Are prohibitions of superluminal causation by stochastic Einstein locality and by absence of Lewisian probabilistic counterfactual causation equivalent?

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Philosophy of Science **60** (1993) 608-618

Abstract

Butterfield's (1992 a-c) claim of the equivalence of absence of Lewisian probabilistic counterfactual causality (**LC**) to Hellman's Stochastic Einstein Locality (**SEL**) is questioned. Butterfield's assumption on which the proof of his claim is based would suffice to prove that **SEL** implies absence of **LC** also for appropriately given versions of these notions in algebraic quantum field theory – but the assumption is not an admissible one. The conclusion must be that the relation of **SEL** and absence of **LC** is open, and that they may be independent.

1 Introduction

The notion of stochastic Einstein locality (**SEL**) was introduced by Hellman (1982a,b) in order to express the prohibition of superluminal causation in physical theories that give a probabilistic description of

*Acknowledgements: This work was supported in part by the Hungarian National Foundation for Scientific Research, grant No. 1900, and was completed while I was staying on a TEMPUS Individual Mobility Grant in the Faculty of Philosophy of the Universty of Groningen. I am indebted to Professor J.D. North and Professor D. Atkinson for their hospitality while staying in Groningen. I also wish to thank an anonymous referee for helpful comments. Special thanks go to Dr. J. Butterfield for providing me with the manuscripts of his forthcoming works and for valuable discussions and correspondence.

physical systems. The specific probabilistic theory Hellman had in mind was a stochastic hidden variable theory of quantum mechanics, and Hellman argued that if one accepts **SEL** as a prohibition of superluminal causation, then the "Bell-locality" condition (also called "factorizability", or "conditional stochastic independence") used in the derivation of Bell's inequalities for *stochastic* hidden variables is not implied by **SEL**. Consequently, argues Hellman (1982b), violation of Bell's inequalities in a stochastic hidden variable theory does not by itself imply physical non-locality (in the sense of superluminal causality).

Butterfield's (1992a-c) motivation to apply Lewis' theory of counterfactual probabilistic causality in a quantum context, was also the problem of Bell's inequalities: based on Jarret's decomposition of factorizability into two conditions called locality and completeness Butterfield (1992a) argues that given endorsement of locality there is nevertheless superluminal causation between outcomes of events involved in the usual Bell experimental setup if superluminal causation is taken to be Lewisian probabilistic counterfactual causation (Lewisian causation, for short). According to Butterfield, under a natural assumption on counterfactual worlds, prohibition of superluminal causation in quantum mechanics by **SEL** is equivalent to the absence of Lewisian causality, as defined in Butterfield (1992a,b), between spacelike separated quantum events involved in the Bell-Bohm system (see Butterfield (1992a,b); Fleming and Butterfield (1991); for a detailed argument, see Butterfield 1992c). The aim of this paper is to question Butterfield's claim and its proof. Before pointing out the problem regarding Butterfield's claim another line of development should be recalled, however.

The question of wether superluminal causation in quantum theory exists needed a re-examination after the discovery of violation of Bell's inequalities in algebraic relativistic quantum field theory (**ARQFT**) (Summers and Werner 1987a,b,c; 1988; also see Summers (1990). Motivated by the results of Summers and Werner, Hellman's **SEL** condition was reformulated in Rédei (1991) so that it would be applicable to **ARQFT**, and it was shown that the relativistic covariance of **ARQFT** implies that **ARQFT** is a **SEL** theory. This result is in line with that of Hellman's (for the interpretation of the **SEL** property of **ARQFT** for the stochastic hidden variable problem see also Rédei (1991); and, in view of Butterfield's argument, the natural question araises as to whether the **SEL** property of **ARQFT** is equivalent to the absence of Lewisian causality in **ARQFT**. It is this question and the framework of **ARQFT** in which problems in connection with Butterfield's claim will be investigated here.

First I recall the **SEL** property as defined for **ARQFT** in Rédei (1991) (Definition 1 below), and then the Lewisian causality between local algebras in **ARQFT** will be defined (Definition 2 and 3). Having the definitions I formulate two claims (Claim 1 and Claim 2) that assert the equivalence of **SEL** and absence of Lewisian causality in **ARQFT**, and for each claim a "proof" will be given. These "proofs" are not real proofs – they are incomplete – but I will argue that Claim 1 asserting that **SEL** implies the absence of Lewisian causation in **ARQFT** can be made complete under the same "natural assumption" (called in Butterfield (1992b) "sophisticated engineering") that Butterfield makes in order to prove his equivalence claim for the original **SEL** and Lewisian causality notions. This assumption is essentially equivalent to the implication to be proven, i.e. the implication to be proven is put in by hand and there does not seem to be any independent reason to make the assumption other than that it does the job. Claim 2, which asserts that absence of Lewisian causation implies **SEL**, will seem to be unprovable even under the assumption Butterfield makes. Finally, I will argue that, in addition to problems regarding Butterfield's equivalence claim on the formal level, if one takes into account Hellman's proviso to **SEL**, then further considerations indicate that the answer to the topic question of this paper is negative.

2 Stochastic Einstein locality and algebraic quantum field theory

Throughout this paper $\mathcal{A}$ is assumed to be a quasilocal C^*- algebra of relativistic quantum field theory satisfying the usual axioms of isotony, local commutativity and Poincaré covariance (for the theory of algebraic quantum field theory we refer to the monograph Horuzhy (1990). It will also be assumed that all strictly local algebras $A(V)$ (V being an open bounded spacetime region in the Minkowski space) are von Neumann algebras. Part of the axioms of relativistic quantum field theory is also the assumption of existence of at least one physical representation of the quasilocal C^*-algebra $\mathcal{A}$, which means mathematically that one postulates the existence of a Poincaré-invariant state such that the spectrum condition holds in the corresponding cyclic representation; however, in what follows the existence and properties of the vacuum do not play a role.

The violation of Bell's inequalities in **ARQFT** for observables belonging to algebras localized in spacelike separated spacetime regions (see Summers and Werner (1987a,b,c; 1988) for precise statements) raises the question whether some kind of superluminal causation exists between such local algebras in **ARQFT**. To answer this question prohibition of superluminal causation must be defined in a general way that is precise enough to enable one to prove either that **ARQFT** satisfies it or violates it. As mentioned above, Hellman (1982a,b) invented **SEL** just in order to give such a definition of prohibition of superluminal causation. Hellmann's idea was to consider a stochastic theory **T** as a formal language **L(T)** and formulate the definition of Einstein locality in terms of models of **T**. To define stochastic Einstein locality for **ARQFT** Hellmann's definition of **SEL** was followed in Rédei (1991): Let **L(ARQFT)** be the formal language describing **ARQFT**. A model **m** of **L(ARQFT)** fixes a concrete Minkowski space M (in the sense of "fixing" used in model theory: given a set M with relations, functions etc. defined on it satisfying axioms so that the structure can be viewed as a representative of what one calls Minkowski space) and a concrete net of local algebras $A(V)$ on M (e.g. a Bose field) such that the axioms of isotony, relativistic covariance etc. are all satisfied by the net $A(V)$. Model **(m,L(ARQFT))** is then a "frame theory" of state of affairs in physically real quantum fields in the sense that a "real" physical situation is described once we fix a state of the quantum field, that is we chose a Φ in **m**, which yields then the probabilities of all possible events E. Let $\ulcorner\Phi(E) = r\urcorner$ stand for the formal formula in **L(ARQFT)** expressing the statement in **L(ARQFT)** that "E has probability r if the quantum field is (prepared) in the state Φ". Since the expectation value of any local observable is well defined in **ARQFT** in every state, **L(ARQFT)** is also assumed to be locally categorical, by which we mean that if $\ulcorner\Phi(Q)\urcorner$ is a formula in **L(ARQFT)** expressing the expectation value of the local observable $Q \in A(V)$ in the state Φ, then for every modell **m**, $\mathbf{m} \models\ulcorner \Phi(Q) = r\urcorner$ holds for a unique real number $r \in [0, 1]$. (I follow Hellmann (1982a,b) in the notations concerning the logical operations: '$\mathbf{m} \models\ulcorner \Phi(Q) = r\urcorner$ ' means that the formula '$\Phi(Q) = r$ ' is satisfied in the model **m**; '$\longleftrightarrow$ ' stands for the "if and only if" in the metalanguage and so forth.) Let $PLC(e)$ be the past light cone of the event $e \in M$ and for a spacetime region V let $PLC(V)$ be given by

$$PLC(V) = (\cup_{e\in V} PLC(e)) \setminus V$$

Furthermore, let $\ulcorner P(Q \in d) = r\urcorner$ be the formula in **L(ARQFT)** which asserts that "the probability that the observable Q has its value in the Borel set d of the real numbers is equal to r".

Definition 1: We say that "**ARQFT** is **SEL** in V" if for all $Q \in A(V)$ the following hold: for every two models **m,m′** of **L(ARQFT)** if

$$\mathbf{m} \models^{\ulcorner} R(V_1, V_2, \ldots, V_n)^{\urcorner} \longleftrightarrow \mathbf{m}' \models^{\ulcorner} R(V_1, V_2, \ldots, V_n)^{\urcorner}$$

$$\mathbf{m} \models^{\ulcorner} f(V_1, V_2, \ldots, V_n) = y^{\urcorner} \longleftrightarrow \mathbf{m}' \models^{\ulcorner} f(V_1, V_2, \ldots, V_n) = y^{\urcorner}$$

for all $V_i \in PLC(V)$ and all functions f and relations R
then

$$\mathbf{m} \models^{\ulcorner} P(Q \in d) = r^{\urcorner} \longleftrightarrow \mathbf{m}' \models^{\ulcorner} P(Q \in d) = r^{\urcorner}$$

for all r and d. **ARQFT** is called stochastic Einstein local if "**ARQFT** is **SEL** in V" (in the sense of the above definition) holds for every V.

This definition is a "smeared" version of Hellman's definition of **SEL** in Hellman (1982b). Hellman emphasizes, however, that his definition of **SEL** must carry a proviso which distinguishes spurious violations of **SEL** from genuine ones due to superluminal causation. Spurious violations of **SEL** arise if one applies **SEL** to probabilities that can be derived solely by logic and mathematics from joint probabilities that are themselves locally determined in a well defined sense (Hellman 1982 b p.470-471). For instance, violations of **SEL** coming from correlations due to (locally determined) conservation principles should not count as genuine violations of **SEL**, for otherwise no probabilistic theory with conservation principles could comply with Einstein locality in the sense of satisfying **SEL**. Since the above definition of **SEL** is just a smeared version of Hellman's **SEL**, it also must be supplemented by the same proviso, and in what follows this will be assumed.

For the proof of the next proposition see (Rédei 1991).

Proposition 1: Algebraic relativistic quantum field theory is stochastic Einstein local.

3 Lewis counterfactual probabilistic causality and algebraic quantum field theory.

Butterfield (1992a) applies Lewis' (1986, 159-184) theory of counterfactual probabilistic causation to the usual Bohm-Bell system. On Lewis' theory an event E causally depends on another event F, if the following two counterfactuals with chance consequents are true:

$$O(F) \rightarrow P(O(E)) = r \qquad \neg O(F) \rightarrow P(O(E)) = s \qquad r \gg s$$

where $O(F)$ (resp. $\neg O(F)$) is the proposition stating that the event F occurs (respectively does not occur), $P()$ is the chance function just after the time (spacelike hypersurface) that F occurs or does not occur, as the case may be, and r and s are real numbers in $[0,1]$ such that r is much greater than s.

Roughly, according to the above definition, E causally depends on F, if the chance (probability) that E does occur in an actual world w is much greater if F indeed occurs in w than it would be if F did not occur in w. If the actual and counterfactual chances of E (i.e. r and s) differ just by a small amount, then we say that *weak* Lewisian causality between F and E exists (see arguments for this weakening in Butterfield (1992a,b).

To raise the question of relation of **SEL** to Lewisian causality, Lewisian causality, too, must be reformulated in terms of models of **L(ARQFT)**. To do that I make the following assumptions/identifications:

1. The local events in a spacetime region V are identified with the projections in the von Neumann algebra $A(V)$ pertaining to V.
2. Probabilities of events are determined by states and states by probabilities; i.e. given an event F and its probability $P(F)$, there is a state Φ on the quasi-local algebra $\mathcal{A}$ such that $P(F) = \Phi(F)$ and each state Φ defines the probabilities of all events via $P(F) = \Phi(F)$.
3. A possible world is identified with the pair $(\mathbf{m}, \Phi)$ where $\mathbf{m}$ is a model of the formal language **L(ARQFT)** and Φ is a state (in the model $\mathbf{m}$) on the quasilocal algebra $\mathcal{A}$.
4. The "probability that the event $Z \in A(V)$ happens in the possible world $(\mathbf{m}, \Phi)$" is identified with the probability assigned by $(\mathbf{m}, \Phi)$ to the event Z, i.e. this probability is equal to r if $\mathbf{m} \models \ulcorner \Phi(F) = r \urcorner$ – and recall that $\mathbf{m} \models \ulcorner \Phi(F) = r \urcorner$ does hold for some r by local categoricity of **L(ARQFT)**). The F-actual worlds are identified with the worlds $(\mathbf{m}, \Phi)$ in which the probability that F happens is equal to 1, i.e. with the worlds $(\mathbf{m}, \Phi)$ that assign probability 1 to F. Accordingly, the counterfactual-for-F worlds, the worlds in which F does not happen, are those $(\mathbf{m}, \Phi)$ for which $\mathbf{m} \models \ulcorner \Phi(F) = 0 \urcorner$ holds.

Equations

$$ACT(F) \equiv \{(\mathbf{m}, \Phi) : \mathbf{m} \models \ulcorner \Phi(F) = 1 \urcorner\}$$

and

$$CFACT(F) \equiv \{(\mathbf{m}, \Phi) : \mathbf{m} \models \ulcorner \Phi(F) = 0 \urcorner\}$$

denote the class of F-actual resp. the class of counterfactual-for-F worlds, respectively.

With these identifications the idea of Lewisian causal dependence can be re-worded in this way: in every F-actual world $(\mathbf{m}, \Phi) \in ACT(F)$ the probability that the event E happens is much greater than in any counterfactual-for-F world $(\mathbf{m}, \Phi) \in CFACT(F)$. Formally, and more explicitly:

Definition 2: Let

$$FACTPROB(E) \equiv \{r : \mathbf{m} \models^{\ulcorner} \Phi(E) = r^{\urcorner}, (\mathbf{m}, \Phi) \in FACT(F)\}$$

and

$$CFACTPROB(E) \equiv \{r : \mathbf{m} \models^{\ulcorner} \Phi(E) = r^{\urcorner}, (\mathbf{m}, \Phi) \in CFACT(F)\}$$

denote the sets of F-actual and conterfactual-for-F probabilities of E; furthermore, let $R > 1$ be a real number (thought to be large and characterizing the strength of causal dependence of E on F). We say that E causally depends on F if the ratio of the F-actual and counterfactual- for-F probabilities of E is larger then R, i.e. if $r/s \geq R$ for all $r \in FACTPROB(E)$ and $s \in CFACTPROB(E)$. If there is no such $R > 1$ but $(r/s) \neq 1$ for any r and s in the respective sets, then we say that E causally depends weakly on F.

Definition 3: There is (weak) Lewisian causality between the algebras $A(V_1)$, $A(V_2)$ in **ARQFT** pertaining to spacelike separated spacetime regions V_1 and V_2 if there are projections $E \in A(V_2)$ and $F \in A(V_1)$ such that E causally (weakly) depends on F in the sense of the above Definition 2. Accordingly, **ARQFT** is said to be free of (weak) Lewisian causality if there are no local algebras pertaining to spacelike separated spacetime regions that have projections with (weak) Lewisian causality between them.

4 Are stochastic Einstein locality and absence of Lewisian causality equivalent?

Now we are in the position of formulating the two claims that spell out, for **ARQFT**, the equivalence of **SEL** and absence of Lewisian causality.

Claim 1: **SEL** implies absence of weak Lewisian causality:

"Proof": Claim 1 is equivalent to the statement "If there is weak Lewisian causality between spacelike separated algebras $A(V_1)$ and $A(V_2)$ then **ARQFT** is not **SEL**". If there is weak Lewisian causality between the said two algebras, then there are projections $F \in A(V_1)$

and $E \in A(V_2)$ such that the F-actual and contrafactual-for-F probablities of E are all different, thus for any two possible worlds $(\mathbf{m}, \Phi)$ and $(\mathbf{m}', \Phi')$ we have that

$$(i) \qquad \text{If} \qquad \mathbf{m} \models^{\ulcorner} \Phi(F) = 1^{\urcorner} \qquad \text{then} \qquad \mathbf{m} \models^{\ulcorner} \Phi(E) = r^{\urcorner}$$

$$(ii) \qquad \text{If} \qquad \mathbf{m}' \models^{\ulcorner} \Phi'(F) = 0^{\urcorner} \qquad \text{then} \qquad \mathbf{m}' \models^{\ulcorner} \Phi'(E) = s^{\urcorner}$$

for some r different from s.

To prove that **ARQFT** is not **SEL** we should now be able to find two models $\mathbf{m}$ and $\mathbf{m}'$ that agree on every (open bounded) spacetime region in the past light cone $PLC(V_2)$ of V_2 but which differ in what probability they assign to the event E. Intuitively, the model $\mathbf{m}'$ should be obtainable from $\mathbf{m}$ satisfying (i) by modifying $(\mathbf{m}, \Phi)$ into $(\mathbf{m}', \Phi)$ on that part of the $PLC(V_1)$ that does not intersect with $PLC(V_2)$, that is on $X \equiv PLC(V_1) \setminus PLC(V_2)$. This modification should be possible on the basis of the assumption of existence of Lewisian causality: One should be able to modify $(\mathbf{m}, \Phi)$ into an $(\mathbf{m}', \Phi)$ such that $\mathbf{m}' \models^{\ulcorner} \Phi(F) = 0^{\urcorner}$ and such that this modification should be within the bounds of X, and then the proof is complete. But, assuming weak (or even strong) Lewisian causality only, there does not seem to be any reason why this modification, if at all possible, can be restricted to the area X. Butterfield assumes that "...for any world w, a supposition, counterfactual relative to $(\mathbf{m}, \Phi)$ about matters... outside E's causal past [assumption such as 'the $(\mathbf{m}, \Phi)$-probability of F is equal to 0'] leads to a class of worlds each of which, (1) matches $(\mathbf{m}, \Phi)$ ... within E's causal past (2) like w, has the same laws as the actual world $(\mathbf{m}, \Phi)$". In the notation of this paper this is equivalent to Butterfield's (1992b) natural assumption ("sophisticated engineering") under which Butterfield proves the equivalence of the original **SEL** and Lewisian causality. Thus he assumes just what is needed, i.e. that $(\mathbf{m}, \Phi)$ *can* be modified as required without modifying it anywhere within $PLC(V_2)$.

Let us formulate the converse of Claim 1:

Claim 2: If there is no pair $(A(V_1), A(V_2))$ of local algebras pertaining to spacelike separated regions V_1, V_2 such that there is Lewisian causality between them, then **ARQFT** is **SEL**.

"Proof": Claim 2 is equivalent to "If **ARQFT** is not **SEL**, then there is a pair $(A(V_1), A(V_2))$ with weak Lewisian causality." If **ARQFT** is not **SEL** then there is a local algebra, $A(V_2)$ say, a projection $E \in A(V_2)$ and there are two models $\mathbf{m}$ and $\mathbf{m}'$ of **L(ARQFT)** that agree on $PLC(V_2)$, but (by local categoricity of **L(ARQFT)**) for some r different from s we have that

$$\mathbf{m} \models^{\ulcorner} \Phi(E) = r^{\urcorner} \qquad \text{and} \qquad \mathbf{m}' \models^{\ulcorner} \Phi(E) = s^{\urcorner}$$

Let us take another spacetime region V_1 spacelike separated from V_2 and pick a projection $F \in A(V_1)$. "Sophisticated engineering" allows a modification of $(\mathbf{m}, \Phi)$ and $(\mathbf{m}', \Phi)$ on X *only* so that $\mathbf{m} \models^{\ulcorner} \Phi(F) = 1^{\urcorner}$ and $\mathbf{m}' \models^{\ulcorner} \Phi(F) = 0^{\urcorner}$ (if this is not the case already), and this means that violation of **SEL** implies that there are possible worlds $(\mathbf{m}, \Phi) \in ACT(F)$ and $(\mathbf{m}', \Phi) \in CFACT(F)$ such that the F-actual and counterfactual-for-F probabilities of E are different. However, to prove that there is at least weak Lewisian causality between $A(V_1)$ and $A(V_2)$, one must show that for *every* pair of possible worlds $(\mathbf{m}, \Phi)$ and $(\mathbf{m}', \Phi')$ if $(\mathbf{m}, \Phi) \in ACT(F)$ and $(\mathbf{m}', \Phi') \in CFACT(F)$ then the corrresponding actual and counterfactual probabilities are different. There is nothing in the negative concept "no **SEL**" to warrant this – not even assuming sophisticated engineering.

Though I formulated above my problem with Butterfiled's claim in terms of the definitions I gave, I do not think that this matters, the problem is the same with Butterfield's less formal proof of the equivalence of the original **SEL** and absence of Lewisian causality.

The assumption of sophisticated engineering is but the assumption of what is to be proven, and, unless there is good, independent reason to make this assumption, it can hardly be considered an admissible assumption. I do not see any independent reason to assume it.

5 Further arguments in favor of inequivalence of stochastic Einstein locality and absence of Lewisian causality

Further problems arise in connection with Butterfield's equivalence claim which relate to the proviso (see Section 2). Butterfield (1992c) seems to acknowledge that without the proviso straightforward counterexamples threaten the appropriateness of **SEL**. If, however, absence of Lewisian causality and **SEL** were equivalent irrespective of the proviso, then the counterexamples to **SEL** would have to show up as counterexamples to the notion of (lack of) Lewisian causality as well. It would be interesting to see in this connection whether the counterexample to **SEL** in Hellman (1982b, 474) can be described in **ARQFT**: If so, then this would explicitly show a deficiency of the absence of Lewisian causality *without* any proviso as a criterion of "no superluminal causation". It remains unclear whether this counterexample can be worked out in **ARQFT**. In any case, there is no proviso[1] to the definition of Lewisian causality, consequently, Lewisian causality can certainly not imply violation of **SEL** *with* the proviso; that is, Lewisian causality by itself does not imply superluminal causation in physical sense. This statement is a stronger, physically interpreted version of invalidity of Claim 1.

This together with the "Proofs" of Claims 1 and 2 indicate that **SEL** and Lewisian causality, as defined here, may be independent. For the following additional reasons I am inclined to take this position despite the fact that I have no formal proof of the independence of **SEL** and Lewisian causality.

"(No) superluminal causation" is a rather informal, intuitive concept. There is always slack between such a principle and a formal expression of it. That this is indeed so is shown by the fact that there are many distinct explications in **ARQFT** of the "no superluminal causation" principle: the different independence conditions put on the local algebras belonging to spacelike separated spacetime regions (such as microcausality, C^*- and W^*-independence, strict locality, split property, primitive causality, etc., see eg. Horuzhy (1990) and Summers (1990) are all expressions of the requirement that spacelike separated quantum events can not influence each other. The point is that the mentioned notions are *not* equivalent, thus even within one and the same theory (**ARQFT**) there are a number of different explications of "no superluminal causation".

In view of the existence of these different and interconnected relativistic locality concepts one should pose the question, as to where **SEL** and absence of Lewisian causality fit into that hierarchy. Investigating this problem should be subject of another paper. Here I restrict myself to one brief comment regarding the relation of **SEL** to the locality concept called *primitive causality.*

Primitive causality is the expression in terms of **ARQFT** of the requirement of hyperbolicity of time evolution in quantum field theory Horuzhy (1990, 14-15). Since we are interested in the relation of primitive causality to **SEL** (taken in the sense of Definition 1), which is a locally formulated concept, we need the notion of *local* primitive causality. This axiom requires that $A(V) = A(V^c)$ for any spacetime region V, where V^c is the causal hull of V (V^c is the set of those spacetime points from which any straight, timelike ray passes through V). Note that primitive causality is independent of the minimal axioms of **ARQFT** (Haag and Schroer 1962), in particular it is independent of the axiom of relativistic covariance. To prove that **ARQFT** is **SEL** (in the sense of Definition 1) the relativistic covariance axiom was sufficient in (Rédei 1991), thus the **SEL** property for **ARQFT** seems to be independent of local primitive causality. That hyperbolicity is not needed to prove **SEL** for **ARQFT** points toward a possible (and needed) refinement of **SEL**, which was argued for by Butterfield. Butterfield suggests[1] refining the **SEL** notion by restricting the class of functions f in terms of which matching of models in $PLC(V)$ is expressed to a subclass Ω of all conceivable functions defined on the subsets of V. If the elements in Ω, which are defined on spacetime regions, are considered as representatives of real physical relations in the respective spacetime regions, they should somehow be coupled to the underlying relativistic causality structure. Such a coupling could be for instance the demand that they be "causally relevant", by which is meant that they should not distinguish causally dependent regions. This idea can be operationalized as follows: One can demand that Ω consists only of those functions for which $f(V) = f(V^c)$ for every spacetime region V. One can now modify Definition 1 by requiring the functions f to lie in Ω. Calling Ω-**SEL** the appropriately strengthened notion of **SEL**, one can ask the question whether **ARQFT** can be proven to be Ω-**SEL**. The relativistic covariance of **ARQFT** is not sufficient for the proof of Proposition 1 in (Rédei 1991) to go through in this case; however, it is not difficult to prove that assuming local primitive causality for **ARQFT** implies that **ARQFT** is Ω-**SEL**. Thus **SEL** can indeed be related to primitive causality in a rather natural way. No direct link seem to connect the absence of Lewisian causality and primitive causality, however. This again indicates the independence of these two notions.

Under the assumption that "no superluminal causality" is richer than allowing just one specification, it is not at all impossible that Lewisian causality is appropriate if we want to express that there *is* superluminal causation, but it might not be suitable if we want to express that there is *not*. For the claim is *not* this: There is superluminal causation *if and only if* there is Lewisian causality; but this: if there is Lewisian causality then there is superluminal causation. Therefore, it should not be surprizing if nothing can be inferred from the assumption that there is no Lewisian causality, in particular, that we may not be able to infer from this that **SEL** holds. Similarly, **SEL** may be appropriate to express that there is *no* superluminal causality, but it may be inappropriate to express that there *is*. For the claim is not this: there is no superluminal causation if and only if **SEL** holds; but: if **SEL** holds then there is no superluminal causality. Consequently, nothing can be expected to be deducible from the assumption that **SEL** does not hold, in particular, we cannot hope to be able to prove that in this case there must be (weak) Lewisian causality between appropriate events.

[1] The idea was formulated correspondence.

Hellman (1982b) points out a limitation even on the inference "if SEL holds then no superluminal causation": This inference is equivalent to "if there is superluminal causation then SEL is violated"; however, argues Hellman (1982b, 492-485), if there is "too much" superluminal causation, then it might not be possible to find models exhibiting violation of SEL. The action- at-a-distance postulates of such "heavily superluminal" theories would be unverifiable, however. Thus SEL should be applied as a sufficient criterion of no superluminal causation to probabilistic theories in which a probabilistic superluminal action would not be everywhere unverifiable (ibid. 485).

If indeed **SEL** does not imply absence of Lewisian causal dependence between spacelike separated events in **ARQFT**, then the question arises as to whether probabilistic counterfactual causal dependence holds between such events in **ARQFT**. In particular, the question should be asked if violation of Bell's inequalities in **ARQFT** implies the existence of Lewisian counterfactual probabilistic causality between spacelike separated events. This question is worth of careful scrutiny and work is underway to investigate it.

Notes

1 Confronted with the problem of provisos in connection with his equivalence claim Butterfield (1992c, 13) seems to indicate that he considers the non-backtracking feature of counterfactuals as a kind of "built-in-proviso" to the definition of Lewisian causality. It remains to be seen, however, how, if at all, this "built-in-proviso" implies just the proviso that is needed for **SEL**.

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