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$\mathfrak{M}$ is a <u>weak model</u> iff (it is coherent and (if it contains a semantical fact $< Tr,\ p,\ t >$ then the "non-semantical" [or better to say "less semantical"] facts in $\mathfrak{M}$ support it. I.e., $\mathfrak{M}$ makes p true resp. false.))

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- Proof: strictly parallel with the (or some) usual argument for the Liar paradox.

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In both cases, left to right is contained in the weak model-property. Right to left: if the facts support a semantical fact, then it is contained in the model.

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  - The Liar proposition f is made false by any weak model. Hence, in an F-closed weak model f should be false, in contradiction to our previous claim that it is not false in any weak model.
- Therefore there are no weak models that are both T-and F-closed (semantically closed in Tarski's terminology).

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- $\mathfrak{M}$ is an almost semantically closed (short: *asc*) weak model if it is T- and N-closed.

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The Closure Theorem

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The idea of the proof:

We extend $\mathfrak{M}$ step by step by semantical facts that are demanded by the *asc* conditions. We have to show that every step yields another weak model that satisfies the witnessing condition.

Obviously, every model should be contained in any *asc* weak model containing $\mathfrak{M}$. Then take the union of the steps. It is an *asc* model containing $\mathfrak{M}$ and every member of it must be a member of an *asc* weak model containing $\mathfrak{M}$.

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- *Asc* models are called hereafter simply models of the world. Maximal model: a model that is not proper part of any other model.
- Corollary: Every *cw* model can be expanded to a maximal model.