How man and nature shake hands: the role of no-conspiracy in physical theories

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Abstract

No-conspiracy is the requirement that measurement settings should be probabilistically independent of the elements of reality responsible for the measurement outcomes. In this paper we investigate what role no-conspiracy generally plays in a physical theory; how it influences the semantical role of the event types of the theory; and how it relates to such other concepts as separability, compatibility and locality.

Keywords: no-conspiracy, separability, compatibility, locality

1 Introduction

As the old *bon mot* has it, in experiment man and nature shake hands. This portrayal of experimentation as the celebration of a good pact between two business men highlights two features of experimentation, namely that both man and nature are equally contributing to its success and that both parties are independent. This independence is the topic of the present paper.

In the foundations of quantum mechanics probably the most significant research project has been for decades to precisely identify and conceptually analyze those assumptions that go into the derivation of the Bell inequalities and can be made responsible for their violation in the EPR scenario. Locality, factorization, Common Cause Principle, determinism—these were the main concepts and principles on the table. There was, however, one additional premise which, though being indispensable in the derivation of the Bell inequalities, remained much more obscure than the others concerning its status, meaning and relation to the other premises.

The palpable evidence for this embarrassment around this assumption is that there has not even been coined a name for it. It has been referred to by many names such as "conspiratorial entanglement" (Bell, 1981), "hidden autonomy" (Van Fraassen, 1982), "independence assumption" (Price 1996), "free will assumption" (Tumulka, 2007), "measurement independence" (Sanpedro, 2013) and—probably in its most well-known form—"no-conspiracy" (Hofer-Szabó, Rédei and Szabó, 1999; Placek and Wroński, 2009). This latter is the phrase we are going to use in this paper.

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The fact that no-conspiracy has been used by so many names attests that there is a wide range of topics which it can be related to. It has been explicitly addressed by Bell in his 1981 paper and its rejection has been qualified as "even more mind boggling than one in which causal chains go faster than light." (Bell, 1981, p. 57) Into the philosophy of physics no-conspiracy made its way via Van Fraassen's 1982 careful analysis of the assumptions leading to the Bell inequalities. Ever since these two influential papers no-conspiracy has been given much attention in the philosophy of science. A topic gaining probably the greatest philosophical interest was how no-conspiracy is related to free will. The first to identify conspiracy as a lack of free will was Bell (1981) himself and has been followed by many others (Price 1996; Conway and Kochen, 2006; Tumulka, 2007).

The present paper does not concern any of the topics mentioned above: neither free will, nor EPR, nor Bell inequalities. It investigates no-conspiracy at a general level. Our aim is to investigate what role no-conspiracy plays in a physical theory. To this aim in Section 2 we will first unfold a general scheme of the ontology of a physical theory. We will discern two event types making the ontology: measurement event types and elements of reality. Measurement event types can be of two types: measurement settings and measurement outcomes. We will clarify how measurement settings and measurement outcomes provide semantics for a physical theory. To illustrate the general scheme we introduce a toy model in Section 3 which will then be used throughout the paper. No-conspiracy enters in Section 4. Here we show how the presence of no-conspiracy deprives measurement settings and measurement outcomes of their semantical role and directs them into pragmatics. In Section 5 some examples will be given for situations when no-conspiracy is violated. In Section 6, 7, and 8 we will investigate in turn the relationship of no-conspiracy to separability, compatibility, and locality. We conclude in Section 9.

This paper is written in the down-to-earth physicalist philosophical style of László E. Szabó to whom I dedicate it.

2 The ontology of experiment

A *physical theory* is a formal system plus a semantics connecting the formal system to the world. The formal system consists of a formal language with some logical axioms and derivation rules, some mathematical and physical axioms. The semantics provides an interpretation for the formalism; it connects the formal system to reality. Though sometimes downplayed or identified with pragmatics, semantics makes an indispensable part of a physical theory. A formal system in itself is not yet a physical theory (Szabó, 2011).

The *semantics* settles the ontology of the theory. This can be done in many ways but every semantics has to minimally fix the ontological *types* or *categories* out there in the world and provide some means to decide when a certain *token* falls in the category of a given type making a certain sentence of the theory true. The types and tokens which we will be interested in here are *event types* and *token events*. The ontology of a physical theory is an *event algebra* constructed from these event types.

Physical theories are verified by *experiments*. The rough picture of an experiment is the following. An experimenter performs a procedure by setting a measurement apparatus in a certain way, obtaining a measurement outcome and repeating this procedure many times. The two essential ontological categories of an experiment are the *measurement settings* and the *measurement outcomes*. These categories are event types just as the other ontological types of the theory.

The token events are the *runs* of the experiment. Measurement settings and measurement outcomes do not appear directly in the textbook form of a theory but they are indispensable part of the semantics (not of pragmatics!): without them the theory cannot be linked to reality. More than that, these two types are the *only* types an experimenter has direct empirical access to. Everything else posited by the theory has to ultimately boil down to some relations between these observable categories. To be more specific, any deductive or inductive relation between the ontological types of the theory has to be accounted for in terms of *correlations* between the token events falling in the category of measurement settings and measurement outcomes. The empiricist thesis is that one has no other access to physical reality than via observation.

Correlations between measurement settings and measurement outcomes can be accounted for in terms of *probabilities*. The probability of an outcome type is simply the *long-run relative frequency* of those runs of the experiment which fall in that type if the experiment is repeated appropriately many times. Similarly, the probability of an outcome *given* a certain measurement setting is simply the number of those runs which fall in both the type of the outcome and the setting divided by the number of those runs which fall in the type of the setting. More importantly, *any* probability assignment to any ontological type to which we have no direct empirical access must be based on *type assignments* to the individual runs of the experiment in the long-run frequency sense: the probability of a given type is p only if the relative frequency of the individual runs falling in the type in question is p. Probability supervenes on the Humean mosaic of token events.

In order to account for the observable measurement outcomes physical theories typically introduce a further, not directly accessible event type, which we will call *elements of reality*. Elements of reality come in two sorts: they can either determine the measurement outcomes for sure for a given measurement setting, or they can fix only the *probability* of the measurement outcomes. We will call the first event type *property* and the second event type *propensity*. Whereas measurement outcomes are clearly causally influenced by and therefore probabilistically dependent on the elements of reality, it is not a priori clear what the relation between the measurement settings and the elements of reality should be. This is what we are going to analyze in what comes.

3 A toy model

Let us make these abstract considerations more concrete on a simple model. (For a general scheme of a physical theory see the Appendix.) Consider a box containing colored dice (Szabó, 2008). Let us try to develop a physical theory of this system. Whatever theory we develop, the semantics of the theory has to minimally specify the measurement settings and measurement outcomes. These are the categories which are directly accessible for an experimenter. Suppose that the measurement settings are the following:

a_1: drawing a dice from the box and checking its *color*
a_2: drawing a dice from the box, throwing it and checking the *number on its upper face*

Suppose furthermore that the measurement outcomes are

A_1^i: the color of the dice is *black* (A_1^1) or *white* (A_1^2)
A_2^j: the number on the *upper face* of the dice is j ($j = 1 \ldots 6$)

So the semantics of the theory posits the following event types: the event type of measurement settings a with two subcategories a_1 and a_2, and the event type of measurement outcomes A with two plus six sub-subcategories A_1^i and A_2^j.

As the experimenter is repeating the experiment, the token events that is the runs falling in the different event types are accumulating giving rise to a probabilistic description of the experiment. She can calculate for example the conditional probability of obtaining a black dice on the condition that she had performed the color measurement:

$$p(A_1^1|a_1) = \frac{\#(A_1^1 \wedge a_1)}{\#(a_1)}$$

This probability is empirically accessible: one just reads off from the relative frequency of the measurement outcomes and measurement settings.

The experimenter can of course try to enrich her theory and introduce a new ontological category into her theory. The motivation behind this move is to obtain an answer to the question: "Why was the outcome of the color measurement black in a certain run of the experiment?" A natural answer to this question is to say: "Because the die itself was black." This answer amounts to introducing a third event type into our ontology, which we will call *property*. What is a property?

The defining feature of the property black is the following: whenever a dice with the property black is subjected to a color measurement, the outcome will always be black. Denote the property black by α_1^1 and the property white by α_1^2. (So our notation is the following: we use lower case Latin letter for the measurement settings (a); capital Latin letters for the measurement outcomes (A); and Greek letters for the elements of reality (α).) The property black is an event type and each token event that is each run of the experiment can be characterized by either falling into this event type or not. Therefore, one can also meaningfully speak about the probability of the property black, $p(\alpha_1^1)$, as the long-run relative frequency of those runs of the experiment which fall into the event type α_1^1. Consequently, one can also express the defining feature of the property black and white in terms of probabilities as follows:

$$p(A_1^i|a_1 \wedge \alpha_1^k) = \delta_{ik} \qquad\qquad i,k = 1,2 \tag{1}$$

That is in each run of the experiment when the dice was black and the color has been measured, the outcome was black and never white; and in each run of the experiment when the dice was white and the color has been measured, the outcome was white and never black. A property is nothing but an event type which, if instantiated and measured in a certain run of experiment, brings with it a definite outcome.

Let us now go over to the case of throwing the dice and ask a similar question to that of the color measurement: "Why does the outcome six come up with a certain probability in the experiment?" Here the natural answer is this: "Because the dice has a certain mass distribution." This leads us to introducing another event type which we will call *propensity*.

Suppose that the box is containing dice with two different mass distributions. Denote them by α_2^1 and α_2^2. Here the lower index 2 indicates that the measurement setting is of the second type, namely checking the upper face of the dice (and not the color), and the upper index discerns the two mass distributions. The mass distinction α_2^1 is again an event type just as α_1^1, the property black was. In every single run of the experiment it is either instantiated or not that is each dice has either the mass distribution α_2^1 or not. Hence one can speak about the probability $p(\alpha_2^1)$ as the relative frequency of those runs which fall into the event type α_2^1. If a dice with mass distribution α_2^1 is drawn from the box and thrown, then let the probability of its coming up j be denoted by q^{j1}. Similarly, if a dice with mass distribution α_2^2 is drawn from the box and thrown, then the probability of coming up j is q^{j2}. This means that the mass distribution of a given dice fixes the *probability* of the dice coming up with a certain face upon throwing. In terms of probabilities this can be expressed as follows:

$$p(A_2^j|a_2 \wedge \alpha_2^l) = q^{jl} \qquad\qquad j = 1\ldots 6,\, l = 1, 2 \tag{2}$$

where $\sum_j q^{jl} = 1$ for $l = 1, 2$.

Metaphysically, the new event type α_2 is the *propensity* of the dice to come up with a certain face in the second type of measurement setting. Note that the propensity here is not something which the notion of probability should be reduced to as in the literature on the interpretations of probability. Here propensity is an event type and probability is simply long-run relative frequency. Their relation is the following: the probability of a certain outcome type is fixed for a certain measurement setting and a certain propensity.

Also observe that a property mathematically differs from a propensity only in that the q^{jl}-s fixing the conditional probabilities are all either 0 or 1 for the properties, whereas they can be any number between 0 and 1 for the propensities. Being black fixes the measurement outcomes for the color measurement, whereas having mass distribution α_2^1 fixes only the probability of obtaining a six. The defining equation (1) of properties is a special case of the defining equation (2) of propensities.

To sum up, in our "theory of dice" we have two measurement event types, the event type of measurement settings and measurement outcomes. Beyond these we can introduce into our ontology two elements of reality for explanatory purposes, the event type of properties, α_1, with two subcategories α_1^1 (black) and α_1^2 (white); and the event type of propensities, α_2, with two subcategories α_2^1 (first mass distribution) and α_2^2 (second mass distribution). From now on we will coin the term *measurement event type* for measurement settings and measurement outcomes and *element of reality* for properties and propensities. The event algebra of the theory will be composed as the Boolean combination of the measurement event types and elements of reality. This algebra will be built up from $2 \cdot (2 \cdot 6) \cdot (2 \cdot 2)$ atomic events associated to the different combinations of measurement settings, measurement outcomes, properties and propensities. Each run of the experiment will instantiate an element of this algebra. Probabilities enter the theory by simply counting how many runs are instantiating certain elements of the algebra.

4 No-conspiracy

So far, so good. But physics is a procedure to move from the observable to the unobservable. Do we have any means to infer from the first two event types to the second two? Can we say something about properties and propensities based on measurement settings and measurement outcomes?

Here is a sufficient condition which entitles us to such an inference. Suppose that the elements of reality, though causally responsible for the measurement outcomes, are causally independent of the measurement settings. Common cause aside, this means that the elements of reality are also probabilistically independent of the measurement settings. In case of the properties this means that

$$p(a_1 \wedge \alpha_1^k) = p(a_1)\, p(\alpha_1^k) \qquad\qquad k = 1,2 \tag{3}$$

in case of the propensities:

$$p(a_2 \wedge \alpha_2^l) = p(a_2)\, p(\alpha_2^l) \qquad\qquad l = 1,2 \tag{4}$$

Putting them together they read as follows:

$$p(a_1 \wedge a_2 \wedge \alpha_1^k \wedge \alpha_2^l) = p(a_1 \wedge a_2)\, p(\alpha_1^k \wedge \alpha_2^l) \qquad\qquad k,l = 1,2 \tag{5}$$

Let us call requirement (5) *no-conspiracy*. Obviously, (3)-(4) are special cases of (5).

No-conspiracy does us a great service: we can reproduce the observable probabilities of the theory in terms of the probabilities of the elements of reality. For example the conditional probability $p(A_1^1|a_1)$ of obtaining a black dice upon color measurement turns out to be just the probability $p(\alpha_1^1)$ of the property black:

$$\begin{aligned} p(A_1^1|a_1) &= \frac{p(A_1^1 \wedge a_1)}{p(a_1)} = \frac{\sum_k p(A_1^1 \wedge a_1 \wedge \alpha_1^k)}{p(a_1)} = \frac{\sum_k p(A_1^1|a_1 \wedge \alpha_1^k) p(a_1 \wedge \alpha_1^k)}{p(a_1)} \\ &= \frac{\sum_k p(A_1^1|a_1 \wedge \alpha_1^k) p(a_1) p(\alpha_1^k)}{p(a_1)} = \sum_k p(A_1^1|a_1 \wedge \alpha_1^k) p(\alpha_1^k) \\ &= \sum_k \delta_{1k} p(\alpha_1^k) = p(\alpha_1^1) \end{aligned} \tag{6}$$

where we used only the theorem of total probability, the defining feature (1) of a property and no-conspiracy (3).

By similar reasoning we can reproduce the conditional probability $p(A_2^6|a_2)$ of obtaining the outcome six upon "upper face" measurement in terms of weighted averages of the probability of propensities $p(\alpha_2^l)$:

$$p(A_2^6|a_2) = q^{61}\, p(\alpha_2^1) + q^{62}\, p(\alpha_2^2) \tag{7}$$

Equations (6) and (7) are of central importance. They explain why in the text book form of a physical theory one need not speak about measurement settings and measurement outcomes. If no-conspiracy holds, then all the conditional probabilities of the measurement event types (settings and outcomes) are mirrored in the probabilities of the elements of reality (properties and propensities). Consequently, the deductive and inductive relations between the measurement event types also reveal similar relations between the elements of reality. For example, observing the relation that the probability of a dice coming up six is higher than that of being black

$$p(A_2^6|a_2) > p(A_1^1|a_1) \tag{8}$$

reveals the unobservable fact that

$$q^{61}\, p(\alpha_2^1) + q^{62}\, p(\alpha_2^2) > p(\alpha_1^1) \tag{9}$$

More than that, the relations between measurement settings and measurement outcomes do not just reveal the hidden relations between the unobservable categories but by the same move they also become superfluous. If the role of these "surface" relations is simply to reflect the deep hidden structural relationships of the unobservable categories with which real physics is concerned—then why one would care about them? Why one would care about measurement settings and measurement outcomes if one can also speak about the "real stuff" directly? Measurement settings and measurement outcomes belong only to pragmatics not semantics.

This is the way how no-conspiracy overshadows the semantical role of measurement settings and measurement outcomes. If no-conspiracy holds, then the very categories which lend empirical meaning to the theory turn to be (seem to be) superfluous.

But does no-conspiracy always hold? What if it does not? In this case the inference from the measurement event types to the elements of reality via (6) and (7) is not possible. But does it make any knowledge of the unobservable categories impossible? Is no-conspiracy a kind of Kantian "condition of the possibility of experience"?

Some seem to think so. In his famous 'cat' paper Schrödinger (1935) likens the free measurement choice of the EPR experiment to a situation when a class of students are asked a set of question such that each student may be asked any of questions. If the answer to the questions are all correct, then one can conclude, so Schrödinger, that *all* students know *all* answers. Analyzing Schrödinger's example Maudlin (2014) writes the following:

> "Recall Schrödinger's class of identically prepared students. We are told they can all answer any of a set of questions correctly, but each can only answer one, and then forgets the answers to the rest. It's an odd idea, but we can still test it: we ask the questions at random, and find that we always get the right answer. Of course it is possible that each student only knows the answer to one question, which always happens to be the very one we ask! But that would require a massive coincidence, on a scale that would undercut the whole scientific method. Or else we are being manipulated: somehow we are led to ask a given question only of the rare student who knows the answer. So we switch our method of choice, handing it over to a random number generator, or the throw of dice, or to be determined by the amount of rainfall in Paraguay. But maybe all of these have been somehow rigged too! Of course, such a purely abstract proposal cannot be refuted, but besides being insane, it too would undercut scientific method. All scientific interpretations of our observations presuppose that they have not have been manipulated in such a way." (Maudlin, 2014 p. 23)

In short, the independence of the measurement choices and the elements of reality is a precondition of pursuing science *per se*. But is it so?

5 When no-conspiracy does not hold

Consider the following examples:

Example 1. Suppose that the black painting on the dice is not durable enough: if you just touch the dice, the color black is wearing off it and it turns white.

Example 2. Suppose that each dice is filled with a high viscosity fluid which can stream and swirl inside the dice. By every throw of the dice the fluid is put in motion which changes the mass distribution of the dice and hence the propensity of the outcome at that very throw.

Example 3 is special case of Example 2. Suppose again that the dice are filled with a fluid which can stream inside it until the dice is landing on the table. But when the dice touches the table, all the fluid flows—due to the heavy shaking, say—to the that side of the dice which is the closest to the table and "freezes out" there. Consequently, the dice will come up with the opposite face.

The above three examples are all illustrating a situation when no-conspiracy is violated. In the first example the property α_1^1 (black) has turned into another property α_1^2 (white) as a result of the measurement setting a_1 (drawing a dice from the box). In the second example the propensity α_2^1 (first mass distribution) has turned into another propensity α_2^2 (second mass distribution) as a result of the measurement setting a_2 (tossing a dice). Finally, in the third example we find a change of category. Recall that properties and propensities differed only in whether they determined the outcome for sure or only up to a certain probability. In the third example there was some non-trivial probability for the different faces of the dice to come up before the throw. After landing the table, however, the dice could come up only with a given face. That means that here a propensity (one sort of mass distribution) has been turned into a property (a special mass distribution exactly fixing the outcome) as a result of the measurement setting a_2 (tossing a dice). In each case no-conspiracy is violated. (For the relevance of these examples to the interpretations of quantum mechanics see (Gömöri and Hofer-Szabó, 2016).)

In all the above examples the violation of no-conspiracy was due to a causal connection between the measurement settings and the elements of reality. So let us first clarify what we mean by a causal connection between two event types, say, the drawing of a dice, a_1, and the property black, α_1^1. It means that these two event types are causally related in a *tokenwise* manner. In other words, there is a *pairing* of token events instantiating the two types such that for each pair of token events the one instantiating a_1 is the cause of the other instantiating α_1^1, or vica versa. But how to create pairs?

Consider a certain run of the experiment which instantiates $a_1 \wedge \alpha_1^1$. Up to now we treated this run of the experiment as *one single* run in which one performed a color measurement *and* the property of the dice which has been drawn was black. How can the color measurement cause the property black in this single run? If this run of the experiment is taken as one single token event, then there can be no tokenwise causal connection; simply because we have only one token. In order to have a causal connection, one needs to decompose the one single run of the experiment instantiating $a_1 \wedge \alpha_1^1$ into a *pair* of token events such that the one token event instantiates a_1 and the other token event instantiates α_1^1. In order to speak about a tokenwise causal relation, one token event is not enough. One (but not the only) possibility to perform this decomposition is to say that the first token event occurred here and the other token event occurred over there. Localization is a typical method for individuation. We come back to the question of localization in Section 8.

With this notion of causal influence in hand let us go back now to above three examples. One can well see that in all the three examples the causal influence is directed *from* the measurement settings *to* the elements of reality. However, this is not the only option. No-conspiracy can fail also due to an opposite causal direction when the elements of reality causally influence the measurement settings. The next example is of this type.

Example 4. Suppose that the black dice are slightly electrifying your hand when you touch them in the box; hence you rather draw white balls.

A next example for the violation of no-conspiracy is a common causal connection between the elements of reality and the measurement settings. It is a combination of example 1 and 4.

Example 5. Suppose again that the black dice are electrically charged. Touching them is unpleasant so you rather draw white balls; *and* if you still draw one, the black painting is wearing off and the dice turns to white.

Finally, there is a further way to violate no-conspiracy which is not related to causation. Two events can be correlating even if they are not causally related: if they are logically not independent. This leads us to the problem of *contextuality*.

A little reflection on the definition of property and propensity can convince us that (1) and (2) say nothing about whether the elements of reality and the measurement settings are *logically* independent or not. It can well be the case that by specifying the measurement setting we partly specify also the elements of reality. Consider the following example.

Example 6. Let $\alpha_2^1(x,p)$ denote the following property of the dice: the mass distribution of the dice is of the first type *and* the initial conditions of the toss is (x,p). Similarly, let $a_2(x,p)$ denote the measurement setting which does not just state the fact that the dice has been thrown but also specifies the initial conditions (x,p) of the toss.

The element of reality $\alpha_2^1(x,p)$ is obviously a property since together with the toss a_2 it determines the upper face for sure. But now instead of a_2 we took a finer description of the measurement settings, namely $a_2(x,p)$. Obviously,

$$p(A_2^j|a_2(x,p) \wedge \alpha_2^1(x,p)) \tag{10}$$

is either 0 or 1 for any j and (x,p). Hence $\alpha_2^1(x,p)$ is a property with respect to also the new measurement setting $a_2(x,p)$.

However, no-conspiracy does not hold simply because the measurement settings and the elements of reality are not logically independent. The measurement setting plays a constitutive role in determining the element of reality. If you toss the dice with a certain initial condition, then you also (partly) specify the elements of reality, namely that the dice started with that specific initial condition.

To sum up, even if the elements of reality and the measurement settings are causally detached, still they can violate no-conspiracy if the measurement settings contribute to the definition of the elements of reality. A "double counting" of some conditions, as in the case of the initial conditions (x,p), cannot be excluded a priori.

Be the above examples as suggestive as they may, they should not be taken too seriously. Properties and propensities are *per definitionem* elements of reality whereas mass distribution is an observable event type. One can directly test whether the mass distribution of a given dice depends on its throwing; good dice are those for which it does not. Therefore, the dependence of the mass distribution of the dice upon its throwing is not a violation of no-conspiracy in the strict sense, but rather the dependence of the outcome of one measurement on another measurement.

Still, disregarding for a moment the observable status of the mass distribution and replacing it by an unobservable propensity, the above examples illustrate that the dependence of the measurement settings on the unobservable categories is well conceivable and cannot be excluded a priori.

But then, under which circumstances can we adopt no-conspiracy in our physical theory, and when are we forced to abandon it? In the upcoming three Sections we investigate three concepts in turn which can qualify the decision. The first is *separability*, the second is *compatibility*, and the third is *locality*.

6 Separability

Niels Bohr's notorious insistence on the use of classical concepts in the description of quantum phenomena is one of the hallmarks of his philosophy. In his contribution to the 1949 Einstein *Festschrift* Bohr writes:

> It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. The argument is simply that by the word "experiment" we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics. (Bohr 1949, p. 209).

Many Bohr scholars have made significant efforts to understand the meaning and role of Bohr's doctrine on the primacy of classical concepts. Camilleri and Schlosshauer (2015) argue that Bohr's doctrine is primarily a general epistemological thesis articulating the epistemology of *experiment* rather than a special interpretation of quantum mechanics (for this see also (Zinkernagel, 2015)). The epistemological problem according to Bohr is that whereas the very notion of experiment presupposes that the measured objects possess a definite state which is independent from the state of the measurement apparatus, quantum mechanics makes this distinction between object and apparatus ambiguous by treating the two as a single, composite, entangled system:

> ...the impossibility of subdividing the individual quantum effects and of separating the behaviour of the objects from their interaction with the measuring instruments serving to define the conditions under which the phenomena appear implies an ambiguity in assigning conventional attributes to atomic objects which calls for a reconsideration of our attitude towards the problem of physical explanation. (Bohr 1948, p. 317).

If entanglement between object and apparatus is *the* obstacle to an unambiguous description of quantum phenomena, then such a description in classical terms can be realized when the subsystems are not entangled, that is when they are *separable*. This is exactly Don Howard's (1994) suggestion for the reconstruction of Bohr's doctrine on classical concepts:

> ...for Bohr, classical concepts are necessary because they embody the assumption of instrument-object separability, and that such separability must be assumed, in spite

> of its denial by quantum mechanics, in order to secure an unambiguous and thus objective description of quantum phenomena. (Howard 1994, p. 209).

Howard's suggestion to analyze classical description in terms of separability boils down to the requirement to reproduce the statistical predictions of a given quantum phenomenon in terms of an "appropriate mixture." The state of a composite system is called *separable*, if it is a *mixture* that is a convex sum of product states of the components. Since product states represent probabilistically independent components, a mixture is simply a convex combination of these states expressing a classical probabilistic correlation between the components. The devise of mixtures gives rise to a classical, ignorance interpretation of the statistics of the phenomenon under investigation. This analysis via the notion of an "appropriate mixture" has been picked up for example by Halvorson and Clifton (2002) who provide an elegant analysis of the EPR experiment from Bohr's perspective along the lines suggested by Howard.

But how separability as a reconstruction of Bohr's demand on classicality relates to no-conspiracy as a kind of independence principle between measurement settings and the elements of reality attributed to the system? Clearly, separability is a broader concept than no-conspiracy: separability simply requires that the relation between the measurement settings and elements of reality should be expressed as a mixture of probabilistic independences; whereas no-conspiracy requires that the two should be probabilistically independent. In our toy model for example separability requires the probability of the color measuring and the system's possessing the property black to be the following:

$$p(a_1 \wedge \alpha_1^1) = \lambda_1\, p(a_1)\, p(\alpha_1^1) + \lambda_1\, p(a_1)\, p(\sim\alpha_1^1) + \lambda_3\, p(\sim a_1)\, p(\alpha_1^1) + \lambda_4\, p(\sim a_1)\, p(\sim\alpha_1^1) \tag{11}$$

with any $\lambda_i \in [0,1]$ and $\sum_{i=1}^{4} \lambda_i = 1$; whereas no-conspiracy requires that

$$p(a_1 \wedge \alpha_1^1) \quad = \quad p(a_1)\, p(\alpha_1^1) \tag{12}$$

But observe that separability (11) does not give any restriction in our case; it simply means that p is a classical probability which we already knew since we took probabilities to be relative frequencies.

All the six examples in the previous Section, though violating no-conspiracy, are completely classical; they provide an unambiguous description of how the unobservable properties or propensities change upon throwing the dice. They even provide a mechanism for the causal dependence. In Example 1 for instance when upon drawing the black color is wearing off the dice, obviously

$$p(a_2 \wedge \alpha_1^1) \quad \neq \quad p(a_2)\, p(\alpha_1^1) \tag{13}$$

Drawing the dice and being black will not be probabilistically independent due to the causal relation between the two event types.

Thus, the "unambiguous language" requires only to attribute *some* properties to the system which stand in *some* classical probabilistic relation to the measurement settings but it does not require them to be probabilistically independent of one another. Hence, separability as a weaker requirement than no-conspiracy cannot be used to back the latter. (In addition, according to Howard even the demand on classicality as separability is too restrictive from perspective of a general epistemology of experiment.)

7 Compatibility

Now, let us go over to our second concept which is *compatibility* of the measurement settings. Up to now we have considered measurement settings only separately. Let us see now what happens when we perform a joint measurement.

Again, consider our toy model and suppose that we perform the measurement $a_1 \wedge a_2$ that is we are drawing a dice from the box, throwing it and checking its color *and* also the number on its upper face. Suppose that after performing both measurements we disregard the upper face and consider only the color. Suppose that we observe that the probability of the outcome black in this joint measurement is not the same as in the measurement a_1. That is we find that

$$p(A_1^1|a_1 \wedge a_2) \neq p(A_1^1|a_1) \tag{14}$$

Let us call (14) *incompatibility* of the two measurements.

What is incompatibility a sign of?

First, observe that the condition a_1 on the right hand side of (14) does not mean that we performed only a_1—this would be $a_1 \wedge \sim a_2$. The condition a_1 means that we consider all the runs in which a_1 has been performed, irrespectively whether a_2 has been performed or not—that is $a_1 = (a_1 \wedge a_2) \vee (a_1 \wedge \sim a_2)$. So what (14) expresses is that whether we perform a_2 or not *does* count in measuring a_1.

One can take here two positions towards incompatibility. I will call the first the *purist* or *Bridgmanian* strategy and the second the *stubborn* strategy. According to the purist strategy if the probability of the outcome of a given measurement can vary depending whether another measurement is performed or not, then this measurement is *not yet well defined*.

Consider the following example.[1] How would you set up a measurement which tests whether a given piece of wood is combustible? Well, just burn it and check what happens. How would you set up a measurement which tests whether this piece of wood is floating? Well, throw it in water and check what happens. But obviously, the two measurements are incompatible; you cannot burn the piece of wood while in water. So the correct definition of the first measurement is this: *keep the piece of wood dry*, burn it and check what happens. Similarly, you should not burn the piece of wood along with throwing it in water—unless you want to test whether the ash is floating.

So the purist attitude towards (14) is that a_1 in itself is not yet a well defined measurement procedure since the probability of the outcomes depends on whether a_2 is performed or not. So instead of taking *two* measurement settings a_1 and a_2 we should take *three* measurement settings, $a_1 \wedge a_2$, $a_1 \wedge \sim a_2$, and $\sim a_1 \wedge a_2$ (the fourth one, $\sim a_1 \wedge \sim a_2$, is that we do nothing). By this move we eliminate the incompatibility from our measurements since the four new measurements are logically mutually excluding; they cannot be co-performed and hence disturb one another. Generally, the purist strategy is to take the the conjunctions of incompatible measurements until they become either compatible or logically excluding.

We call this strategy Bridgmanian since it is in tune with Bridgman's ideas on the correct definition of measurement unfolded for example in *The Logic of Modern Physics*:

> Implied in this recognition of the possibility of new experience beyond our present range, is the recognition that no element of a physical situation, no matter how

[1] This example is due to Márton Gömöri.

apparently irrelevant or trivial, may be dismissed as without effect on the final result until proved to be without effect by actual experiment. (Bridgman 1958, p. 3)

Returning to no-conspiracy, the Bridgmanian strategy makes all co-measurable measurements compatible with one another. Therefore, the problem of incompatibility disappears and we are back to our single case measurement scenario. The purist strategy teaches nothing new about no-conspiracy.

Let us go over to the stubborn strategy. I call it stubborn since it keeps a_1 and a_2 as the correct measurement settings in spite of their incompatibility (14)? What does then (14) say about no-conspiracy?

This is a point where we need to go one step further concerning the relation between measurement event types and elements of reality. We need to specify how the elements of reality behave when jointly measured. Therefore suppose that the following relation also holds (in addition to (1) and (2)):

$$p(A_1^i \wedge A_2^j | a_1 \wedge a_2 \wedge \alpha_1^k \wedge \alpha_2^l) = \delta_{ik}\, q^{jl} \qquad i,k,l = 1,2;\ j = 1\ldots 6 \tag{15}$$

Requirement (15) expresses a kind of non-disturbance relation between the measurements which can be better seen if we sum up first for i then for j:

$$p(A_1^i | a_1 \wedge a_2 \wedge \alpha_1^k \wedge \alpha_2^l) = \delta_{ik} = p(A_1^i | a_1 \wedge \alpha_1^k) \tag{16}$$

$$p(A_2^j | a_1 \wedge a_2 \wedge \alpha_1^k \wedge \alpha_2^l) = q^{jl} = p(A_2^j | a_2 \wedge \alpha_2^l) \tag{17}$$

(Here the second equation in both rows are due to the defining equation (1) of the property and (2) of the propensity, respectively.) (16) and (17) express that the probability of an outcome conditioned on an element of reality and a measurement setting does not change by further conditioning it on other elements of reality or measurement settings. From (16) (where the element of reality is a property) it also follows that

$$p(A_1^i | a_1 \wedge a_2 \wedge \alpha_1^k) = p(A_1^i | a_1 \wedge \alpha_1^k \wedge \alpha_2^l) = p(A_1^i | a_1 \wedge \alpha_1^k) \tag{18}$$

Now, suppose that no-conspiracy also holds that is

$$p(a_1 \wedge a_2 \wedge \alpha_1^k \wedge \alpha_2^l) = p(a_1 \wedge a_2)\, p(\alpha_1^k \wedge \alpha_2^l) \qquad k,l = 1,2 \tag{19}$$

From (15) and (19) it is easy to show (via a derivation similar to (6)) that

$$p(A_1^1 | a_1 \wedge a_2) = p(A_1^1 | a_1) \tag{20}$$

in contradiction to incompatibility (14). This means that if we observe incompatibility between the measurements, then we have to abandon either the non-disturbance of the measurement procedures (15) or no-conspiracy (19).

So in case of the stubborn strategy compatibility of the measurement settings is a good sign of that both non-disturbance and no-conspiracy hold; and incompatibility is a good sign of that either the one or the other is violated. Whether to blame the one or the other is a question for further investigation.

8 Locality

No-conspiracy is a probabilistic independence relation between the measurement settings and the elements of reality. What

$$p(a_1 \wedge \alpha_1^1) = p(a_1)\, p(\alpha_1^1) \tag{21}$$

is expressing is that the relative frequency of those runs of the experiment which instantiate the event type "the color has been measured *and* the dice was black" is equal to the product of the relative frequencies of those runs which instantiate these event types "the color has been measured" *and* "the dice was black" separately.

Now, probabilistic independence is a sign of causal independence and correlation is a sign of causal connection. Due to Reichenbach's Common Cause Principle, if two event types are correlated, then there is either a direct or a common causal relation between them. *Vica versa* (assuming that causal effects do not cancel each other) if two event types are probabilistically independent, then there is neither a direct nor a common causal relation between them. Hence, no-conspiracy can be ensured if causal relations between the measurement settings and the elements of reality can be excluded.

Can we exclude causal connections? Do locality considerations help us in that? Is there a spatiotemporal arrangement of the event types in question such that one can safely say that all possible causal connections between the measurement settings and the elements of reality are shielded off? As one expects, the answer to this question is *no*.

Recall what we said about the causal connection in Section 5. Two event types are causally related if there is a pairing of token events instantiating the two types such that for each pair the one token is the cause of the other; or—in case of a common cause—there is a third token instantiating a third event type which is the common cause of both.

Consider a certain run of the experiment which instantiates $a_1 \wedge \alpha_1^1$. This means that in this run one performed a color measurement *and* the property of the dice was black. As said in Section 5 in order to meaningfully raise the question of a causal connection, one needs first to decompose this one single run into a *pair* of token events. Suppose we individuate the two token events by localizing the first token event at one spacetime locus and the other token event at another one. Localization is a typical method for individuation. If the color measurement and the dice with property black are localized in different regions of the spacetime, then one can meaningfully ask whether they are in tokenwise causal connection or not.

Suppose now that the two token events are spacelike separated. Does it tell us something about their causal relation? No. Even if they are spacelike separated, they can still be causally related to one another both in a direct and also in a common causal way. As for direct causal connection, just note that in order to produce a measurement outcome these two token events need to interact somewhere in spacetime. Hence even is they are spacelike separated at a certain moment, they will not be so at the moment of bringing about the outcome. Therefore their direct causal effect on one another cannot be excluded based on the fact that at a certain time they were localized in a spacelike separated way. The situation is similar or even worse in case of a common cause. Even if the two token events are spacelike separated, there well can be a common cause in their common past causally influencing both. To sum up, locality considerations do not help is in excluding causal mechanisms and hence to ensure no-conspiracy.

9 Conclusions

In this paper we have argued for the following.

A physical theory is a formal system plus a semantics connecting the formal system to the world. The semantics has to minimally specify what event types inhabit the world. Event types can be of two sorts: measurement event types and elements of reality. Typically we have direct access to the former but not to the latter. There are two measurement event types: measurement settings and measurement outcomes and there are also two elements of reality: properties and propensities. The probability of an event type is understood as simply the long-run relative frequency of the token events instantiating the event type in question. In an experiment the token events are the runs of the experiment.

No-conspiracy is the requirement that elements of reality should be probabilistically independent of the measurement settings. There is no a priori guarantee that no-conspiracy does hold. If it does, probabilistic relations between the measurement event types mirrors only the relations between the elements of reality. This licenses physics to incorrectly forget about measurement settings and measurement outcomes and to talk directly about elements of reality.

No-conspiracy can be naturally related to the concepts of separability, compatibility and locality. However, none of them brings us closer to no-conspiracy. Separability is a weaker concept than no-conspiracy, so one cannot back the latter by the former. Compatibility of measurement settings is empty in case of a purist strategy and only a partial motivatation in case of the stubborn strategy. Finally, locality cannot be used to support no-conspiracy at all.

Neither being an analytic nor a transcendental truth, all we can do is to check no-conspiracy on a case-by-case basis. Life is hard.

Appendix

Here we provide a general mathematical picture of a physical theory.

Let a_i $(i = 1 \dots I)$ be the measurement settings in a given theory and let $A_i^{j_i}$ $(j_i = 1 \dots J_i)$ denote the jth outcome of the ith measurement. Suppose furthermore that there is an element of reality $\alpha_i^{k_i}$ $(k_i = 1 \dots K_i)$ (either a property or a propensity) associated to each measurement setting a_i such that

$$p(A_i^{j_i} | a_i \wedge \alpha_i^{k_i}) = q_i^{j_i k_i} \tag{22}$$

where $\sum_{j_i=1}^{J_i} q_i^{j_i k_i} = 1$ for any $i = 1 \dots I$ and $k_i = 1 \dots K_i$. For a given $i \in I$ the element of reality $\alpha_i^{k_i}$ is a property iff $J_i = K_i$ and $q_i^{j_i k_i} = \delta_{j_i k_i}$. Otherwise $\alpha_i^{k_i}$ is a propensity.

Suppose that the elements of reality are related nicely to the measurement event types not only in case of a single measurement but also in case of a joint measurement. (Note the the word "single" does not mean that the other measurements are not performed; it means rather that it is not taken into consideration whether they are performed or not.) Therefore, select I' measurement settings out of the possible I and let now the index i run from 1 to I'. What we require is that for any such selection (among them the no-selection) the following should hold:

$$p(A_1^{j_1} \wedge \dots \wedge A_{I'}^{j_{I'}} | a_1 \wedge \dots \wedge a_{I'} \wedge \alpha_1^{k_1} \wedge \dots \wedge \alpha_{I'}^{k_{I'}}) = q_1^{j_1 k_1} \times \dots \times q_{I'}^{j_{I'} k_{I'}} \tag{23}$$

Now, the elements of reality $\{\alpha_i^{k_i}\}$ are said to satisfy no-conspiracy iff

$$p(a_1 \wedge \ldots \wedge a_I \wedge \alpha_1^{k_1} \wedge \ldots \wedge \alpha_I^{k_I}) = p(a_1 \wedge \ldots \wedge a_I)\, p(\alpha_1^{k_1} \wedge \ldots \wedge \alpha_I^{k_I}) \tag{24}$$

from which it follows that they also satisfy no-conspiracy for all selections, among them

$$p(a_i \wedge \alpha_i^{k_i}) = p(a_i)\, p(\alpha_i^{k_i}) \tag{25}$$

By means of (23) and no-conspiracy (24) one can transform for any selection the probabilistic relations among the measurement event types into probabilistic relations among elements of reality as follows:

$$p(A_1^{j_1} \wedge \ldots \wedge A_{I'}^{j_{I'}} | a_1 \wedge \ldots \wedge a_{I'}) = \sum_{k_1 \ldots k_{I'}} q_1^{j_1 k_1} \times \cdots \times q_{I'}^{j_{I'} k_{I'}} p(\alpha_1^{k_1} \wedge \ldots \wedge \alpha_{I'}^{k_{I'}}) \tag{26}$$

Specifically, if all the event types $\{\alpha_i^{k_i}\}$ are properties, then (26) reads as

$$p(A_1^{j_1} \wedge \ldots \wedge A_{I'}^{j_{I'}} | a_1 \wedge \ldots \wedge a_{I'}) = p(\alpha_1^{j_1} \wedge \ldots \wedge \alpha_{I'}^{j_{I'}}) \tag{27}$$

and in the special case of a single measurement as

$$p(A_i^{j_i} | a_i) = p(\alpha_i^{j_i}) \tag{28}$$

for all $i = 1 \ldots I$. (26) shows that the probability of the outcomes conditioned on the measurement settings is mirrored in the probability of the properties.

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References

J. S. Bell, "Bertlmann's socks and the nature of reality," *Journal de Physique*, Colloque C2, **42**, 41–61. (1981).

N. Bohr, "On the Notions of Causality and Complementarity," *Dialectica*, **2**, 312-319 (1948).

N. Bohr, "Discussions with Einstein on epistemological problems in atomic physics," In: Schilpp, P. A. (Ed.), *Albert Einstein: Philosopher–Scientist. Vol. 7. Library of Living Philosophers*, Evanston, Illinois, 201–241 (1949).

P. W. Bridgman, *The Logic of Modern Physics*, (New York: The Macmillan Company, 1958).

K. Camilleri and M. Schlosshauer, "Niels Bohr as philosopher of experiment: Does decoherence theory challenge Bohr's doctrine of classical concepts?,"*Stud. Hist. Phil. Sci.*, **49(1)**, 73-83, (2015).

J. Conway and S. Kochen, "The free will theorem," *Found. Phys.*, **36**, 1441–1473 (2006).

L. E. Szabó, "The Einstein-Podolsky-Rosen argument and the Bell inequalities," *Internet Encyclopedia of Philosophy*, URL= http://www.iep.utm.edu/epr/ (2008).

L. E. Szabó, “Mathematics in a physical world,” in M. Stannet et al. (eds.), *Pre-proceedings of the 3rd Intentional Hypercomputation Workshop* (HyperNet 11), TUCS Lecture Notes 14, Turku (2011).

M. Gömöri and G. Hofer-Szabó, “On the meaning of EPR’s Criterion of Reality,” (in preparation).

A. Hájek and J. Bub, “EPR,” *Found. Phys.*, **22**, 313-331 (1992).

H. Halvorson and R. Clifton, “Reconsidering Bohr’s reply to EPR,” in J. Butterfield and T. Placek (eds). *Non-Locality and Modality*, Kluwer Academic Publishers, 3-18 (2002).

G. Hofer-Szabó, M. Rédei, and L. E. Szabó, “On Reichenbach’s common cause principle and on Reichenbach’s notion of common cause,” *Brit. J. Phil. Sci.*, **50**, 377-399 (1999).

D. Howard, “What makes a classical concept classical? Toward a reconstruction of Niels Bohr’s philosophy of physics,” in: Niels Bohr and Contemporary Philosophy. Vol. 158 of Boston Studies in the Philosophy of Science. Kluwer, Dordrecht, pp. 201–229. (1994).

T. Maudlin, “What Bell did,” *J. Phys. A: Math. Theor.*, **47**, 424010 (2014).

T. Placek and L. Wroński, “On infinite EPR-like correlations,” *Synthese*, **167 (1)**, 1-32 (2009).

H. Price, *Time’s Arrow and Archimedes’ Point*, (Oxford University Press, 1996)

E. Schrödinger, “The present situation in quantum mechanics,” in: J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement* (Princeton Series in Physics vol 23) (Princeton, NJ: Princeton University Press) (Engl. transl. of “Die gegenwärtige Situation in der Quantenmechanik,” *Naturswissenschaften*, **23**, 807–49) (1935).

I. Sanpedro, “On Free Will and No-Conspiracy,” in: T. Sauer, A. Wüthrich (eds.), *New Vistas on Old Problems. Recent Approaches to the Foundations of Quantum Mechanics*, (Edition Open Access, 2013).

R. Tumulka, “Comment on ‘The free will theorem’,” *Found. Phys.*, **37**, 186–197 (2007).

B. C. van Fraassen, “The charybdis of realism: Epistemological implications of Bell’s inequality,” *Synthese*, **52**, 25–38.

H. Zinkernagel, “Are we living in a quantum world? Bohr and quantum fundamentalism,” *One hundred years of the Bohr atom: Proceedings from a conference* (Edited by F. Aaserud and H. Kragh). Scientia Danica. Series M: Mathematica et physica, vol. 1., 2015.