Quantum mechanics without operational equivalence

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Main messages

Ontological models for quantum mechanics can be different with respect to causal structure, contextuality, fine-tuning, etc. when operators are realized by different measurements

Outline

I. Operational theories and ontological models

II. The Bridgmannian perspective

III. Trivialization: removing simultaneous measurements

Operational theories and ontological models

- **Operational theory:**
  - $\{P, P'...\}$: preparations
  - $\{M, M'...\}$: measurements
  - $\{X, X'...\}$: outcomes

$$\{p(X|M \wedge P) : \text{ for all } M, P\}$$

- **Ontological (hidden variable) model:**
  - $\{\Lambda, \Lambda'...\}$: ontic (hidden) states

$$\{p(\Lambda|P) : \text{ for all } P\}$$

$$\{p(X|M \wedge \Lambda) : \text{ for all } M, \Lambda\}$$

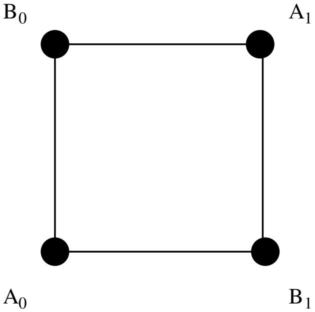
Operational theories and ontological models

$$p(X|M \wedge P) = \sum_{\Lambda} p(X|M \wedge \Lambda)\, p(\Lambda|P)$$

An operational theory

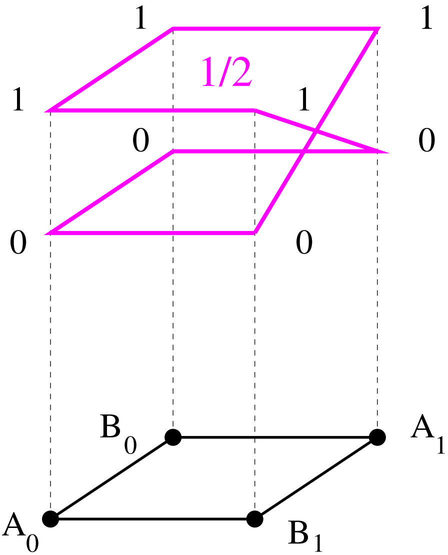
- **Measurements:**

$$\{\underbrace{A_0,\ A_1,\ B_0,\ B_1}_{\text{Basic}},\ \underbrace{A_0 \wedge B_0,\ A_0 \wedge B_1,\ A_1 \wedge B_0,\ A_1 \wedge B_1}_{\text{Simultaneous}}\}$$

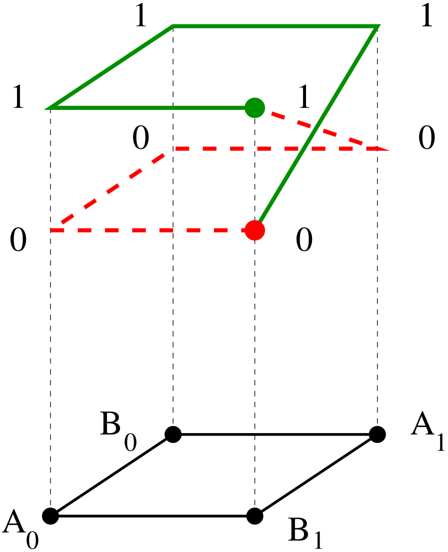


- **Outcomes**: $X, Y = 0, 1$

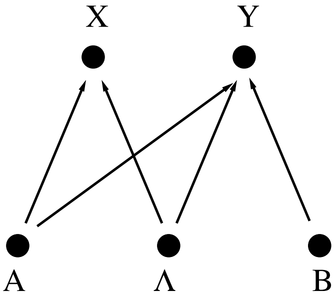
An operational theory: the PR box



An ontological model: the PR box

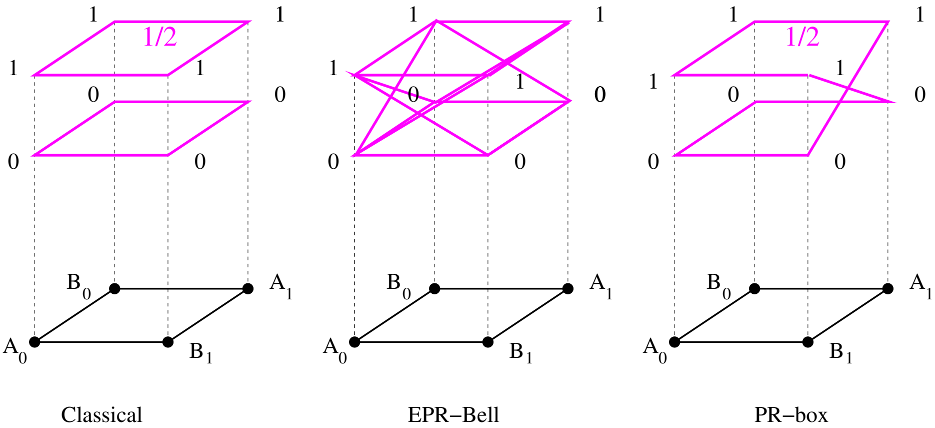


Causal structure of the PR box

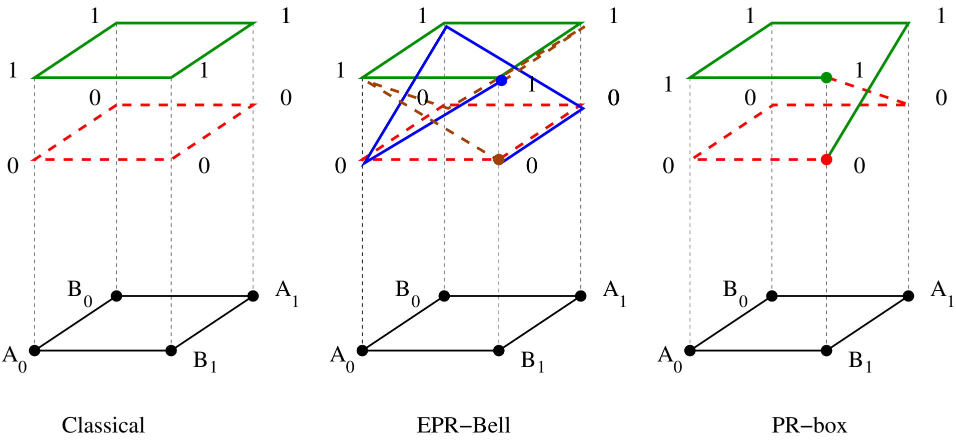


- CHSH inequality
- Contextuality
- Fine-tuning

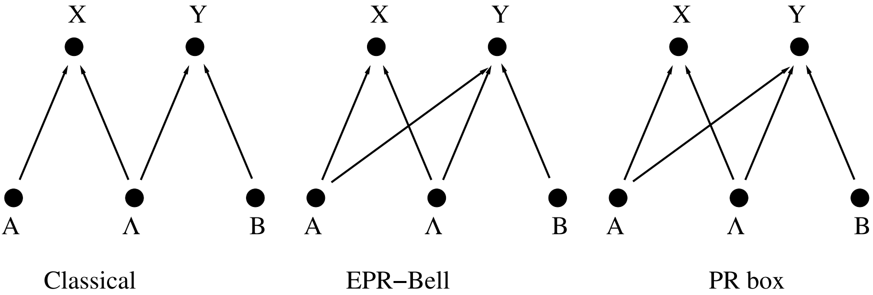
Three operational theories



Three ontological models



Causal structure of the models



- CHSH inequality
- Contextuality
- Fine-tuning

- **Representation:**

$$p(X|M \wedge P) = \mathrm{Tr}(\boldsymbol{\rho}\,\mathbf{P})$$

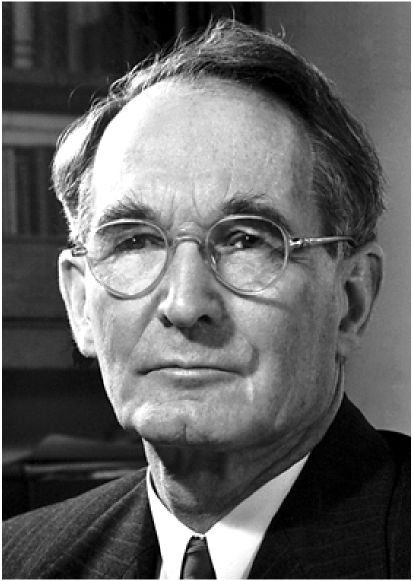
- $\{P, P'...\} \longrightarrow \{\boldsymbol{\rho}, \boldsymbol{\rho}'...\}$: density operators
- $\{M, M'...\} \longrightarrow \{\mathbf{O}, \mathbf{O}'...\}$: self-adjoint operators
- $\{X, X'...\} \longrightarrow \{\mathbf{P}, \mathbf{P}'...\}$: spectral projections

Operational equivalence

- $M_1 \sim M_2 \quad \text{if} \quad p(X|M_1 \wedge P) = p(X|M_2 \wedge P)$ for all P
- Then M_1 and M_2 are represented by the same operator:

$$p(X|M_1 \wedge P) = p(X|M_2 \wedge P) = \mathrm{Tr}(\boldsymbol{\rho}\,\mathbf{P})$$

Percy Bridgman (1882-1961)



Bridgman's question

When do two measurements measure the same observable?

Observables

	Bridgmannian	**Standard**
Operator:	$\mathbf{O}$	$\mathbf{O}$
	$\swarrow \quad \searrow$	$\downarrow$
Observable:	$\mathcal{O}_1 \qquad \mathcal{O}_2$	$\mathcal{O}$
	$\downarrow \qquad \downarrow$	$\swarrow \quad \searrow$
Measurement:	$M_1 \quad \sim \quad M_2$	$M_1 \quad \sim \quad M_2$

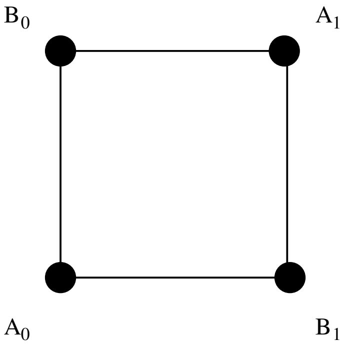
Project

- Keep the operators, change the measurements
- Replace measurements by operationally equivalent measurements
- Remove simultaneous measurements
- Show that the new operational theory is different with respect to the causal structure, contextuality, fine-tuning, etc.

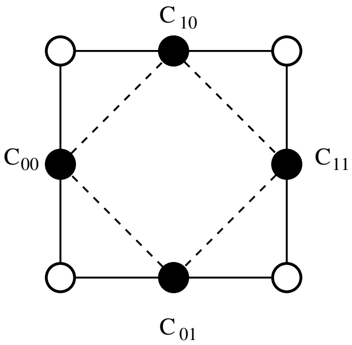
Trivialization

Replace the simultaneous measurements by operationally equivalent new basic measurements

Trivialization



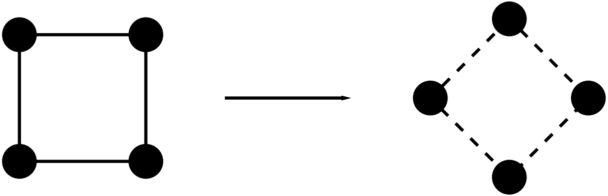
Trivialization



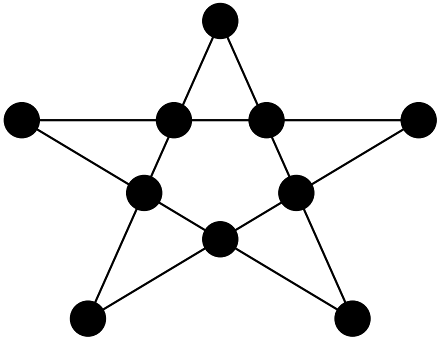
- The old basic basic measurements will be operationally equivalent to certain marginalizations of the new basic measurements

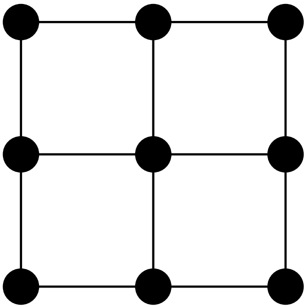
Trivialization

$$
\begin{aligned}
\textbf{Non-trivial theory} \quad &\longrightarrow \quad \textbf{Trivial theory} \\
\{A_0,\ A_1,\ B_0,\ B_1,\ A_0 \wedge B_0 \quad &\longrightarrow \quad \{C_{00},\ C_{01},\ C_{10},\ C_{11}\} \\
A_0 \wedge B_1,\ A_1 \wedge B_0,\ A_1 \wedge B_1\} \quad & \quad\quad\quad\ \ C_{00}^{(1)} \sim C_{01}^{(1)} \sim A_0 \\
& \quad\quad\quad\ \ C_{10}^{(1)} \sim C_{11}^{(1)} \sim A_1 \\
& \quad\quad\quad\ \ C_{00}^{(2)} \sim C_{10}^{(2)} \sim B_0 \\
& \quad\quad\quad\ \ C_{01}^{(2)} \sim C_{11}^{(2)} \sim B_1
\end{aligned}
$$

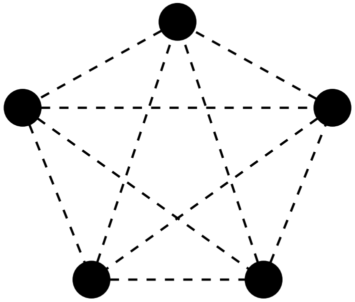


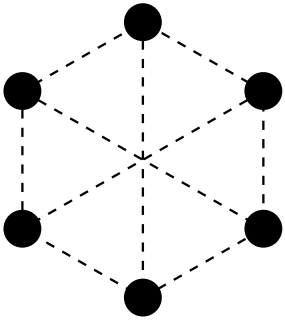
The GHZ and the Peres-Mermin graph



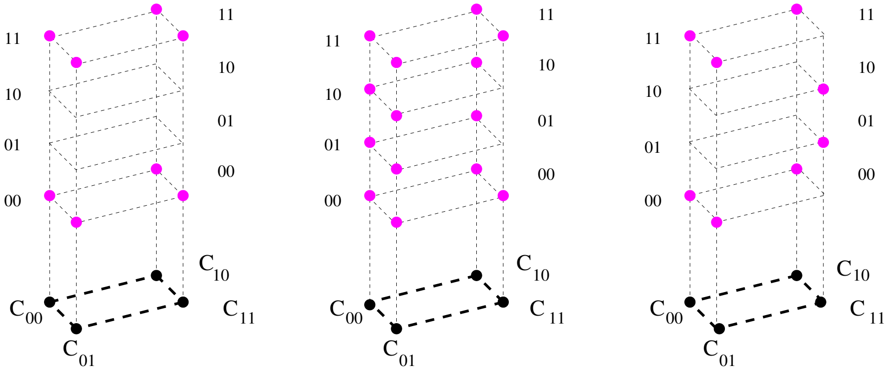


The GHZ and the Peres-Mermin line graph



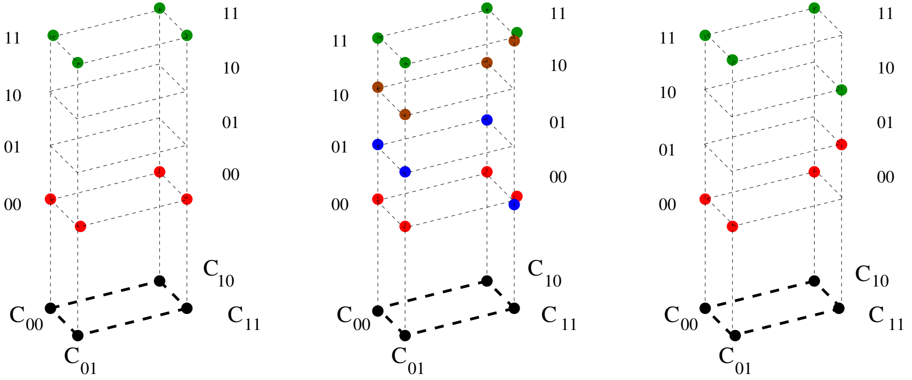


Three operational theories



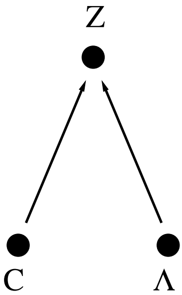
Classical EPR−Bell PR−box

Three ontological models



Classical — EPR–Bell — PR–box

Causal structure of the models



- CHSH inequality
- Contextuality
- Fine-tuning

Quantum mechanics

$$p(X, Y|A, B, P_{\text{EPR}}) \quad = \quad \langle \Psi_s | (\mathbf{X}^{\mathbf{A}} \otimes \mathbf{Y}^{\mathbf{B}}) \Psi_s \rangle$$

Observables

Operator: $\mathbf{A_0} \otimes \mathbf{B_0}$

$$\swarrow \qquad \searrow$$

Measurement: $A_0 \wedge B_0 \qquad \sim \qquad C_{00}$

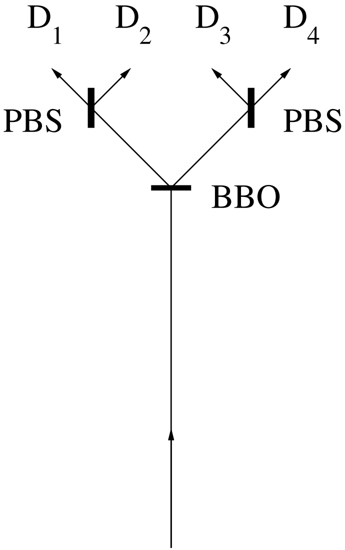
Quantum mechanics

$A_0 \wedge B_0$: Measure the linear polarization of the left photon along a transverse axis a_0

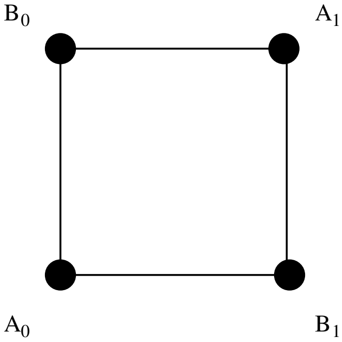
and

measure the linear polarization of the right photon along a transverse axis b_0

Quantum mechanics

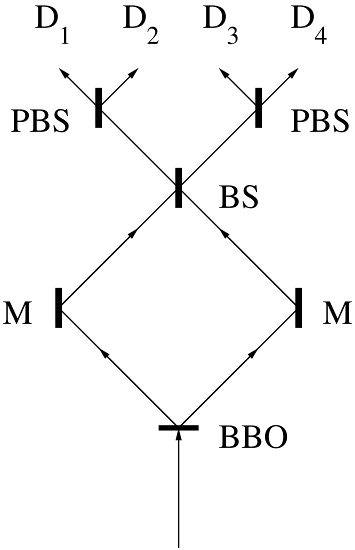


Quantum mechanics

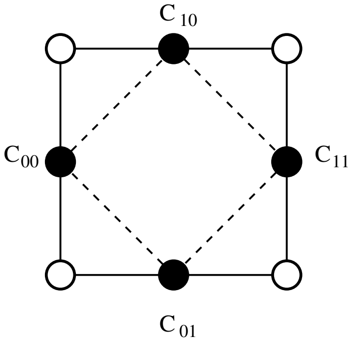


C_{00}: Perform a *global* polarization measurement on the photon pair with four outcomes corresponding to the four eigenvectors of $\mathbf{A_0} \otimes \mathbf{B_0}$ in $H_2 \otimes H_2$

Quantum mechanics



Quantum mechanics



Conclusions

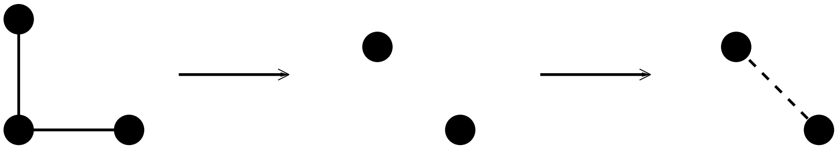
The realization of an operator in quantum mechanics by different measurements can give rise to different ontological models with respect to contextuality, causal structure, fine-tuning, etc.

More on that . . .

- Gábor Hofer-Szabó, "Quantum mechanics without operational equivalence" (in preparation).

Trivialization of a simple theory

Non-trivial theory	$\longrightarrow$	Trivial theory
$\{M_1, M_2, M_3, M_1 \wedge M_2, M_1 \wedge M_3\}$	$\longrightarrow$	$\{M_{12}, M_{13}\}$
		$M_{12} \sim M_1 \wedge M_2$
		$M_{13} \sim M_1 \wedge M_3$
		$M_{12}^{(1)} \sim M_{13}^{(1)} \sim M_1$



Noncontextuality

An ontological model for QM is **noncontextual** if

- every ontic state determines the probability distribution of outcomes of every measurement independently of what other measurements are simultaneously performed **(simultaneous noncontextuality)**
- any two measurements which are represented by the same self-adjoint operator have the same probability distribution of outcomes in every ontic state **(measurement noncontextuality)**

Noncontextuality

- **Simultaneous noncontextuality:**

$$p(X|M \wedge \Lambda) = p(X|M \wedge M' \wedge \Lambda) \qquad \text{for all } \Lambda$$

- **Measurement noncontextuality:**

$$\text{If} \quad p(X|M \wedge P) = p(X'|M' \wedge P) \qquad \text{for all } P$$

$$\text{then} \quad p(X|M \wedge \Lambda) = p(X'|M' \wedge \Lambda) \qquad \text{for all } \Lambda$$