Repeat some definitions and facts

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if $(True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if ($True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).
- $\mathfrak{M}$ is N-closed if ($False_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models \overline{p}$)

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if $(True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).
- $\mathfrak{M}$ is N-closed if $(False_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models \overline{p})$
  - From left to right: witnessing condition.

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if ($True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).
- $\mathfrak{M}$ is N-closed if ($False_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models \overline{p}$)
  - From left to right: witnessing condition.
- $\mathfrak{M}$ is an *asc* model if it is both T- and N-closed.

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if ($True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).
- $\mathfrak{M}$ is N-closed if ($False_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models \overline{p}$)
  - From left to right: witnessing condition.
- $\mathfrak{M}$ is an *asc* model if it is both T- and N-closed.
- Closure Theorem: If $\mathfrak{M}$ is a weak model satisfying the witnessing condition (a closable weak, *cw* model), then there is a(n unique) smallest *asc* weak model $\mathfrak{M}^*$ such that $\mathfrak{M}^* \supseteq \mathfrak{M}$.

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if ($True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).
- $\mathfrak{M}$ is N-closed if ($False_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models \overline{p}$)
  - From left to right: witnessing condition.
- $\mathfrak{M}$ is an *asc* model if it is both T- and N-closed.
- Closure Theorem: If $\mathfrak{M}$ is a weak model satisfying the witnessing condition (a closable weak, *cw* model), then there is a(n unique) smallest *asc* weak model $\mathfrak{M}^*$ such that $\mathfrak{M}^* \supseteq \mathfrak{M}$.
- *Asc* models are called simply models (of the world). A model is maximal if it is not the proper part of some other model.

Repeat some definitions and facts

- A weak model $\mathfrak{M}$ is a class of *soas* satisfying some minimal coherence conditions.
- $\mathfrak{M}$ is T-closed if ($True_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models p$).
- $\mathfrak{M}$ is N-closed if ($False_{\mathfrak{M}}(p)$ iff $\mathfrak{M} \models \overline{p}$)
  - From left to right: witnessing condition.
- $\mathfrak{M}$ is an *asc* model if it is both T- and N-closed.
- Closure Theorem: If $\mathfrak{M}$ is a weak model satisfying the witnessing condition (a closable weak, *cw* model), then there is a(n unique) smallest *asc* weak model $\mathfrak{M}^*$ such that $\mathfrak{M}^* \supseteq \mathfrak{M}$.
- *Asc* models are called simply models (of the world). A model is maximal if it is not the proper part of some other model.
- Every *cw* model can be expanded to a maximal model (not uniquely).

Parallels with the Kripkean account

Parallels with the Kripkean account

- At a Kripkean fixed point, we don't need to extend the interpretation of the truth- and the falsity-predicate any more because the required semantical facts are in the model now. This is exactly the same as the closure conditions required for the *asc* models.

Parallels with the Kripkean account

- At a Kripkean fixed point, we don't need to extend the interpretation of the truth- and the falsity-predicate any more because the required semantical facts are in the model now. This is exactly the same as the closure conditions required for the *asc* models.
- An $\mathfrak{M}$ containing no semantical facts at all satisfies vacuously the conditions for *cw* models. It corresponds to the special case of Kripke's sequences beginning with the pair $< \emptyset, \emptyset >$ and leading to the least fixed point.

Parallels with the Kripkean account

- At a Kripkean fixed point, we don't need to extend the interpretation of the truth- and the falsity-predicate any more because the required semantical facts are in the model now. This is exactly the same as the closure conditions required for the *asc* models.
- An $\mathfrak{M}$ containing no semantical facts at all satisfies vacuously the conditions for *cw* models. It corresponds to the special case of Kripke's sequences beginning with the pair $<\emptyset, \emptyset>$ and leading to the least fixed point.
- The least fixed point $<T_\infty, F_\infty>$ constructed by Kripke is by and far the same as the minimal model constructed from an $\mathfrak{M}$ containing no semantical facts in the proof of the Closure theorem.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (sub*sets* of SOA) with the following constraints (p is a member of $dom(w)$):

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (subsets of SOA) with the following constraints (p is a member of $dom(w)$):

- If $p = [a\ H\ c]$, then $< H,\ a,\ c,\ 1 > \in w(p)$ and similarly for the negation.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (subsets of SOA) with the following constraints (p is a member of $dom(w)$):

- If $p = [a\ H\ c]$, then $< H,\ a,\ c,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [a\ Bel\ q]$, then $< Bel,\ a,\ q,\ 1 > \in w(p)$ and similarly for the negation.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (subsets of SOA) with the following constraints (p is a member of $dom(w)$):

- If $p = [a\ H\ c]$, then $< H,\ a,\ c,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [a\ Bel\ q]$, then $< Bel,\ a,\ q,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [\bigwedge P]$, then $P \subseteq dom(w)$ and for each $q \in P$, $w(q) \subseteq w(p)$.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (subsets of SOA) with the following constraints (p is a member of $dom(w)$):

- If $p = [a\ H\ c]$, then $< H,\ a,\ c,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [a\ Bel\ q]$, then $< Bel,\ a,\ q,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [\bigwedge P]$, then $P \subseteq dom(w)$ and for each $q \in P$, $w(q) \subseteq w(p)$.
- If $p = [\bigvee P]$, then for some $q \in P \cap dom(w)$, $w(q) \subseteq w(p)$.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (subsets of SOA) with the following constraints (p is a member of $dom(w)$):

- If $p = [a\ H\ c]$, then $< H,\ a,\ c,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [a\ Bel\ q]$, then $< Bel,\ a,\ q,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [\bigwedge P]$, then $P \subseteq dom(w)$ and for each $q \in P$, $w(q) \subseteq w(p)$.
- If $p = [\bigvee P]$, then for some $q \in P \cap dom(w)$, $w(q) \subseteq w(p)$.
- If $p = [Tr\ q]$, then $q \in dom(w)$ and $w(q) \cup \{< Tr,\ p,\ 1 >\} \subseteq w(p)$.

Witnessing functions

General idea: determine the *soas* that a model must contain in order to make a proposition true.

A witnessing function w is a partial function from propositions to situations (subsets of SOA) with the following constraints (p is a member of $dom(w)$):

- If $p = [a\ H\ c]$, then $< H,\ a,\ c,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [a\ Bel\ q]$, then $< Bel,\ a,\ q,\ 1 > \in w(p)$ and similarly for the negation.
- If $p = [\bigwedge P]$, then $P \subseteq dom(w)$ and for each $q \in P$, $w(q) \subseteq w(p)$.
- If $p = [\bigvee P]$, then for some $q \in P \cap dom(w)$, $w(q) \subseteq w(p)$.
- If $p = [Tr\ q]$, then $q \in dom(w)$ and $w(q) \cup \{< Tr,\ p,\ 1 >\} \subseteq w(p)$.
- If $p = [Fa\ q]$, then $\overline{q} \in dom(w)$ and $w(\overline{q}) \cup \{< Tr,\ q,\ 0 >\} \subseteq w(p)$.

Witnessing functions 2.

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p) | p \in dom(w)\}$ is coherent.

Witnessing functions 2.

- The witnessing function w is <u>coherent</u> if $\mathfrak{M} = \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- w is <u>coherent with $\mathfrak{M}_0$</u> if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p) | p \in dom(w)\}$ is coherent.

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: .

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.

Witnessing functions 2.

- The witnessing function w is <u>coherent</u> if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is <u>coherent with $\mathfrak{M}_0$</u> if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is <u>consistent</u> if it is in the domain of some coherent witnessing function.
- p is <u>consistent with $\mathfrak{M}_0$</u> if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$:

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$
- Witnessing function for $f = [Fa\ f]$:

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$
- Witnessing function for $f = [Fa\ f]$: $\bar{f} \in dom(w)$;

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p) | p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$
- Witnessing function for $f = [Fa\ f]$: $\bar{f} \in dom(w)$; $< Tr,\ f,\ 0 > \in w(f)$;

.

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$
- Witnessing function for $f = [Fa\ f]$: $\bar{f} \in dom(w)$; $< Tr,\ f,\ 0 > \in w(f)$; $w(\bar{f}) \subseteq w(f)$;

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$
- Witnessing function for $f = [Fa\ f]$: $\bar{f} \in dom(w)$; $< Tr,\ f,\ 0 > \in w(f)$; $w(\bar{f}) \subseteq w(f)$; $< Tr,\ f,\ 1 > \in w(\bar{f})$.

Witnessing functions 2.

- The witnessing function w is coherent if $\mathfrak{M} = \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- w is coherent with $\mathfrak{M}_0$ if $\mathfrak{M} = \mathfrak{M}_0 \cup \bigcup\{w(p)|p \in dom(w)\}$ is coherent.
- p is consistent if it is in the domain of some coherent witnessing function.
- p is consistent with $\mathfrak{M}_0$ if it is in the domain of some witnessing function coherent with $\mathfrak{M}_0$.

Examples:

- Witnessing function for $t = [Tr\ t]$: $w(t) = \{< Tr,\ t,\ 1 >\}$.
- Witnessing function for $\bar{t}$: $w(\bar{t}) = \{< Tr,\ t,\ 0 >\}$
- Witnessing function for $f = [Fa\ f]$: $\bar{f} \in dom(w)$; $< Tr,\ f,\ 0 > \in w(f)$; $w(\bar{f}) \subseteq w(f)$; $< Tr,\ f,\ 1 > \in w(\bar{f})$. No such coherent witnessing function. The same for $\bar{f}$.

Model existence theorem

Model existence theorem

- A proposition p is consistent (with a model $\mathfrak{M}_0$) iff there is a model $\mathfrak{M}(\supseteq \mathfrak{M}_0)$ such that p is true in $\mathfrak{M}$.

Model existence theorem

- A proposition p is consistent (with a model $\mathfrak{M}_0$) iff there is a model $\mathfrak{M}(\supseteq \mathfrak{M}_0)$ such that p is true in $\mathfrak{M}$.
  From right to left: We have a model $\mathfrak{M}$ and need a witnessing function w. Let $w(q) = \mathfrak{M}$ for every q that is true in $\mathfrak{M}$.

Model existence theorem

- A proposition p is consistent (with a model $\mathfrak{M}_0$) iff there is a model $\mathfrak{M}(\supseteq \mathfrak{M}_0)$ such that p is true in $\mathfrak{M}$.
  From right to left: We have a model $\mathfrak{M}$ and need a witnessing function w. Let $w(q) = \mathfrak{M}$ for every q that is true in $\mathfrak{M}$.
  From left to right: We have a w and need an $\mathfrak{M}$. Let us extend w to w': $w'([Tr\ p]) = w(p) \cup \{< Tr,\ p,\ 1 >\}$. It is a witnessing function compatible with $\mathfrak{M}_0$ again. The union of the range of w' is a *cw*-model and by the Closure theorem, it can be extended to a model.

Model existence theorem

- A proposition p is consistent (with a model $\mathfrak{M}_0$) iff there is a model $\mathfrak{M}(\supseteq \mathfrak{M}_0)$ such that p is true in $\mathfrak{M}$.
  From right to left: We have a model $\mathfrak{M}$ and need a witnessing function w. Let $w(q) = \mathfrak{M}$ for every q that is true in $\mathfrak{M}$.
  From left to right: We have a w and need an $\mathfrak{M}$. Let us extend w to w': $w'([Tr\ p]) = w(p) \cup \{< Tr,\ p,\ 1 >\}$. It is a witnessing function compatible with $\mathfrak{M}_0$ again. The union of the range of w' is a *cw*-model and by the Closure theorem, it can be extended to a model.
- Exc. 37, 38 (p. 91/105), 42, 43 (p.101/115) : homework. We return to them next week.

Paradoxical and classical propositions

Paradoxical and classical propositions

- A proposition p is paradoxical in $\mathfrak{M}$ if there is no maximal model $\mathfrak{N} \supseteq \mathfrak{M}$ such that p has a truth value in $\mathfrak{N}$.

Paradoxical and classical propositions

- A proposition p is paradoxical in $\mathfrak{M}$ if there is no maximal model $\mathfrak{N} \supseteq \mathfrak{M}$ such that p has a truth value in $\mathfrak{N}$.
- p is classical if it is not paradoxical in any model.

Paradoxical and classical propositions

- A proposition p is paradoxical in $\mathfrak{M}$ if there is no maximal model $\mathfrak{N} \supseteq \mathfrak{M}$ such that p has a truth value in $\mathfrak{N}$.
- p is classical if it is not paradoxical in any model.
- p is contingently paradoxical if it is paradoxical in some but not in every model. Propositions paradoxical in every model are called intrinsically paradoxical.

Paradoxical and classical propositions

- A proposition p is paradoxical in $\mathfrak{M}$ if there is no maximal model $\mathfrak{N} \supseteq \mathfrak{M}$ such that p has a truth value in $\mathfrak{N}$.
- p is classical if it is not paradoxical in any model.
- p is contingently paradoxical if it is paradoxical in some but not in every model. Propositions paradoxical in every model are called intrinsically paradoxical.

Examples:
The proposition $p = [a\ H\ A\spadesuit] \vee [Fa\ p]$ is true in models containing $< H,\ a,\ A\spadesuit,\ 1 >$ and paradoxical in the others.

Paradoxical and classical propositions

- A proposition p is paradoxical in $\mathfrak{M}$ if there is no maximal model $\mathfrak{N} \supseteq \mathfrak{M}$ such that p has a truth value in $\mathfrak{N}$.
- p is classical if it is not paradoxical in any model.
- p is contingently paradoxical if it is paradoxical in some but not in every model. Propositions paradoxical in every model are called intrinsically paradoxical.

Examples:
The proposition $p = [a\ H\ A\spadesuit] \vee [Fa\ p]$ is true in models containing $< H,\ a,\ A\spadesuit,\ 1 >$ and paradoxical in the others.
An earlier example was that both t and $\bar{t}$ have coherent witnessing functions. Therefore, there are models where t is true and other models where $\bar{t}$ is true. In a maximal model, one of these two must hold. Hence the Truth-teller is classical. But every *cw*-model containing no semantical facts can be extended both to a maximal model where t is true and to another one where it is false.

Classify the classical propositions

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.
- p has a determinate truth value over $\mathfrak{M}$ if it has the same truth value in every maximal model containing $\mathfrak{M}$.

  f p is grounded, then it has a determinate truth value but nut vice versa.

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.
- p has a determinate truth value over $\mathfrak{M}$ if it has the same truth value in every maximal model containing $\mathfrak{M}$.
  If p is grounded, then it has a determinate truth value but nut vice versa.

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.
- p has a determinate truth value over $\mathfrak{M}$ if it has the same truth value in every maximal model containing $\mathfrak{M}$.
  If p is grounded, then it has a determinate truth value but nut vice versa.

Examples:

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.
- p has a determinate truth value over $\mathfrak{M}$ if it has the same truth value in every maximal model containing $\mathfrak{M}$.
  If p is grounded, then it has a determinate truth value but nut vice versa.

Examples:
If $\mathfrak{M}$ contains some non-semantical facts, e.g. $< H,\ a, K\Diamond,\ 1 >$, then the corresponding proposition ($q = [H\ a\ K\Diamond]$) is grounded over $\mathfrak{M}$. $[Fa\ q]$ is grounded, too, etc.

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.
- p has a determinate truth value over $\mathfrak{M}$ if it has the same truth value in every maximal model containing $\mathfrak{M}$.
  If p is grounded, then it has a determinate truth value but nut vice versa.

Examples:
If $\mathfrak{M}$ contains some non-semantical facts, e.g. $< H,\ a, K\diamondsuit,\ 1 >$, then the corresponding proposition ($q = [H\ a\ K\diamondsuit]$) is grounded over $\mathfrak{M}$. $[Fa\ q]$ is grounded, too, etc.
If $\mathfrak{M}$ contains no semantical facts, then t has no determinate truth value over it.

Classify the classical propositions

$\mathfrak{M}$ is a *cw*-model, p is a classical proposition.

- p is grounded over $\mathfrak{M}$ if it has a truth value in the smallest model containing $\mathfrak{M}$.
- p has a determinate truth value over $\mathfrak{M}$ if it has the same truth value in every maximal model containing $\mathfrak{M}$.
  If p is grounded, then it has a determinate truth value but nut vice versa.

Examples:
If $\mathfrak{M}$ contains some non-semantical facts, e.g. $< H,\ a, K\diamondsuit,\ 1 >$, then the corresponding proposition ($q = [H\ a\ K\diamondsuit]$) is grounded over $\mathfrak{M}$. $[Fa\ q]$ is grounded, too, etc.
If $\mathfrak{M}$ contains no semantical facts, then t has no determinate truth value over it.
The proposition $[Tr\ t] \vee [Fa\ t]$ has a determinate truth value, but it is not grounded (under the same conditions).