Toward a Quantum Theory of Cognition: History, Development and Perspectives

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Example: Modeling Concept Conjunction and Negation

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  where $\mu(\cdot)$ represents the probability of an outcome, and AB is the outcome '(yes,yes)'of the joint experiment $E_A E_B$.
- Ex. $\mu(A) = 0.7$, $\mu(B) = 0.42$, and $\mu(AB) = 0.1$ is not a possible experience.

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  6. **ETC.**
- However, there is a body of experimental evidence that challenges the validity of the conditions of possible experience in cognition

Example: Overextension of Conjunction

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- However, psychological findings show strong 'overextensions' (inversions of (3)) in experimental data (Osherson & Smith, 1981; Hampton, 1988).

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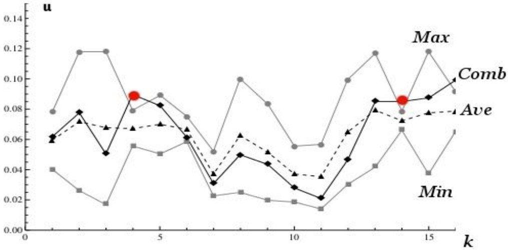

- We would expect $\mu(AB) \leq \mu(X)$, $X = A, B$. However, all objects are overextended (red points are *doubly* overextended)

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1. **Vagueness**: Fuzzy boundaries
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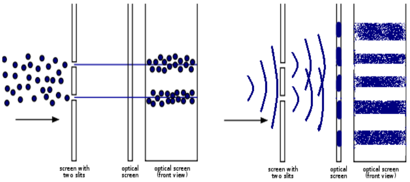
3. **Non-compositionality**: Combinations are structurally different than the parts

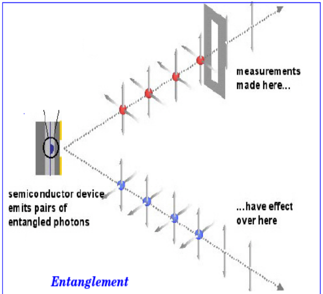
- These three features have been largely discussed in cognitive science, but no formal tools have proven to be satisfactory

Violations of Possible Experience and Quantum Probability

- Vagueness, Contextuality, and Non-compositionality are fundamental features of quantum theory



Interference



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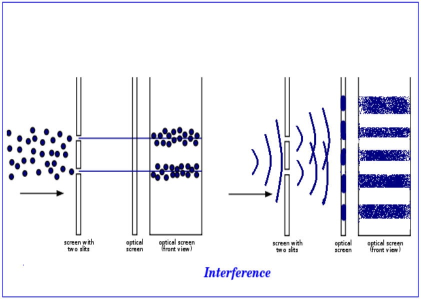

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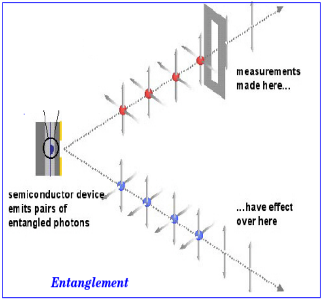

Entanglement

- Vagueness→ State Superposition
- Context→ Measurement-Induced Collapse
- Non-compositionality→ Interference, Entanglement

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Intensional meaning	State
Context	Measurement Operator
Extensional meaning	Collapsed State
Combination	Entanglement / Interference

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- Quantum Cognition **does not** follow or take a position w.r.t. *Quantum brain* hypothesis

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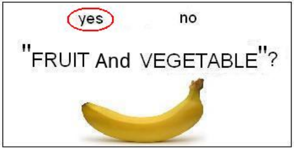
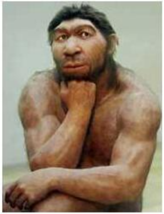
Modes of Thought

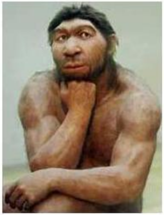
Emergent Mode of Thought (Hilbert Space)





YES

Logical Mode of Thought (Tensor Product)



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- In particular, we can model the two interpretations underlying the combination of concepts in $\mathcal{H} \oplus (\mathcal{H} \otimes \mathcal{H})$
- The conjuntion concept is described by the superposition of modes of thought

$$|AB\rangle = ne^{i\phi}|AB_1\rangle + \sqrt{1-n^2}e^{i\theta}|AB_2\rangle$$

- $n = 1$ implies first mode and $n = 0$ implies second mode of thought

A Simple Model Illustrating the General Scheme

- The membership operator is $\mathbf{M}^F = \mathbf{M} \oplus (\mathbf{M} \otimes \mathbf{M})$. Then

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- Can we assume that *conditions of possible experience* of this type apply in cognition?

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- Conditions of possible experience:

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\begin{aligned}
I_A &= \mu(A) - \mu(AB) - \mu(A\bar{B}) = 0, && (5)\\
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I_{AB\bar{A}\bar{B}} &= 1 - \mu(AB) - \mu(\bar{A}B) - \mu(A\bar{B}) - \mu(\bar{A}\bar{B}) = 0.
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- We tested 4 pairs of concepts, 24 exemplars for each pair.

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- Next, when we tested the 95% confidence intervals of the classical rule for combinations I_X, $X = \{A, B, \bar{A}, \bar{B}\}$, we obtain

95% CI	$j = 1$	$j = 2$	$j = 3$	$j = 4$
I_A	$(-0.469, -0.406)$	$(-0.476, -0.390)$	$(-0.426, -0.349)$	$(-0.463, -0.398)$
I_B	$(-0.482, -0.427)$	$(-0.443, -0.376)$	$(-0.429, -0.368)$	$(-0.495, -0.446)$
$I_{\bar{A}}$	$(-0.458, -0.393)$	$(-0.375, -0.326)$	$(-0.332, -0.259)$	$(-0.359, -0.302)$
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- What about trying the quantum model developed for conjuntions in this case?

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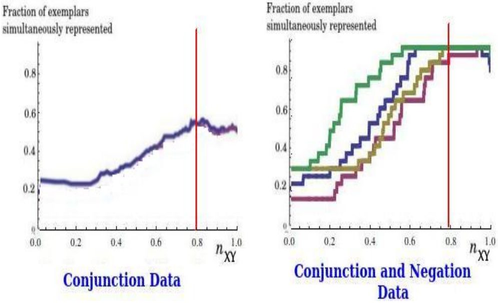
- Non classical data can be modeled in this scheme!

Comparing different datasets

Comparing the model's performance w.r.t data set on conjunction (Hampton), and on conjunction and negation (Aerts, Sozzo, Veloz)



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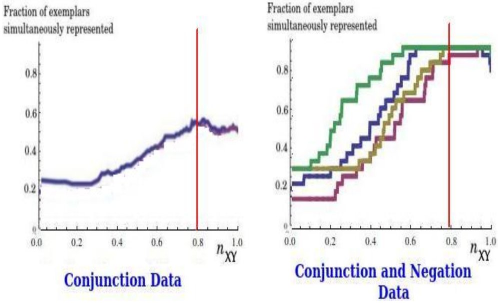

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Emergent mode of thought is dominant in these two distinct datasets!

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- Data reveals emergent mode of thought is dominant
- Shall we think of new (quantum?) conditions of possible experience in cognition?

Thank you!...questions?

- Boole, G. On the Theory of Probabilities. Philosophical Transactions of the Royal Society of London (1962)
- Boole, G. The Laws of Thought. New York: Dover Edition (1958)
- Pitowsy, I. George Boole's Conditions of Possible Experience and the Quantum Puzzle. Brit. J. Phil. Sci. (1995)
- Van Rijsbergen, C. J. (2004). The geometry of information retrieval (Vol. 157). Cambridge: Cambridge University Press.
- Proceedings of the n-th Quantum Interaction Symposium-Qi-n, n=2007,...,2015.
- **These and the other references can be requested to me, space problems to put them all!**