous clarification of such concepts as "existing", "real", "determined", "settled", etc.

Denote $p \sim_\gamma q$ if p and q are simultaneous relative to inertial observer γ. I shall also use the standard notations $J^+(p)$, $J^-(p)$ for the causal future, causal past of $p \in \mathbb{M}^4$ (e.g., Wald 1984, p. 188). Introduce the following notation: $C(p) = \mathbb{M}^4 \setminus (J^+(p) \cup J^-(p))$. One can extend these notions for an arbitrary subset $X \subseteq \mathbb{M}^4$:

$$J^\pm(X) = \bigcup_{p \in X} J^\pm(p)$$

$$C^\pm(X) = \bigcup_{p \in X} C^\pm(p)$$

Putnam's Theorem Let A be a non-empty subset of $\mathbb{M}^4$, such that

$$(\forall p, q)\left[(p \in A \;\&\; (\exists \gamma)\left[q \sim_\gamma p\right]) \rightarrow q \in A\right] \tag{65}$$

Then $A = \mathbb{M}^4$.

Proof If $p \in A$ then $C(p) \subseteq A$. Indeed, if $r \in C(p)$ then p and r are spatially separated. Therefore, there exists an inertial observer γ such that $r \sim_\gamma p$, that implies $r \in A$. Similarly, $C(C(p)) \subseteq A$. On the other hand, in Minkowski's space-time, $(\forall p)\left[C(C(p)) = \mathbb{M}^4\right]$. Consequently, $A = \mathbb{M}^4$.

So, no matter how the property defining set A is exactly specified, if the assignment of this property to space-time events satisfies condition (65), then—according to the theorem—all space-time events share this property.

Consider only one example. Assume that

1. There is, as presentism claims, a subset A of space-time events, singled out as the set of events belonging to the "existing" entities, more exactly, to the entities "co-existing with us, now".

2. If p belongs to an "existing" entity and p and q are simultaneous to any observer—that is to say, if there is an observer who witnesses their "co-existence"—then q also belongs to an "existing" entity.

Then, the lemma says, $A = \mathbb{M}^4$. In other words, in their ontological status, there cannot be any difference between the events of space-time.

82. One might think that this kind of argument from contemporary physics loses its strength if, as it turned out from our analysis, relativistic physics is not necessarily committed to the relative spatiotemporal concepts $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$, and can be equally well formulated in terms of $\widehat{\text{space}}$ and $\widehat{\text{time}}$ or absolute space and time. And this is true.

But before jumping to the other extreme I hasten to add that this is true also for the opposite argument from the neo-Lorentzian interpretation (e.g., Craig 2001), and because of the *same* reason. They both are based on the same delusion: namely, that there are two different theories having different claims about the spatiotemporal features of reality. For, if Reichenbach's dictum is true that "there is no any other way to solve the problem of time than the way through physics", it is even more true that one has to take into account what physical theories mean by time (and space) in various contexts. This is what we actually did. And the conclusion was that special relativity and the Lorentz theory provide completely identical accounts of $\widehat{\text{space}}$, $\widehat{\text{time}}$, $\widetilde{\text{space}}$, $\widetilde{\text{time}}$ (and absolute space and absolute time if (E1)–(E14) are true). So if the fact of the observer-dependence of $\widetilde{\text{time}}$ provides enough reason to draw the metaphysical conclusion from special relativity that there is no objective becoming, then the same fact provides enough reason to draw the same metaphysical conclusion from the Lorentz theory. And *vice versa*, if presentism finds support in the observer-independence of $\widehat{\text{time}}$ stated by the Lorentz theory, then the observer-independence of $\widehat{\text{time}}$ stated by special relativity provides the same support for presentism.

In other words, the metaphysical conclusions depend not on the physicist's alleged choice between the Lorentz theory and special relativity, but on the philosopher's choice whether $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ or $\widehat{\text{space}}$ and $\widehat{\text{time}}$ (or absolute space and absolute time) are the metaphysically relevant spatiotemporal notions.

83. Does this all mean that none of the above mentioned metaphysical theses can find support in what physics claims about space and time? Not exactly. As we will see, the actual metaphysical gap is not between non-relativistic versus relativistic theories, but between the Newtonian/Kantian aprioristic concept of "true and mathematical time" versus the empirically defined time of physics. Here is how Newton describes the distinction between "absolute" and "relative" time:

> Absolute, true, and mathematical time, of itself, and from its own nature flows equably without regard to anything external, and by another name is called duration: relative, apparent, and common time, is some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time; such as an hour, a day, a month, a year. (in Scholium, chapter "Definitions" of the *Principia*.)

However, even the observer-independent absolute time is "relative" according to Newton's terminology. It is an ordinary *empirically* defined physical quantity (see (A1) in Point **53**), and as such cannot be "absolute" in the Newtonian sense. (It is not really clear how any empirically meaningful physical quantity can be "absolute" in the Newtonian sense.) For, the empirical definition of a physical quantity is basically of conventional nature; even if the physical quantity in question describes an objective feature of physical reality; and even if there are objective constraints on the range of the possible conventions.

84. The choice of an inertial *etalon* clock is, for example, an inevitable conventional element of the definition providing meaning for the physical term "absolute time". As we will see, this element of semantical conventionality leads to the same metaphysical conclusions as those that are meant to be supported by the argument from special relativity, no matter if the observer-independent absolute space and absolute time are regarded as the relevant spatiotemporal physical concepts.

For, from the metaphysical point of view, the essence of the argument from special relativity is that such fundamental concepts as "becoming determined", "becoming existing", etc., must be independent of *any contingent human perspective*. Namely, they must be independent of the choice of the reference frame of a particular observer. In other words, all these perspectives are metaphysically equivalent. This equivalence is expressed by conditions (65) and (**??**).

By the same token, however, these fundamental metaphysical concepts must be independent of any other instances of contingent human perspective; for example, they must be independent of the conventional ingredients of spatiotemporal notions, such as the choice of the *etalon* clock. Accordingly, Putnam's Theorem can be modified by exploiting the conventionality of the choice of an *etalon* clock.

85. The following assumption will be made:

> **Assumption** We chose an *etalon* clock and defined absolute time tags, according to (A1). The *etalon* clock proved inertial and it was possible to go through definitions (A2)–(A13). That is, all the empirical facts (E1)–(E13) proved true. In addition, empirical fact (E14) proved true.

Consequently, the behaviors of moving physical objects are exactly the same as described by our believed special relativistic physical theories; the restricted relativity principle holds; and the spatiotemporal relations, in terms of the corresponding $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags, can be described by the Minkowski geometry—at least in our neighborhood. For example, it follows from the above assumption that an identical physical copy of the *etalon* clock, moving along a time-like straight line Γ of absolute velocity $\mathbf{v}$, will read $\chi(t) = t\sqrt{1 - \mathbf{v}^2/c^2}$, that is the Minkowskian path-length of Γ. (This is what we call $\widetilde{\text{time}}$ relative to the observer of world line Γ.)

Thus, under the above assumption, on the bases of our believed special relativistic theories, we can predict what the absolute time and space tags would be like if the *etalon* clock were different; if it were an arbitrary "clock-like" equipment moving along an arbitrary inertial trajectory Γ, reading an arbitrary "time" $\chi(t)$.

86. Let us denote by τ_Γ the "absolute time tag" defined via operational procedure (A1), by means of an equipment which is an identical copy of the *etalon* clock, moving along inertial trajectory Γ of velocity $\mathbf{v}$, and, therefore, reading a "time" equal to $t\sqrt{1-\mathbf{v}^2/c^2}$. (Let us call it absolute time$_\Gamma$ tag.) Denote $p \sim_\Gamma q$ if p and q are τ_Γ-absolute-simultaneous, that is $\tau_\Gamma(p) = \tau_\Gamma(q)$.

Putnam's Theorem v2 Let A be a non-empty set of events, such that

$$(\forall p, q)\left[\left(p \in A \,\&\, (\exists \Gamma)\left[q \sim_\Gamma p\right]\right) \rightarrow q \in A\right] \tag{66}$$

Then $A = \mathbb{M}^4$.

Proof If $p \in A$ then $C(p) \subseteq A$. Indeed, if $r \in C(p)$ then p and r are spatially separated. Therefore, there exists an inertial clock Γ—as an *etalon* clock in (A1)—such that $r \sim_\Gamma p$, that implies $r \in A$. (To prove the theorem, it is enough to vary only the inertial trajectory of the clock.) Similarly, $C(C(p)) \subseteq A$. On the other hand, in a Minkowski's geometry, $(\forall p)\left[C(C(p)) = \mathbb{M}^4\right]$. Consequently, $A = \mathbb{M}^4$.

87. Mindebből tehát az következik, hogy a prezentistának (és metafizikai rokonainak) két választása van:

1. **Meg kell indokolnia az *etalon* óra egyetlen konkrét választásának metafizikai kitüntetettségét.** It is however not easy to explain where this privileged status comes from. For, no doubt, the rotation of the Earth and its motion around the Sun have been of vital importance for living beings throughout the biological evolution. This explains why we traditionally describe the physical world in terms of a "time" defined by means of an *etalon* clock running in approximate tune with these *particular* astronomical phenomena; and it also explains the "pre-established harmony" between the physical time and the Kantian "pure form of our sensible intuition" of time. But it seems quite implausible that—of all the possible clock-like phenomena in the universe—the Earth's

motion is exactly the one which has a distinguished *metaphysical* significance (not to mention the lucky choice of electromagnetic signals in definition (A1)); that our "time" not only reflects how the sunlit area changes across the Earth, not only reflects a useful objective feature of the physical world by which we and our fellow creatures on the Earth can organize and tune our life—when to be born, when to eat, when to go to the badminton club, when to sleep, and when to die—but it is exactly *the* parameter by which the things become existing, become determined, and so on.

2. (Sokkal kínálkozóbb lehetőség:) **Bele kell törődnie abba, hogy az olyan temporális kifejezéseink, mint „létrejön", „bekövetkezik", „van", „determinálódik", „megszűnik", stb. – bármennyire is szeretnénk ettől metafizikailag elvonatkoztatni – relatívak e kifejezéseknek jelentést adó szemantikai konvencióra nézve, vagyis arra az empirikus/operácionális eljárásra nézve, melynek segítségével a fizikus az „idő" fogalmát definiálja.** Vigasztalásul: a temporális fogalmainknak ez az elkerülhetetlen konvencionális eleme nem zárja ki, hogy e fogalmak a fizikai világ objektív tulajdonságát írják le. Más szemantikai konvenció esetén azonban a fizikai világ más objektív tulajdonságával foglalkoznánk.

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