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The propositions [...] are sets that are different for different propositions and contain their constituents in their transitive closure.

Russellian propositions 2.

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PrePROP contains arbitrarily long liar-cycles.

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- p is *nonsubstantive* iff it is a member of a nonsubstantive set; *substantive* in the other case.
- *PROP* – the class of *Russellian propositions* – is the largest subset of *PrePROP* such that every $p \in PROP$ is substantive and every immediate constituent of p is in *PROP*, too.