phy—for the same reasons. Mathematical models are distinct bodies of knowledge that can be and in fact are negotiated across boundaries, like hammers can be used for different purposes in different contexts. However, the criteria for assessing which model is most accurate, or most true, belong to the image of knowledge. Corry's framework that Weintraub so convincingly uses throughout his book is regrettably forgotten for a moment when it comes to discussing the role of models in the process of mathematization. Maybe it isn't that odd to recall (and paraphrase) Lakatos's (1976) view on the relationship between history and philosophy: the history of economics, lacking the guidance of (not necessarily normative) philosophy, is in danger of becoming blind, while the philosophy of science, turning its back on the most intriguing phenomena in the history of science, is in danger of becoming empty.

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Miklós Rédei and Michael Stölzner (eds.), ***John von Neumann and the Foundations of Physics.*** Dordrecht: Kluwer Academic Publishers (2001), ix + 371 pp., $119.00 (cloth).

The name of 'von Neumann' is known to all who work in the foundations of quantum theory. While one could make the case for Heisenberg, Schrödinger, or Dirac, von Neumann would probably receive the most votes for the one person most responsible for putting the mathematical foundations of quantum theory on firm ground. Yet at the same time, apart from the famous chapter on measurement, even von Neumann's major work in the area, *Mathematische Grundlagen der Quantenmechanik* is rarely read in its entirety. Far less are his other relevant works studied, or even read.

This book nicely illustrates the richness (though also the limitations) of material on the foundations of physics to be found in von Neumann's corpus. While not every contribution to the volume could truly be called "philosophically motivated historical research" (v), all of the works on von Neumann do contribute to a complete understanding of von Neumann's approach to the foundations of physics—especially quantum theory—and its impact on contemporary research in the field. A few brief comments on some of the papers appear below.

In addition to these contributions, the first part of the book includes several previously unpublished documents from the von Neumann archive in the Library of Congress. What appears here is not a complete selection, but a few documents selected by the editors, and therefore one must be very careful not to suppose that they are representative of the archive as a whole. (Certainly the editors do not claim so.) Nonetheless, there are some quite revealing documents, for example, a letter from 1945 in which von Neumann admits his inability to write an article on quantum logic, citing distractions by the war, but also the 'exceedingly great' difficulties he encountered while trying to achieve the unification of quantum propositional logic, quantificational logic, probability logic, and projective geometry that he had originally intended. Also included are typescripts of two lectures, 'Unsolved Problems in Mathematics' and 'Quantum Mechanics of Infinite Systems'. Of these, the former especially illustrates what von Neumann took to be the outstanding foundational problems facing quantum theory.

Part one of the book, comprising the papers on von Neumann and the archival material, is roughly 270 pages long, certainly enough to constitute a book in its own right. But there is a second part, containing material, it must be said, utterly unrelated to part one. The editors explain (or excuse?) thus: "As in each Vienna Circle Institute Yearbook the special topic is rounded off by papers emerging from the Institute's annual lecture series and by a review section" (vi). This 'rounding off' consists of roughly one hundred pages, from an article raging against theological interpretations (past and present) of cosmology, to a review of a book on the sociology of music. While there may be nothing wrong with these papers in themselves, their presence in this book is mostly a distraction, and given the title and likely audience of the book, almost assures that they will not be noticed except by accident. (A few of these contributions make a little more sense in this book than the rest—one might make a case, for example, for the review of Paolo Mancosu's book on the foundations of mathematics in the 1920s.) It does seem that everybody—authors, editors, and readers—would have been better served by finding a more suitable home for the material of part two. The remainder of this review shall therefore focus exclusively on the material from part one.

That material does indeed make a unified work. Moreover, the editors have done a nice job of organizing the material so that, for the most part, each essay leads naturally into the next. Perhaps the most exciting example occurs in the two articles following the initial overview of von Neumann's influence on the foundations of physics by Thirring. In the first of this pair of articles, Majer provides a fascinating account of Hilbert's view of the role of axiomatics in the foundations of science. While many perceive Hilbert as a formalist mathematician with little regard for or understanding of physical issues (even when he was concerned with physical theories such as general relativity or statistical mechanics), Majer argues convincingly that Hilbert instead did understand the physical issues, and had a view according to which axiomatics would lead to a deeper understanding of those issues. Such an argument provides the perfect precursor to the fine article by Stöltzner, who outlines von Neumann's view (heavily influenced by Hilbert's view) of the role of axiomatics in mathematical physics. Stöltzner argues that von Neumann's view of this role—according to which axiomatics is not a post hoc analysis of an ossified physical theory, but an effective impetus to new discovery in science—is pertinent to contemporary debates about the foundations of physics. In the course of this argument, one gets considerable insight into the foundational motivations behind some of von Neumann's most famous results. For example, the suggestion is made (in passing, but nonetheless convincingly) that von Neumann's proof that representations of the canonical commutation relations are equivalent up to unitary transformation was motivated by a concern with the possible failure of categoricity with regard to the axioms of quantum theory. (An excellent summary of the relevant results, and related results, appears later by Summers, himself one of the giants in the field of algebraic quantum theory.)

Of course, von Neumann famously also considered the consequences of the axioms of quantum theory in the context of measurements, and perhaps more clearly than previous authors illustrated the problem that arises, the so-called 'problem of measurement.' In his 1932 book, von Neumann showed that in a simple model of quantum measurements, the problem of 'Schrödinger's cat' inevitably arises. Bub reviews these points in his article, and considers the argument given by von Neumann in 1932 and again in 1954 to the effect that no 'hidden variables' theory can resolve the problem. Bub offers his own solution to the problem, which is one form of the so-called 'modal interpretation' of quantum theory.

Several of the contributors consider von Neumann's infamous 'proof', which was severely criticized in later years, notably by John Bell. While the criticisms are correct, it is worth keeping in mind the deeper motivations behind von Neumann's argument, and Breuer provides us with the material to do so. He suggests that von Neumann's approach to the problem of measurement is closely analogous to Tarski's reaction to Gödel. In an extension of Gödel's incompleteness theorem, Tarski showed that one cannot define a truth predicate within an axiom system that is sufficiently rich to comprise arithmetic. Similarly, argues Breuer, letting 'having a result of a measurement' play the role of 'having a proof of a statement', von Neumann argues that in the context of the axioms of quantum theory, one cannot say that measurements have results. Instead, one must, as von Neumann does, introduce a process that is *extra-axiomatic,* and which is responsible for the actual results of measurements. Whether one finds von Neumann's 'collapse postulate' convincing, or even coherent, it is quite helpful to view it in this light. In a somewhat different analysis, Giuntini and Laudisa also argue that von Neumann's 'no-go' theorem has been misunderstood. On the one hand, they (correctly!) agree with current orthodoxy that the theorem does not establish the impossibility of any hidden-variables theory. On the other hand, they argue that von Neumann's proof *does* show that "any deterministic completion, whatever form it might assume, will be a theory very remote from a classical theory" (182). Their discussion recalls the usually forgotten close connection between von Neumann's reasoning and the relative-frequency interpretation of probability, and is valuable for doing so, but one would have liked to see additional discussion of the meaning of the conclusion noted above. Precisely how must such completions be non-classical? After all, this conclusion is quite trivial in one sense, because the completion is a completion of a non-classical theory. Nonetheless, despite wanting a lengthier discussion, the paper by Giuntini and Laudisa is a valuable and careful commentary on von Neumann's theorem, at a time when most comments simply rehearse Bell's incredulity at von Neumann's assumption of additivity.

There are several other worthwhile papers in the collection. Here we consider just one more, the paper by Redei on von Neumann's approach to quantum logic. Apart from his purely mathematical work, and his analysis of measurement in quantum theory, von Neumann's work on quantum logic is probably most influential on contemporary philosophy of physics. (Other papers in the volume deal with von Neumann's work on entropy and infinite systems, admittedly important in its own right, but arguably less central to the philosophy of quantum theory in the decades after von Neumann's death.) However, as Redei points out, von Neumann's own conception of quantum logic differed quite significantly (and was significantly more subtle than) the usual contemporary understanding. Perhaps the most striking example of a mismatch between von Neumann's view and the usual view of quantum logic concerns the meaning (if any) of the meet (lattice-theoretic greater lower bound) of two non-commuting subspaces. In most accounts, it is taken to represent the conjunction of the two propositions, but as Redei shows, von Neumann took the meet of two subspaces *A* and *B* "*not* as an experimentally meaningful proposition stating that 'both *A* and *B* is the case'" (166).

So what *does* it mean? I encourage the reader of this review to read the article (and the book) to find out. While doing so will reveal why, in the many areas to which he contributed, philosophy of physics has had to go beyond von Neumann's work, it will also reveal why von Neumann continues to be one of the fathers of contemporary foundations of quantum theory, and it might even reveal a few little-known gems of insight from von Neumann that have not managed to survive to the present day, but should have.

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