Einstein meets von Neumann: Operational independence and operational separability in algebraic quantum field theory

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Main Message

Einstein and von Neumann meet in algebraic relativistic quantum field theory in the following metaphorical sense:

- AQFT emerged in the late fifties/early sixties and was based on the theory of “rings of operators", which von Neumann established in 1935-1940 (partly in collaboration with J. Murray).
- In the years 1936-1949 Einstein criticized standard, non-relativistic quantum mechanics, arguing that it does not satisfy certain criteria that he regarded as necessary for any theory to be compatible with a field theoretical paradigm.
- AQFT satisfies those criteria and hence it can be viewed as a theory in which the mathematical machinery created by von Neumann made it possible to express in a mathematically explicit manner the physical intuition about field theory formulated by Einstein.

Two-parts of Main Message

Historical:

(controversial)

Algebraic, relativistic, local quantum field theory
is compatible with the field theoretical paradigm
Einstein articulated in his critique of standard,
non-relativistic quantum mechanics
(mainly) because

Systematic:

(uncontroversial ?)

operational independence and operational separability
(independent notions that are interesting in their own right)
typically hold in
algebraic quantum field theory

Outline

- Einstein's 1948 description of the field theoretical paradigm (quotations)
- Interpretation of field theoretical paradigm
  Three requirements:
  Spatio-temporality, Independence, Local Operations
- **Spatio temporality**: main idea of AQFT
- **Independence** in AQFT – review
- **Local Operations**
  - The notion of operational separability in AQFT
  - Operational independence and operational separability
  - Operational separability holds in AQFT

Einstein contrasting QM and field theory

If one asks what is characteristic of the realm of physical ideas independently of the quantum theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, i.e. ideas are posited of things that claim a ‘real existence’ independent of the perceiving subject (bodies, fields, etc.), and these ideas are, on the other hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a spacetime continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things ‘lie in different parts of space’.

Einstein contrasting QM and field theory

Without such an assumption of mutually independent existence (the 'being-thus') of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible. Nor does one see how physical laws could be formulated and tested without such a clean separation. Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four dimensional) space-elements the elementary things existing independently of one another that it takes as basic as well as the elementary laws it postulates for them.

For the relative independence of spatially distant things (A and B), this idea is characteristic: an external influence on A has no immediate effect on B; this is known as the 'principle of local action', which is applied consistently only in field theory. The complete suspension of this basic principle would make impossible the idea of the existence of (quasi-)closed systems and, thereby, the establishment of empirically testable laws in the sense familiar to us.

Einstein contrasting QM and field theory

Matters are different, however, if one seeks to hold on principle II – the autonomous existence of the real states of affairs present in two separated parts of space R_1 and R_2 – simultaneously with the principles of quantum mechanics. In our example the complete measurement on S_1 of course implies a physical interference which only effects the portion of space R_1. But such an interference cannot immediately influence the physically real in the distant portion of space R_2. From that it would follow that every measurement regarding S_2 which we are able to make on the basis of a complete measurement on S_1 must also hold for the system S_2 if, after all, no measurement whatsoever ensued on S_1. That would mean that for S_2 all statements that can be derived from the postulation of ψ_2 or ψ_2', etc. must hold simultaneously. This is naturally impossible, if ψ_2, ψ_2', are supposed to signify mutually distinct real states of affairs of S_2,...

A. Einstein: Quantenmechanik un Wirklichkeit, Dialectica ***2*** *(1948) 320-324*

translation by D. Howard

(blue my emphasis, red original emphasis)

Einstein’s three major points

Three requirements for a physical theory to be compatible with a field theoretical paradigm:

Spatio-temporality “... physical things [...] are conceived of as being arranged in a spacetime continuum..."

Independence “... essential for this arrangement of the things introduced in physics is that, at a specific time, these things claim an existence independent of one another, insofar as these things ‘lie in different parts of space’."

Local Operation “... an external influence on A has no immediate effect on B; this is known as the ‘principle of local action’";

“... measurement on S_1 of course implies a physical interference which only effects the portion of space R_1. But such an interference cannot immediately influence the physically real in the distant portion of space R_2."

Three claims

Algebraic relativistic quantum field theory (AQFT)
satisfies all three requirements Einstein formulates:

- **Spatio-temporality**

  Algebraic quantum field theory:

  Spacetime $\supset V \mapsto \mathcal{A}(V)$ C^*-algebra

  $\mathcal{A}(V)$ = set of observables measurable in spacetime region V

  Local net with physically motivated properties

  isotony, local commutativity, existence of vacuum state with the spectrum condition, weak additivity

- **Independence**

  There is a rich hierarchy of independence notions that hold for $\mathcal{A}(V_1), \mathcal{A}(V_2)$ with V_1 and V_2 spacelike separated (see next)

  C^*- and W^*-independence, logical independence, split property etc.

- **Local Operation**

  Will be defined precisely and argued that it holds

General idea of independence

Assume that S_1 and S_2 are two subsystems of a larger system S

- Anything which is possible in principle for S_1 as a system
  in its own right
  and
- anything which is possible in principle for S_2 as a system
  in its own right
  are
- also jointly possible in principle for the pair (S_1, S_2) viewed as subsystems of S

C^*-independence

Definition A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{C}$ is called C^*-independent if for any state ϕ_1 on $\mathcal{A}_1$ and for any state ϕ_2 on $\mathcal{A}_2$ there exists a state ϕ on $\mathcal{C}$ such that

$$\begin{aligned} \phi(X) &= \phi_1(X) \quad \text{for any } X \in \mathcal{A}_1 \\ \phi(Y) &= \phi_2(Y) \quad \text{for any } Y \in \mathcal{A}_2 \end{aligned}$$

Any two partial (C^*-) states can be jointly prepared

C^*-independence in the product sense

Definition A pair $(\mathcal{A}_1, \mathcal{A}_2)$ of C^*-subalgebras of C^*-algebra $\mathcal{C}$ is called C^*-independent in the product sense if the map

$$\eta(XY) \doteq X \otimes_{min} Y$$

extends to an C^*-isomorphism of $\mathcal{A}_1 \vee \mathcal{A}_2$ with $\mathcal{A}_1 \otimes_{min} \mathcal{A}_2$

W^*-independence

Definition Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{M}$ are called W^*-independent if for any normal state ϕ_1 on $\mathcal{N}_1$ and for any normal state ϕ_2 on $\mathcal{N}_2$ there exists a normal state ϕ on $\mathcal{M}$ such that

$$\begin{aligned} \phi(X) &= \phi_1(X) \quad \text{for any} \quad X \in \mathcal{N}_1 \\ \phi(Y) &= \phi_2(Y) \quad \text{for any} \quad Y \in \mathcal{N}_2 \end{aligned}$$

Any two partial normal states
can be jointly prepared as a normal state

W^*-independence in the product sense

Definition Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{M}$ are called W^*-independent in the product sense if for any normal state ϕ_1 on $\mathcal{N}_1$ and for any normal state ϕ_2 on $\mathcal{N}_2$ there exists a normal product state ϕ on $\mathcal{M}$, i.e. a normal state ϕ on $\mathcal{M}$ such that

$$\phi(XY) = \phi_1(X)\phi_2(Y) \text{ for any } \quad X \in \mathcal{N}_1, Y \in \mathcal{N}_2$$

Any two partial normal states

can be jointly prepared as a normal product state

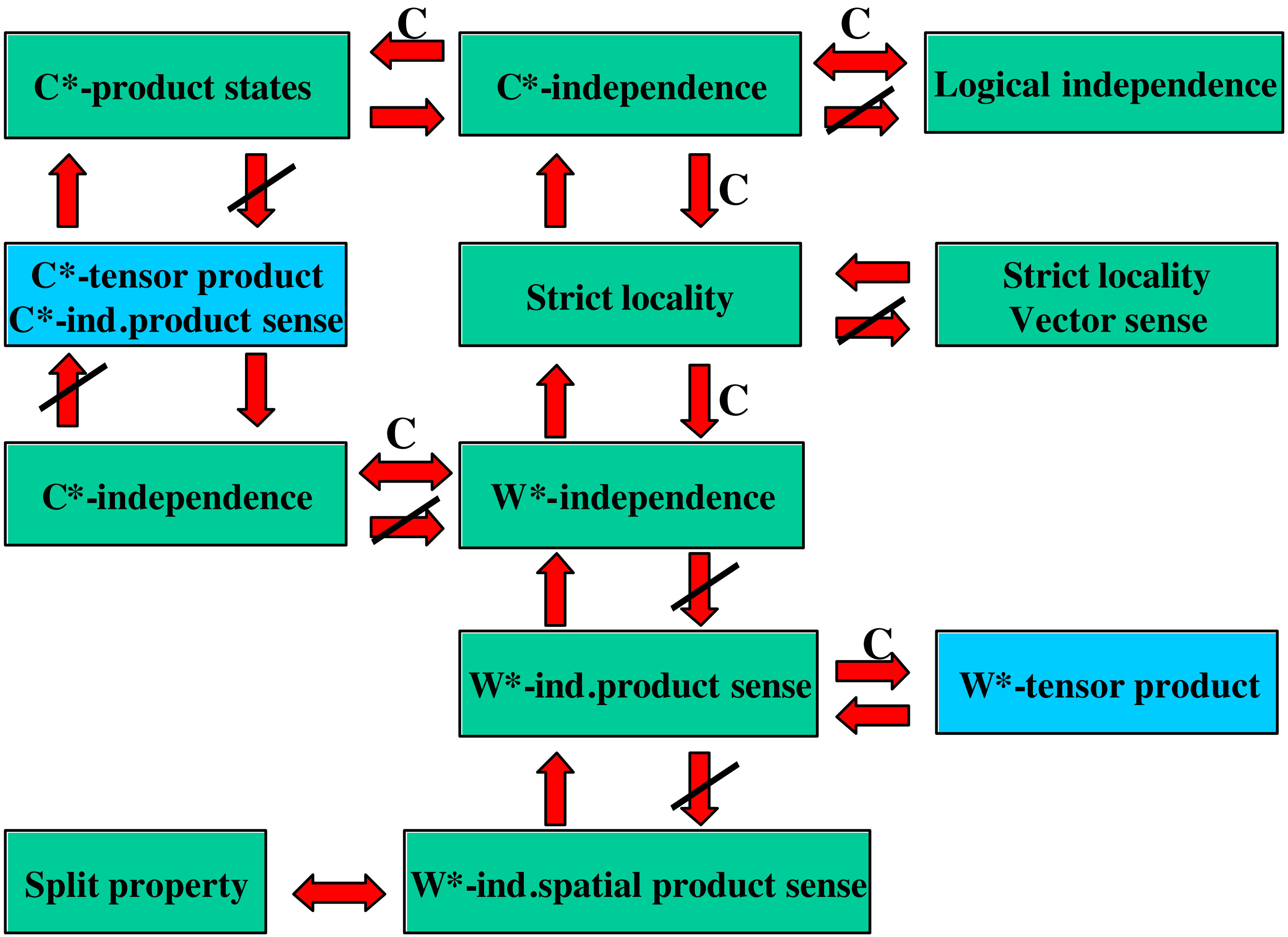
W^*-independence – spatial product sense

Definition Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{M}$ are called W^*-independent in the spatial product sense if there exist faithful normal representations $(\pi_1, \mathcal{H}_1)$ of $\mathcal{N}_1$ and $(\pi_2, \mathcal{H}_2)$ of $\mathcal{N}_2$ such that the map

$$\mathcal{M} \subseteq \mathcal{N}_1 \vee \mathcal{N}_2 \ni XY \to \pi_1(X) \otimes \pi_2(Y) \qquad X \in \mathcal{N}_1 \quad Y \in \mathcal{N}_2$$

extends to a spatial isomorphism of $\mathcal{N}_1 \vee \mathcal{N}_2$ with $\pi(\mathcal{N}_1) \otimes \pi(\mathcal{N}_2)$

Interdependence of independence



C = assuming commutativity

Independence in AQFT

Proposition $\mathcal{A}(V_1), \mathcal{A}(V_2)$ satisfy the strongest independence property in the hierarchy (are W^*-independent in the spatial product sense – hence W^*-independent too) if V_1 and V_2 are strictly spacelike separated double cone regions

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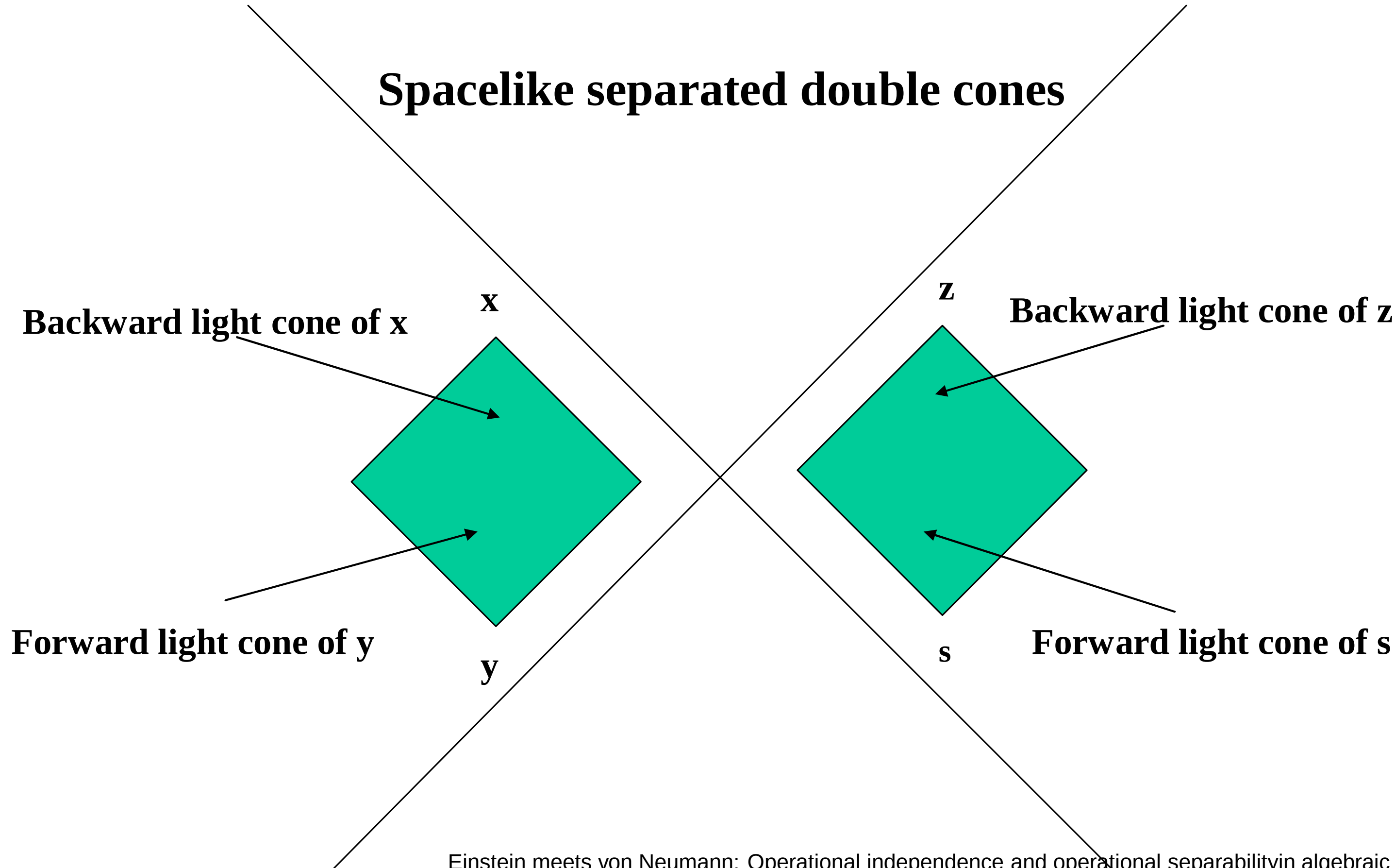

Operations and normal operations

Definition:

- A completely positive unit preserving map T on $\mathcal{A}$ is called an operation
- An operation T on a von Neumann algebra $\mathcal{N}$ is called normal operation if it is $(\sigma - wo)$ continuous

Note: the dual

$$T^*: E(\mathcal{A}) \to E(\mathcal{A}) \qquad\qquad T^*\phi = \phi \circ T$$

maps the state space $E(\mathcal{A})$ into itself (normal states get mapped into normal states if T is a normal operation)

Examples of CP maps

- States
- Conditional expectations
- In particular the conditional expectation:

$$\mathcal{N} \ni X \mapsto T(X) \quad = \quad \sum_i P_i X P_i$$
$$T : \mathcal{N} \quad \to \quad \{P_i\}' \subset \mathcal{N}$$

P_i one dimensional projections in $\mathcal{N}$
$\sum_i P_i = I$

“projection postulate"

Operations on local algebras

Definition

- If $\{\mathcal{A}(V)\}$ is a local net in the sense of AQFT and $V_1, V_2 \subset V$ then

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$$

  is called a local system if

  - V_1 and V_2 are spacelike separated
  - ϕ is a state on $\mathcal{A}(V)$
  - T is an operation on $\mathcal{A}(V)$:

$$T\colon \mathcal{A}(V) \to \mathcal{A}(V)$$

- The operation $T\colon \mathcal{A}(V) \to \mathcal{A}(V)$ is said to represent an operation on $\mathcal{A}(V_1)$ if T is an extension of an operation T_1 on $\mathcal{A}(V_1)$; i.e. if there is an operation T_1 on $\mathcal{A}(V_1)$ such that

$$T(A) = T_1(A) \quad \text{whenever} \quad A \in \mathcal{A}(V_1)$$

Operations on local algebras

If $T: \mathcal{A}(V) \to \mathcal{A}(V)$ represents an operation on the small system $\mathcal{A}(V_1)$ then this
does entail that

- performing the operation T on $\mathcal{A}(V)$ will change any given state ϕ_1 of the small system $\mathcal{A}(V_1)$ into a definite other state $T^*\phi_1$ given by

$$[T^*\phi_1](A) = \phi_1(T(A)) \qquad\qquad A \in \mathcal{A}(V_1)$$

does not entail that

- a given (or any) state ϕ_2 of system $\mathcal{A}(V_2)$ is left unchanged while the operation T is carried out, i.e. it is not necessarily the case that

$$[T^*\phi_2](A) = \phi_2(A) \qquad\qquad A \in \mathcal{A}(V_2)$$

Operational separation

Definition The local system

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$$

with an operation T on $\mathcal{A}(V)$ that represents an operation in $\mathcal{A}(V_1)$ is operationally separated if the operation conditioned state

$$T^*\phi = \phi \circ T$$

coincides with ϕ on $\mathcal{A}(V_2)$, i.e. if

$$\phi(T(A)) = \phi(A) \text{ for all } A \in \mathcal{A}(V_2)$$

Operational separation expresses that an interaction (measurement etc.) with system $\mathcal{A}(V_1)$ does not change the state of remote system $\mathcal{A}(V_2)$

Violation of operational separation

Proposition If $\{\mathcal{A}(V)\}$ is a net in AQFT satisfying the standard axioms then there exist local systems

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$$

that are **NOT** operationally separated
in spite of local commutativity!!!

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Recall that V_1 and V_2 are spacelike separated!

Prop $\Downarrow$ **?**

AQFT violates (Einstein's) **Local Operation** requirement

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Too quick!!

Operational separation is too strong a condition:
Operational nonseparatedness might be due to a “wrong choice" of T:
there might exist operations on the large system that represent the same operation on the small system and which are “better behaving"

Operational separability

Definition The local system

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$$

is called operationally separable if it is operationally separated, or, if it is not operationally separated, then there exists an operation

$$T' : \mathcal{A}(V) \to \mathcal{A}(V)$$

such that T coincides with T' on $\mathcal{A}(V_1)$:

$$T'(X) = T(X) \text{ for all } X \in \mathcal{A}(V_1)$$

and the system

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T')$$

is operationally separated.

Operational C^*-and W^*-separability

Depending on whether one requires the operations to be normal, one can distinguish two versions of operational separability:

- Operational C^*-separability
  operations are not assumed to be normal
- Operational W^*-separability
  operations are assumed to be normal

This leads to the the

Problem What is the relation of operational C^*-and operational W^*-separability ?

The Problem is open

Conjecture ?

Local Operations requirement in AQFT

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Comments

- If AQFT violates the **Local Operation** requirement then Einstein's (1948) objection against QM (that QM is not compatible with the field theoretical paradigm) is valid against relativistic quantum field theory

  Would be very ironic and distressing

- If AQFT does satisfy the **Local Operation** requirement then the notion of operation is compatible with locality and causality as expressed in AQFT and this entails that AQFT satisfies Einstein's 1948 criteria for a theory to be compatible with the field theoretical paradigm

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Does AQFT satisfy the **Local Operation** requirement?

YES

next few slides

Operational C^*-and W^*-independence

Definition $\mathcal{A}_1, \mathcal{A}_2$ are operationally C^*- (W^*-) independent in $\mathcal{A}$ if any two (normal) operations

$$T_1 \;:\; \mathcal{A}_1 \to \mathcal{A}_1$$
$$T_2 \;:\; \mathcal{A}_2 \to \mathcal{A}_2$$

have a joint extension to a (normal) operation on $\mathcal{A}$, i.e. there exists a unit preserving (normal) CP map

$$T : \mathcal{A} \to \mathcal{A}$$

such that

$$T(X) = T_1(X) \quad \text{for all } X \in \mathcal{A}_1$$
$$T(Y) = T_2(Y) \quad \text{for all } Y \in \mathcal{A}_2$$

in the product sense if

$$T(XY) = T(X)T(Y) \qquad X \in \mathcal{A}_1,\ Y \in \mathcal{A}_2$$

Comments on operational independence

- States are special cases of operations; yet, operational C^*-and W^*-independence are not special cases of C^*-and W^*-independence:

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- Operational independence requires the extendibility of a larger class of CP maps but the extension is allowed to be in a larger class of CP maps
- C^*-and W^*-independence express that any two partial states are co-possible
- Operational independence of $\mathcal{A}_1, \mathcal{A}_2$ in $\mathcal{A}$ expresses that any two state transitions of the form

$$\phi_1 \mapsto T_1^* \phi_1 \qquad \phi_2 \mapsto T_2^* \phi_2$$

are co-possible

Operational C^*-and W^*-independence

Proposition [Redei & Summers 2010]: If $\mathcal{A}_1, \mathcal{A}_2$ ($\mathcal{N}_1, \mathcal{N}_2$) are mutually commuting C^*-(von Neumann) algebras acting on a separable Hilbert space, then

- the pair $(\mathcal{A}_1, \mathcal{A}_2)$ is C^*-independent in the product sense if and only if it is operationally C^*-independent in $\mathcal{A}_1 \vee \mathcal{A}_2$ in the product sense;

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Problem Do operational C^*-and W^*-independence (without the product assumption) entail C^*-and W^*-independence in the product sense?
Conjecture: No. Counterexample?

Op. ind. and op. separability

Proposition If the pair $\mathcal{A}(V_1), \mathcal{A}(V_2)$ is operationally (C^*-/W^*-) independent in $\mathcal{A}(V)$, then for every ϕ and every T that represents an operation on $\mathcal{A}(V_1)$ the local system

$$(\mathcal{A}(V), \mathcal{A}(V_1), \mathcal{A}(V_2), \phi, T)$$

is operationally (C^*-/W^*-) separable

Proposition shows that the **Local Operations** requirement (interpreted as operational separability) is not independent of the **Independence** requirement: Operational (C^*-/W^*-) separability in AQFT holds if operational (C^*-/W^*-) independence holds in AQFT

Injectivity and operational independence

Recall: $\mathcal{A}_1, \mathcal{A}_2$ were defined operationally independent

in a given algebra $\mathcal{A}$

The specification of $\mathcal{A}$ within which operational independence holds (or does not) is important

Reason:

$(\mathcal{A}_1, \mathcal{A}_2)$ to be operationally independent in $\mathcal{A}$

every operation T_1 on $\mathcal{A}_1$

and

every operation T_2 on $\mathcal{A}_2$

must be extendible to an operation on $\mathcal{A} \supseteq \mathcal{A}_1, \mathcal{A}_2$

BUT

Operations defined on subalgebras

might not be extendible in general

Sharp contrast with states

Arveson's Theorem

Proposition Let $\mathcal{A}_0$ be a C^*-subalgebra of C^*-algebra $\mathcal{A}$ and

$$T\colon \mathcal{A}_0 \to \mathcal{B}(\mathcal{H})$$

a completely positive, unit preserving linear map. Then T can be extended from $\mathcal{A}_0$ to $\mathcal{A}$ to a completely positive, unit preserving, linear map

- It is important in Arweson's Theorem that T is assumed to take its values in $\mathcal{B}(\mathcal{H})$
  very strong assumption !
- From the perspective of extendability of operations $\mathcal{B}(\mathcal{H})$ behaves like the set of complex numbers for states

Injectivity and operational independence

Definition: A C^*-algebra $\mathcal{C}$ is called injective if for any C^*-algebra $\mathcal{A}$ and for any C^*-subalgebra $\mathcal{A}_0$ of $\mathcal{A}$ it holds that if $T\colon \mathcal{A}_0 \to \mathcal{C}$ is a completely positive, unit preserving, linear map then T can be extended from $\mathcal{A}_0$ to $\mathcal{A}$ into a completely positive, unit preserving, linear map

Definition: A von Neumann algebra is injective if it is injective as a C^*-algebra

- Hahn-Banach Theorem = the set of complex numbers (as a commutative C^*-algebra) is injective
- Arweson's Theorem = the set of all bounded operators on a Hilbert space is injective

Injectivity and operational independence

- A von Neumann algebra $\mathcal{N}$ on a separable Hilbert space is injective if and only if it is approximately finite dimensional (AHF) i.e. if $\mathcal{N} = wo-$ closure of $\Big(\cup_n M_n \Big)$
- There is exactly one (up to isomorphism) AHF factor of type II_1, II_∞, III_λ, $\lambda \in (0, 1]$
- “ ... the local algebras of relativistic quantum field theory in the vacuum sector are of type III_1. In fact ... these algebras are isomorphic as W^*-algebras to the (unique) hyperfinite factor of type III_1."

  R. Haag: Local Quantum Physics (Springer 1992) p. 225

Operational independence in AQFT

- Injectivity of $\mathcal{M}$ is sufficient (but not necessary) in general to ensure a necessary (but not sufficient) condition for operational W^*-independence of $(\mathcal{N}_1, \mathcal{N}_2)$ in $\mathcal{M}$
  Therefore

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  Therefore
- Hyperfiniteness (= injectivity) of local algebras in AQFT can be interpreted as a sufficient condition that ensures a necessary condition for operational C^*-independence to hold for local algebras in AQFT
  And so

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  And so
- There is hope that operational C^*-and W^*-independence holds in AQFT

Operational independence in AQFT

Proposition: Let $\{\{\mathcal{N}(V)\}_{V\subseteq M}\}$ be a net of local von Neumann algebras on a Hilbert space $\mathcal{H}$ satisfying the standard axioms (isotony, local commutativity, covariance, existence of vacuum with the spectrum condition and strong additivity) and let $\mathcal{N}(D_1)$, $\mathcal{N}(D_2)$ and $\mathcal{N}(D)$ be local von Neumann algebras associated with strictly spacelike separated double cones D_1, D_2 and double cone $D \supset D_1 \cup D_2$. Then

$$\Big(\mathcal{N}(D_1), \mathcal{N}(D_2)\Big)$$

(A) are operationally C^*-independent in the product sense in $\mathcal{N}(D)$
Injectivity of $\mathcal{N}(D)$ is crucial in this !!

(B) are operationally W^*-independent in the product sense
in $\mathcal{N}(D_1) \vee \mathcal{N}(D_2) = \mathcal{N}(D_1)\overline{\otimes}\mathcal{N}(D_2)$
Also in $\mathcal{N}(D)$? – Don't know; is injectivity of $\mathcal{N}(D)$ enough?

Op. separability in AQFT

Proposition If $\mathcal{A}(D_1), \mathcal{A}(D_2)$ are local von Neumann algebras associated with strictly spacelike separated double cone regions D_1 and D_2 in a local net of von Neumann algebras satisfying the standard axioms and D is a double cone containing D_1 and D_2 then (since $\mathcal{A}(D_1), \mathcal{A}(D_2)$ are operationally C^*-independent in $\mathcal{A}(D)$)

the typical local systems in AQFT

$$(\mathcal{A}(D), \mathcal{A}(D_1), \mathcal{A}(D_2), \phi, T)$$

are operationally C^*-separable for every ϕ and T

and so

AQFT typically satisfies the **Local Operation** requirement

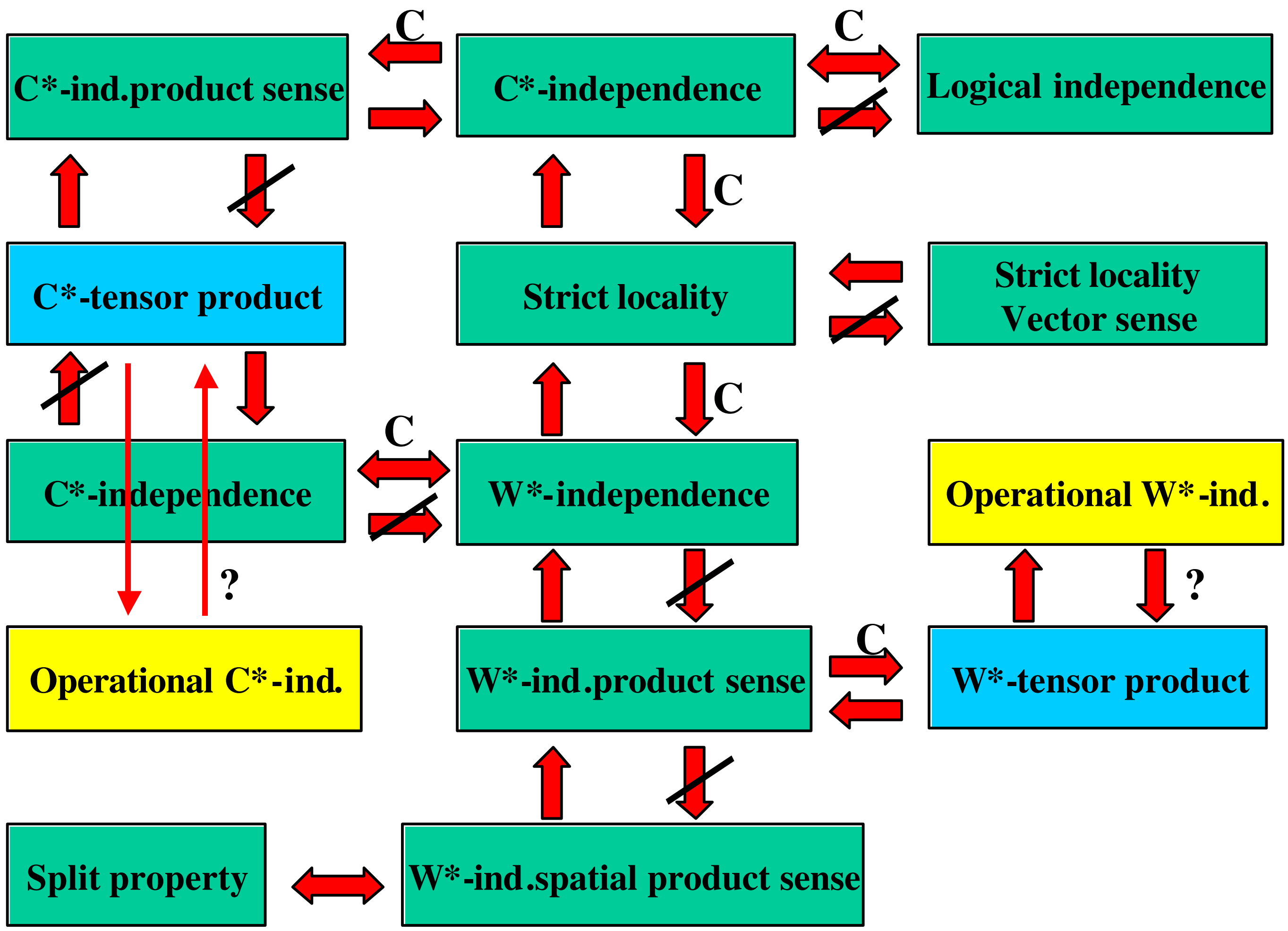
Typically:

– for strictly spacelike separated double cones

– status of W^*-separability is unknown

Some open problems

Characterize operational C^*-and W^*-independence in the hierarchy:



Summary

- Operational independence and operational separability are natural, non-independent independence concepts that can be given technically explicit definitions in terms of AQFT
- Einstein's **Local Operation** requirement formulated in 1948 as part of a field theoretical paradigm can be naturally interpreted in AQFT as operational separability
- Operational C^*-independence and operational C^*-separability holds in AQFT for strictly spacelike separated double cone algebras

Summary

- Operational independence and operational separability are natural, non-independent independence concepts that can be given technically explicit definitions in terms of AQFT
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Algebraic relativistic, local quantum field theory
can be viewed as a
research program
that was suggested/formulated informally by Einstein in 1948
started in the late 50's
(Haag, Kastler, Araki)
and is still active

References

[2], [4], [5], [1], [3]

References

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