Reichenbach's common cause principle and quantum field theory

Miklós Rédei*

Faculty of Natural Sciences, Loránd Eötvös University

Rákóczi út 5., H-1088 Budapest, Hungary

email: redei@ludens.elte.hu

Abstract

Reichenbach's principle of a probabilistic common cause of probabilistic correlations is formulated in terms of relativistic quantum field theory and the problem is raised whether correlations in relativistic quantum field theory between events represented by projections in local observable algebras $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ pertaining to spacelike separated spacetime regions V_1 and V_2 can be explained by finding a probabilistic common cause of the correlation in Reichenbach's sense. While this problem remains open, it is shown that if all superluminal correlations predicted by the vacuum state between events in $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ have a genuinely probabilistic common cause, then the local algebras $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ must be statistically independent in the sense of C^*-independence.

1   Introduction

As a consequence of violation of Bell's inequality in algebraic relativistic quantum field theory (ARQFT) [16], [17], [18], [19] [20], [23] (see [22] and [21] for reviews) ARQFT predicts *superluminal* correlations, i.e. correlaions between events represented by projections belonging to von Neumann algebras that pertain to *spacelike separated* spacetime regions. Unless one takes the position that correlations need not be explained at all, a position taken by Van Fraassen for example [25], one would like to say either of the following

1. There exists a direct causal connection between the correlated events

2. There exists a probabilistic common cause of the correlation

   In fact, in the case of *superluminal* correlations one would *not* like to say 1. – and consider it true, too, since spacelike separated events are not supposed to causally influence each other. Yet, option 1. is not a priori impossible, for it can happen that ARQFT does not comply with the no-action-at-a-distance principle, despite the fact that this theory was constructed precisely with the aim of creating a quantum theory that complies with the no-action-at-a-distance principle. However, to claim that there is (or that there is not) causal connection between spacelike separated events, one has to specify "causal connection" in terms of ARQFT precisely enough to be able to prove absence/presence of a causal link. There exist reasonable (and apparently different [10]) definitions (of absence) of a causal link between projections as events, such as Stochastic Einstein Locality [6], [9], its strengthening called Stochastic Haag Locality [8] and counterfactual probabilistic causal connection [2], [11]. While there remain some open questions concerning the status in ARQFT of these (and of different versions [3] of these) notions of prohibition of superluminal causal link, results have been obtained showing that ARQFT is free from a direct superluminal causal connection between spacelike events in the sense of most of these definitions – despite presence of superluminal correlations in the theory. These negative results on 1. leave one with option 2. The aim of this paper is to investigate this option 2. More precisely, we wish to specify

Reichenbach's notion of a probabilistic common cause [13] in terms of ARQFT in order to raise the problem of whether superluminal correlations predicted by ARQFT can be causally explained in field theory in the sense of Reichenbach's probabilistic theory of common cause suitably adapted to ARQFT. In what follows, first we summarize briefly Reichenbach's common cause principle, and in particular his notion of "screening off" (section 2). We shall distinguish two types of screening off: the strong one, in which the causing event actually implies both of the correlated events; and the genuinely probabilistic case, in which the probabilistic cause does not entail any of the correlated events. In section 3 the basic notions of ARQFT will be recalled, together with results showing the existence of superluminal correlations. This is followed in section 4 by a definition of Reichenbach's principle of common cause in ARQFT. We wish to stress that we are not able to give an answer to the apparently difficult question (Problem in section 4) of whether the superluminal correlations predicted by ARQFT have a probabilistic common cause in general. It is shown in section 4, however, that if each single superluminal correlation predicted by the *vacuum state* between events in $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ has a *genuinely probabilistic* common cause, then the local algebras $\mathcal{A}(V_1)$ and $\mathcal{A}(V_2)$ must be statistically independent in the sense of C^*-independence. In view of the recent results on the statistical independence of algebras [4] it follows then that the existence of truly probabilistic common causes entails that the algebras satisfy a number of equivalent independence conditions (see the concluding remarks in section 5).

2 Reichenbach's common cause principle

Let A and B be two events and $p(A)$ and $p(B)$ be their probabilities. If the joint probability $p(AB)$ of A and B is greater than the product of the single probabilities, i.e. if

$$p(AB) > p(A)p(B) \tag{1}$$

then the events A and B are said to be *correlated*. According to Reichenbach ([13], Section 19), a probabilistic common cause explanation of a correlation like (1) means finding a third event C (cause) such that the following (independent) conditions hold:

$$
\begin{align}
p(AB|C) &= p(A|C)p(B|C) \tag{2} \\
p(AB|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) \tag{3} \\
p(A|C) &> p(A|C^{\perp}) \tag{4} \\
p(B|C) &> p(B|C^{\perp}) \tag{5}
\end{align}
$$

where $p(X|Y)$ denotes here the conditional probability of X on condition Y, and it is assumed that none of the probabilities $p(X)$, $(X = A, B, C)$ is equal to zero.

Reichenbach shows that conditions (2)-(5) imply (1), if one assumes that the conditional probabilities are defined in the standard way as $p(A|C) = p(AC)/p(C)$ etc. Condition (2) has become known as "screening off", it expresses "... the fact that relative to the cause C the events A and B are mutually independent" ([13] p. 159); that is to say the common cause event C "screens off" the correlation in the sense that conditionalizing the probability measure p by C, the conditioned probability $p(\bullet|C)$ renders the two events A and B statistically independent. One way to interpret the screening off condition (2) is to re-write it as

$$
\begin{align}
p(A|BC) &= p(A|C) \tag{6} \\
p(B|AC) &= p(B|C) \tag{7}
\end{align}
$$

Conditions (6)-(7) can be read as saying that "knowing the cause C already yields enough information to predict the probability of the event $A(B)$, information on $B(A)$ is redundant".

Notice that there exist two opposite ways the screening off condition (2) can be satisfied: (i) It can happen that, in addition to being a probabilistic common cause, the event C (thought of as an element in a Boolean algebra) is contained both in A and in B, $C \subseteq A$, $C \subseteq B$, and, as another extreme, (ii) it can also happen that C is a probabilistic cause that is contained neither in A nor in B. Case (i) means that the event C is not simply a *probabilistic* common cause but a cause that necessarily entails the events A and B, and the screening off condition (2) holds then in a trivial way. Given a correlation between A and B, if a probabilistic common cause C can be found such that (in addition to the conditions (2)-(5)) $C \subseteq A$ and $C \subseteq B$ also is the case, then we say that the correlation can be screened off in the *strong sense*. We refer to the situation (ii) by calling C a *truly (genuinely) probabilistic* common cause.

3 Superluminal correlations in quantum field theory

Recall that $\mathcal{A}$ is a quasilocal C^*-algebra of relativistic quantum field theory if $\mathcal{A}$ is the uniform closure of a net $\{\mathcal{A}(V)\}$ of (strictly) local C^*-algebras $\mathcal{A}(V)$ (with common unit) associated with the open, bounded subsets V of the Minkowski space M, where the net has the following properties:

(i) isotony: if V_1 is contained in V_2, then $\mathcal{A}(V_1)$ is a subalgebra of $\mathcal{A}(V_2)$;

(ii) microcausality: if V_1 is spacelike separated from V_2, then every element of $\mathcal{A}(V_1)$ commutes with every element of $\mathcal{A}(V_2)$;

(iii) relativistic covariance: there is a representation α of the identity-connected component $\mathcal{P}$ of the Poincaré group by automorphisms on $\mathcal{A}$ such that $\alpha(g)\mathcal{A}(V) = \mathcal{A}(gV)$ for all V and $g \in \mathcal{P}$;

Part of the axioms of relativistic quantum field theory is also the assumption of existence of at least one physical representation of the quasilocal C^*-algebra $\mathcal{A}$, which means mathematically that one postulates the existence of an α-invariant state ϕ_0 (vacuum state) such that

(iv) the spectrum condition holds in the corresponding cyclic (GNS) representation $(\mathcal{H}_0, \Omega_0, \pi_0)$.

In this representation one can identify the local algebras representing the observables with the von Neumann algebras $\pi_0(\mathcal{A}(V))''$. One then has a net of von Neumann algebras having the properties (i)-(iv). In what follows we assume that $\{\mathcal{A}(V)\}$ is a net of von Neumann algebras having properties (i)-(iv), and it also is assumed that the net is in fact an *irreducible* vacuum representation of a net of local C^*-algebras. (For these axioms, see [5] and [7].)

Let V_1 and V_2 be two spacelike separated spacetime regions and $A \in \mathcal{A}(V_1)$ and $B \in \mathcal{A}(V_2)$ be two projections. If ϕ is a state on the quasilocal algebra $\mathcal{A}$, then it can happen very well that

$$\phi(AB) > \phi(A)\phi(B) \tag{8}$$

If (8) is the case, then we say that there is *superluminal correlation* between A and B in state ϕ.

A typical example of superluminal correlation is the one predicted by the vacuum state ϕ_0: If V_1 and V_2 are two spacelike separated tangent double cone regions, or two spacelike separated complementary wedge regions in the Minkowski spacetime, then

$$\phi_0(AB) > \phi_0(A)\phi_0(B) \tag{9}$$

for some projections $A \in \mathcal{A}(V_1), B \in \mathcal{A}(V_2)$.

The existence of such A, B is a consequence of the fact that the vacuum state violates Bell's inequality for the said regions in "every" field theory; that is to say, the Bell correlation $\beta(\phi_0, \mathcal{A}(V_1), \mathcal{A}(V_2))$ defined below by (11) takes on its maximal value ($\sqrt{2}$), so the Bell correlation violates Bell's inequality, which in this notation reads:

$$\text{Bell's inequality:} \quad \beta(\phi_0, \mathcal{A}(V_1), \mathcal{A}(V_2)) \leq 1 \tag{10}$$

$$\beta(\phi_0, \mathcal{A}(V_1), \mathcal{A}(V_2)) \equiv sup_{-I \leq X_i, Y_j \leq I} \phi_0(X_1(Y_1 + Y_2) + X_2(Y_1 - Y_2)) \tag{11}$$

(The supremum in (11) is taken over selfadjoint contractions in the respective algebras: $X_i \in \mathcal{A}(V_1), Y_j \in \mathcal{A}(V_2)$. For further details and for a review of the precise statements concerning Bell's inequality in ARQFT see [22] and [21].) Since a product state satisfies Bell's inequality, ϕ_0 cannot be a product state across the algebras $\mathcal{A}(V_1), \mathcal{A}(V_2)$, i.e. there exist selfadjoint contractions $X \in \mathcal{A}(V_1), Y \in \mathcal{A}(V_2)$ such that $\phi_0(XY) \neq \phi_0(X)\phi_0(Y)$, which implies that $\phi_0(P_1P_2) \neq \phi_0(P_1)\phi_0(P_2)$ for some spectral projections P_1, P_2 of X and Y respectively, hence either $\phi_0(P_1P_2) > \phi_0(P_1)\phi_0(P_2)$ or $\phi_0(P_1^\perp P_2) > \phi_0(P_1^\perp)\phi_0(P_2)$ holds.

4 Do superluminal correlations have a probabilistic common cause?

We wish to raise the problem whether the correlations of the type (8) can be explained by finding a common cause in Reichenbach's sense. To make this problem precise we have to adopt Reichenbach's notion of common cause to the situation in ARQFT. This is done in the next definition.

Definition: Let V_1 and V_2 be two spacelike separated (open, bounded) spacetime regions, $BLC(V_1)$ and $BLC(V_2)$ be their backward light cones, and $\{\mathcal{A}(V)\}$ be a net of local algebras satisfying the standard axioms. We say that the pair of algebras $\mathcal{A}(V_1)\mathcal{A}(V_2)$ satisfies (Reichenbach's) Screening off Principle iff for any state ϕ over the quasilocal algebra $\mathcal{A}$ and for any pair of projections $A \in \mathcal{A}(V_1)$ $B \in \mathcal{A}(V_2)$ we have the following: if $\phi(AB) > \phi(A)\phi(B)$ then there exists a projection C in the von Neumann algebra $\mathcal{A}(V)$ that is associated with a region V lying within the intersection $BLC(V_1) \cap BLC(V_2)$ such that $\phi(C) \neq 0 \neq \phi(C^\perp)$ and C satisfies the following conditions:

(i) C commutes with both A and B

(ii) the conditions below (analogous to (2), (3), (4) and (5)) hold:

$$\frac{\phi(ABC)}{\phi(C)} = \frac{\phi(AC)}{\phi(C)}\frac{\phi(BC)}{\phi(C)} \tag{12}$$

$$\frac{\phi(ABC^\perp)}{\phi(C^\perp)} = \frac{\phi(AC^\perp)}{\phi(C^\perp)}\frac{\phi(BC^\perp)}{\phi(C^\perp)} \tag{13}$$

$$\frac{\phi(AC)}{\phi(C)} > \frac{\phi(AC^\perp)}{\phi(C^\perp)} \tag{14}$$

$$\frac{\phi(BC)}{\phi(C)} > \frac{\phi(BC^\perp)}{\phi(C^\perp)} \tag{15}$$

We say that the Screening off Principle holds in ARQFT iff for every pair of spacelike separated spacetime regions V_1, V_2 the Screening off Principle holds for the pair $\mathcal{A}(V_1)\mathcal{A}(V_2)$. Just like in the case of Reichenbach's formulation, one can distinguish the strong and genuinely probabilistic versions of probabilistic common cause in ARQFT and one can speak accordingly of the Screening off Principle holding in ARQFT in the strong and genuinely probabilistic sense.

The Screening off Principle as specified above differs slightly from Reichenbach's in two respects: First, since ARQFT is a non-commutative theory, one has to require explicitly the commutativity of the events involved – unless one is willing to expand Reichenbach's scheme and replace it by a theory of "non-commutative screening off", involving non-commutative conditionalization, which we do not wish to consider here. (See the paper [24] for an analysis of some technical difficulties concerning the generalization of Reichenbach's scheme to non-distributive event structures.) Second, the common cause event C is required in the above definition to lie in the common causal past of the two correlated events. This latter condition was not part of Reichenbach's original theory. It could not be because that theory was not formulated within the framework of Minkowski spacetime. But it is clear that as soon as one is in a theory, where there is an underlying causal structure to consider, like in ARQFT, the condition that C be causally not disconnected from either A or B must be required, otherwise one could hardly talk about a *common* cause explanation in *relativistic* sense.

We are now in the position to ask

Problem: Does ARQFT satisfy the Screening off Principle?

As we have indicated already, we are not able to answer this question, nor do we know of any result that would give a partial answer, positive or negative. What will be seen below is that the existence of a genuinely probabilistic common cause of every vacuum correlation entails C^*-independence of the algebras involved. To show this we recall first the relevant definitions.

Two C^*-subalgebras $\mathcal{A}_1, \mathcal{A}_2$ of the C^*-algebra $\mathcal{C}$ are called C^*-independent, if for any state ϕ_1 on $\mathcal{A}_1$ and for any state ϕ_2 on $\mathcal{A}_2$ there is a state ϕ on $\mathcal{C}$ such that $\phi(A) = \phi_1(A)$ and $\phi(B) = \phi_2(B)$ ($A \in \mathcal{A}_1, B \in \mathcal{A}_2$). The C^*-independence of the pair $(\mathcal{A}_1, \mathcal{A}_2)$ means that no preparation in any state of the system described by $\mathcal{A}_1$ can exclude any preparation of the system described by $\mathcal{A}_2$. That is to say, any two partial states ϕ_1 and ϕ_2 can be prepared by the same preparation procedure. Algebras pertaining to (regular shaped) spacelike separated spacetime regions are C^*-independent (see precise statements in [22]); in particular algebras over spacelike separated tangent double cones are C^*-independent. A closely related notion of independence is W^*-independence: Two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ of the von Neumann algebra $\mathcal{N}$ are called W^*-independent, if for any *normal* state ϕ_1 on $\mathcal{N}_1$ and for any *normal* state ϕ_2 on $\mathcal{N}_2$ there is a *normal* state ϕ on $\mathcal{N}$ such that $\phi(A) = \phi_1(A)$ and $\phi(B) = \phi_2(B)$ ($A \in \mathcal{N}_1, B \in \mathcal{N}_2$). The two von Neumann subalgebras $\mathcal{N}_1, \mathcal{N}_2$ are called W^*-independent *in the product sense* if they are

W^*-independent and ϕ can be chosen such that $\phi(AB) = \phi(A)\phi(B)$ ($A \in \mathcal{N}_1, B \in \mathcal{N}_2$). The ordered pair $(\mathcal{N}_1, \mathcal{N}_2)$ of von Neumann subalgebras of the von Neumann lagebra $\mathcal{N}$ is called strictly local if for any projection $A \in \mathcal{N}_1$ and any normal state ϕ_2 on $\mathcal{N}_2$ there exists a normal state ϕ on $\mathcal{N}$ that extends ϕ_2 and such that $\phi(A) = 1$. (For the origin and a detailed analysis of the interrelation of these and other statistical independence notions see the review [21] and the references therein.)

Proposition: Let V_1, V_2 be two open, bounded spacelike separated spacetime regions and $\mathcal{A}(V_1), \mathcal{A}(V_2)$ be the two von Neumann algebras in a net of von Neumann algebras satisfying the standard conditions. If each single correlation between projections of $\mathcal{A}(V_1), \mathcal{A}(V_2)$ predicted by the vacuum state has a genuinely probabilistic common cause explanation in the sense described in the Definition, then the two algebras $\mathcal{A}(V_1), \mathcal{A}(V_2)$ are C^*-independent.

Proof: The statement is an easy consequence of the powerful, non-trivial Schlieder-Roos and Reeh-Schlieder theorems. The Schlieder-Roos theorem [15], [14] says that if $\mathcal{A}_1$ and $\mathcal{A}_2$ mutually commute (i.e. $XY = YX$ for all $X \in \mathcal{A}_1, Y \in \mathcal{A}_2$) then C^*-independence of $\mathcal{A}_1, \mathcal{A}_2$ is equivalent to the following condition ("Schlieder property"): $XY \neq 0$ whenever $0 \neq X \in \mathcal{A}_1, 0 \neq Y \in \mathcal{A}_2$ (recently Florig and Summers have proved that the Schlieder-Roos theorem remains valid without assuming mutual commutativity of the algebras; see the Proposition 3 in [4]). The Reeh-Schlieder theorem says that the vacuum vector Ω_0 is both cyclic and separating for any local algebra belonging to a region V with non-empty causal complement; in other words, no non-zero positive element in $\mathcal{A}(V)$ can annihilate the vacuum vector: if $0 \leq X \in \mathcal{A}(V)$ and $X\Omega_0 = 0$ then $X = 0$. By the Schlieder-Roos theorem it is enough to show that the assumptions in the proposition imply the Schlieder property. Using ideas from [15], Summers shows in [21] that to prove the Schlieder property, it is enough to prove it for projections only (for a quick argument also see [12]); so let $A \in \mathcal{A}(V_1), B \in \mathcal{A}(V_2)$ be arbitrary non-zero projections. We must show that $AB \neq 0$. Consider now the vacuum state $X \mapsto \langle \Omega_0, X\Omega_0 \rangle = \phi_0(X)$. One of the following three equations holds.

$$\langle \Omega_0, AB\Omega_0 \rangle \quad > \quad \langle \Omega_0, A\Omega_0 \rangle \langle \Omega_0, B\Omega_0 \rangle \tag{16}$$

$$\langle \Omega_0, AB\Omega_0 \rangle \quad = \quad \langle \Omega_0, A\Omega_0 \rangle \langle \Omega_0, B\Omega_0 \rangle \tag{17}$$

$$\langle \Omega_0, AB\Omega_0 \rangle \quad < \quad \langle \Omega_0, A\Omega_0 \rangle \langle \Omega_0, B\Omega_0 \rangle \tag{18}$$

The right hand sides of all of the above equations is strictly positive by the Reeh-Schlieder theorem, therefore if either (16) or (17) is the case then $\langle \Omega_0, AB\Omega_0 \rangle > 0$, and so $AB \neq 0$. If equation (18) is the case, then one checks easily that

$$\langle \Omega_0, A^\perp B\Omega_0 \rangle > \langle \Omega_0, A^\perp \Omega_0 \rangle \langle \Omega_0, B\Omega_0 \rangle \tag{19}$$

By assumption there is a genuinely probabilistic common cause of the correlation (19), i.e. there exists a C projection in a local algebra $\mathcal{A}(V)$, $V \subseteq BLC(V_1) \cap BLC(V_2)$ such that C commutes with both A and B, $C \not\subseteq A^\perp$, $C \not\subseteq B$ and such that

$$\frac{\langle \Omega_0, A^\perp BC\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \quad = \quad \frac{\langle \Omega_0, A^\perp C\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \frac{\langle \Omega_0, BC\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \tag{20}$$

$$\frac{\langle \Omega_0, A^\perp BC^\perp\Omega_0 \rangle}{\langle \Omega_0, C^\perp\Omega_0 \rangle} \quad = \quad \frac{\langle \Omega_0, A^\perp C^\perp\Omega_0 \rangle}{\langle \Omega_0, C^\perp\Omega_0 \rangle} \frac{\langle \Omega_0, BC^\perp\Omega_0 \rangle}{\langle \Omega_0, C^\perp\Omega_0 \rangle} \tag{21}$$

$$\frac{\langle \Omega_0, A^\perp C\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \quad > \quad \frac{\langle \Omega_0, A^\perp C^\perp\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \tag{22}$$

$$\frac{\langle \Omega_0, BC\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \quad > \quad \frac{\langle \Omega_0, BC^\perp\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \tag{23}$$

By an elementary rewriting of (20) one can verify easily that the following also holds:

$$\frac{\langle \Omega_0, ABC\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} = \frac{\langle \Omega_0, AC\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \frac{\langle \Omega_0, BC\Omega_0 \rangle}{\langle \Omega_0, C\Omega_0 \rangle} \tag{24}$$

$\langle \Omega_0, BC\Omega_0 \rangle$ is non-zero by (23), hence, if $\langle \Omega_0, AC\Omega_0 \rangle$ is shown to be non-zero, then the right hand side of (24) is not equal to zero, and the proof is then complete. If $\langle \Omega_0, AC\Omega_0 \rangle$ were equal to zero then (since AC is a projector, hence non negative) $AC = 0$ would follow by the Reeh-Schlieder Theorem, but $AC = 0$ implies $C \subseteq A^\perp$, which can not be the case, since C was assumed to be a genuinely probabilistic common cause of the correlation (19).

5

5   Concluding remarks

Statistical independence is a property that is typically expected to hold for local algebras pertaining to spacelike separated, i.e. causally disconnected spacetime regions. The Screening off Principle, on the other hand, involves causally connected regions/algebras. The Proposition connects the two notions, and it shows that C^*-independence of spacelike separated algebras is necessary for the Screening off Principle (in the genuinely probabilistic sense) to hold in ARQFT. (It s also clear from the proof that Proposition remains valid by replacing the vacuum state by any other *faithful* state.) It has been proved recently that in the context of ARQFT C^*-independence, W^*-independence and strict locality are equivalent [4]. It follows then that validity of the Screening off Principle in ARQFT (in the probabilistic sense) implies both W^*-independence and strict locality of the local algebras confined in spacelike separated spacetime regions. However, the proof of the Proposition also indicates that C^*-independence (hence also W^*-independence and strict locality) is unlikely to be sufficient for the Screening off Principle to hold: One of the properties of the probabilistic common cause, namely that the common cause C belongs to the common causal past of the correlated events, was not used in inferring the C^*-independence property.

Since the Screening off Principle appears to be stronger than C^*-independence, a natural question is, whether it implies stronger independence conditions. It is known [4] that W^*-independence in the product sense (hence also the so-called "split property", see [21] for the definition) is a strictly stronger independence condition than W^*-independence. If W^*-independence in the product sense or the split property could be inferred from the Screening off Principle, then one could conclude that the Screening off Principle does not hold in general, since it is known that the split property fails for tangent spacetime regions (see [20]). It is not known, whether the Screening off Principle implies any of the stronger statistical independence conditions. Most pressing would be to know, however, whether the Screening off Principle can hold at all, at least for some pairs of spacelike separated regions.

In a recent paper Belnap and Szabó have proved that the non-probabilistic superluminal correlations occurring in the Greenberger-Horne-Zeilinger (GHZ) situation do not have a non-probabilistic *common* common cause [1]. The results on the violation of Bell's inequality in ARQFT imply that the "same" is true also in the present case: If for a given state ϕ there exists a *single, common* probabilistic common cause C (in the sense of Definition) of *all* correlations predicted by ϕ, then the C-conditioned state $\phi(\bullet|C)$ is a product state across the algebras $\mathcal{A}(V_1), \mathcal{A}(V_2)$. Since a product state satisfies Bell's inequality, and since for tangent spacelike separated wedge and double cone regions *every* normal state maximally violates Bell's inequality [20], there exists no normal state over local algebras in the said regions such that the correlations predicted by it have a *common* common cause. But the assumption that *all* the superluminal correlations predicted by a given state in ARQFT have a *common* common cause, seems totally unwarranted. Not only isn't there anything in the Reichenbachian notion of common cause that would justify this assumption, the common cause principle doesn't even seem to contain any hint as to how the different common causes $C^{A',B'}, C^{A'',B''} \dots$ of different correlated pairs $(A', B'); (A'', B'') \dots$ (possibly containing even incompatible elements) might be related to each other. This dependence of the common cause on the pair of the correlated events and the unrelatedness of the causes of correlations of different event-pairs not only simply blocks the inference from the assumption of existence of common causes to the value of the Bell correlation, but it makes unclear in which state one should check the value of the Bell correlation: given a state, the vacuum ϕ_0 say, and assuming that there exist probabilistic common causes $C^{A',B'}, C^{A'',B''} \dots$ of all the correlated pairs $(A', B'); (A'', B'') \dots$ we have the conditioned states $\phi_0(\bullet|C^{A',B'}), \phi_0(\bullet|C^{A'',B''}) \dots$. Which of these states should satisfy Bell's inequality (10)? In fact we know (since all these states are normal) that *each* violates Bell's inequality (10) (for complementary wedges and spacelike separated tangent double cones). But why shouldn't they – assuming only (12)-(15) to hold with $A', B', C'; A'', B'', C'' \dots$? In short, under the present specification of Bell's inequality and Reichenbachian common cause, it is impossible to give meaning to the claim "Bell's inequality is implied by Reichenbach's common cause principle"; hence, on the present interpretation, violation of Bell's inequality does not imply the impossibility of Reichenbachian common causes of superluminal correlations. Whether such (not common) Reichenbachian probabilistic common causes exist in ARQFT remains an open question, just like in the case of the non-probabilistic GHZ correlations.

It should be noted that this conclusion remains valid also in connection with Bell's inequality in standard, non-relativistic quantum mechanics. If the notion of common cause is specified in Reichenbach's spirit as a non-common common cause, i.e. if it is *not* assumed that different correlated event pairs in the standard Bell-Bohm setting have the same common cause, then the usual Bell's inequality cannot be derived from the assumption of existence of common causes of correlations: The standard arguments

in support of the claim that "Bell's inequality is implied by Reichenbach's common cause principle" (for instance the proof given by Van Fraassen [26]) assume (more or less tacitly) and make essential use of the assumption that each *single* hidden variable (interpreted as common cause) screens off *all* correlations, i.e. that the hidden variable is a *common* "common cause". While this assumption might be justifiable in a hidden variable framework, the justification must involve considerations that go beyond Reichenbach's notion of a statistical common cause.

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