Ramsey's Logicism

András Máté

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Frank Plumpton Ramsey (1903-1930)

Mathematician

Fundamental works in mathematical logic, combinatorics, economy, metaphysics

Ramsey's program

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Another basic principle of logicism, which includes some criticism of formalism: the numbers of arithmetics are the same as the numbers used for counting in everyday life, therefore expressions of arithmetics are not just symbols devoid of any content.

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$\forall x f(x)$ is the logical product [conjunction], $\exists x f(x)$ is the logical sum [disjunction] of all the propositions resulting by substitution from $f\hat{x}$. I.e., they are truth functions.

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A second fundamental thesis: mathematics is essentially extensional. E.g. set equivalence means that there exists a mapping between the two sets, and this is independent of whether the mapping can be expressed (defined) in some way.

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Semantical paradoxes involve the notion of 'meaning' (denoting) and are irrelevant to mathematics. Logical paradoxes, on the other hand, involve only logical and mathematical concepts and show that something went wrong in our logic (mathematics).

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Unquestionably correct and sufficient to remove the logical paradoxes.

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But it blocks important mathematical ideas and arguments (math. induction, Dedekind cut). That is why the axiom of reducibility is needed for Russell.

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E. g.

$$F(a), \quad \exists x(F(x) \wedge x = a)$$

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Ramsey: this is the root of the whole problem of reducibility.

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A function is predicative$_{Ramsey}$ iff it is the truth function of arguments that are either atomic functions of individuals or propositions.

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Ramsey: yes, but this sort of circularity is not vicious. The proposition Fa, the value of F for an individual a is the conjunction of all the propositions of the form ϕa - including Fa itself. No reference to a class of which F is a member, but to the members of that class only.

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Such cases of denoting are more difficult and the relation R is not legitimately extended to them.

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E. g. in the paradox of the least natural number not nameable in fewer than nineteen syllables we create a new definition that is of higher order than the definitions it refers to.

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Choice: in the framework of the *Principia* it is empirical. But in Ramsey's interpretation, it becomes a tautology. But not necessarily provable.

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Therefore, the Axiom of Infinity is a tautology if it is true, but it can't be proved. It must be postulated.