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- An equation for the indeterminate $\mathbf{x}$ in $\mathscr{X}$ is an expression $\mathbf{x} = a(\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots)$, where a is some term in the indeterminates $\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots$.

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- A system of equations in the indeterminates $\mathscr{X}$ is a set of equations that contains exactly one equation for each member of $\mathscr{X}$.

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- Solution lemma: Every system of equations in $\mathscr{X}$ has exactly one solution.
- If a system of equations is finite, and each term is in HF_1, then the value of the solution for every indeterminate will be in HF_1, too.