Published in: *Observation and Experiment in the Natural and Social Sciences,* M.C. Galavotti (ed.), Kluwer Academic Publishers, 2003.

From Theory to Experiments and Back Again ... and Back Again ...

Comments to Patrick Suppes: *From Theory to Experiments and Back Again*

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"The picture of theory often presented by philosophers of science is too austere, abstract and self-contained" Professor Suppes writes. While, as it turns out from the two substantive examples considered in the paper, a closer analysis of the experimental details, the method of data processing and the most important features of the measuring equipments can be fruitful in understanding the basic concepts and the metaphysical conclusions drawn from the theoretical description of the experimental scenario.

Since my field of interest is closer to quantum mechanics, I would like to focus on Suppes' second example based on de Barros and Suppes (2000) general analysis of the realistic GHZ experiments, where experimental error reduces the perfect correlations of the ideal GHZ case. The following important question motivated their analysis: *"How can one verify experimentally predictions based on correlation-one statements, since experimentally one cannot obtain events perfectly correlated?"* De Barros and Suppes' analysis makes use of inequalities which are said to be *"both necessary and sufficient for the existence of a local hidden variable"* for the experimentally realizable GHZ correlations. In applying their analysis to the Innsbruck experiment, however, they only count events in which all the detectors fire. While necessary for the analysis of that experiment, they recognize that this selective procedure weakens the argument for the non-existence of local hidden variables.

In Szabó and Fine (2002) we pointed out that **their analysis does not rule out a whole class of local hidden variable models in which the detection inefficiency is not (only) the effect of the random errors in the detector equipment, but it is a more fundamental phenomenon, the manifestation of a predetermined hidden property of the particles**. This conception of local hidden variables was first suggested in Fine's *prism model* (1982) and, arguably, goes back to Einstein.

Both, de Barros and Suppes' analysis and our polemics, confirm, however, Suppes' thesis about the continuing interaction in science between theory and experiment.

Theory ⇒ Experiment

De Barros and Suppes approach the problem in the following way. Without loss of generality, the space of hidden variable can be identified with $\mathcal{O} = \{+,-\}^6$, the set of the $2^6 = 64$ different 6-tuples of possible combinations of the values of $\sigma_{1x}, \sigma_{1y}, \ldots \sigma_{3y}$. Then the GHZ contradiction amounts to the assertion that no probability measure over $\mathcal{O}$ reproduces the expectation values.

De Barros and Suppes demonstrate this by concentrating on the product observables (A, B, C and ABC) for which they derive a system of inequalities that play the same role for GHZ that the general form of the Bell inequalities do for EPR-Bohm type experiments; namely, they provide necessary and sufficient conditions for a certain class of local hidden variable models. Their inequalities are just

$$\begin{array}{rcccl} -2 & \leq & E(A)+E(B)+E(C)-E(ABC) & \leq & 2 \\ -2 & \leq & E(A)+E(B)-E(C)+E(ABC) & \leq & 2 \\ -2 & \leq & E(A)-E(B)+E(C)+E(ABC) & \leq & 2 \\ -2 & \leq & E(A)+E(B)+E(C)+E(ABC) & \leq & 2 \end{array}$$

and clearly this is violated by

$$\begin{aligned} E(A)=E(B)=E(C) &= 1 \\ E(ABC) &= -1 \end{aligned}$$

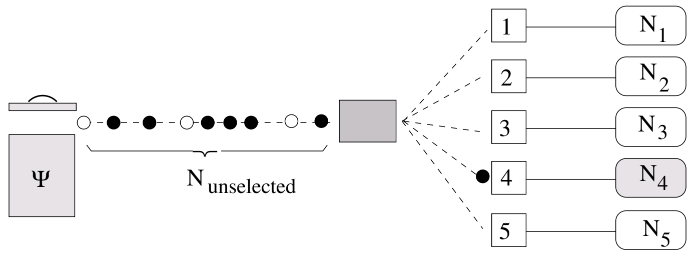
Experiment ⇒ Theory

In the realistic experiments, due to **inefficiencies in the detectors** or to **dark photon detection**, the observed correlations were reduced by some factor ε; that is

$$\begin{aligned} E(A)=E(B)=E(C) &= 1-\varepsilon \\ E(ABC) &= -1+\varepsilon \end{aligned}$$

Theory ⇒ Experiment

Then, it follows immediately from the inequalities that, *"the observed correlations are only compatible with a local hidden variable theory"* if $\varepsilon > \frac{1}{2}$. De Barros and Suppes (2000) translated this condition into the language of the dark-count rate and the detector efficiency.



$$N_{\text{unselected}} \neq \sum_i N_i$$

$$tr\,(WP_i) = \frac{N_i}{\sum_i N_i}$$

Figure 1: *In a typical quantum measurement, quantum mechanical "probabilities" are equal to the relative frequencies taken on a* sub-ensemble of *objects producing any outcome*

Experiment ⇒ Theory

Estimating the realistic values of the dark-count rate and the detector efficiency, they found that the **Innsbruck experiment is not compatible with a local hidden variable theory**.

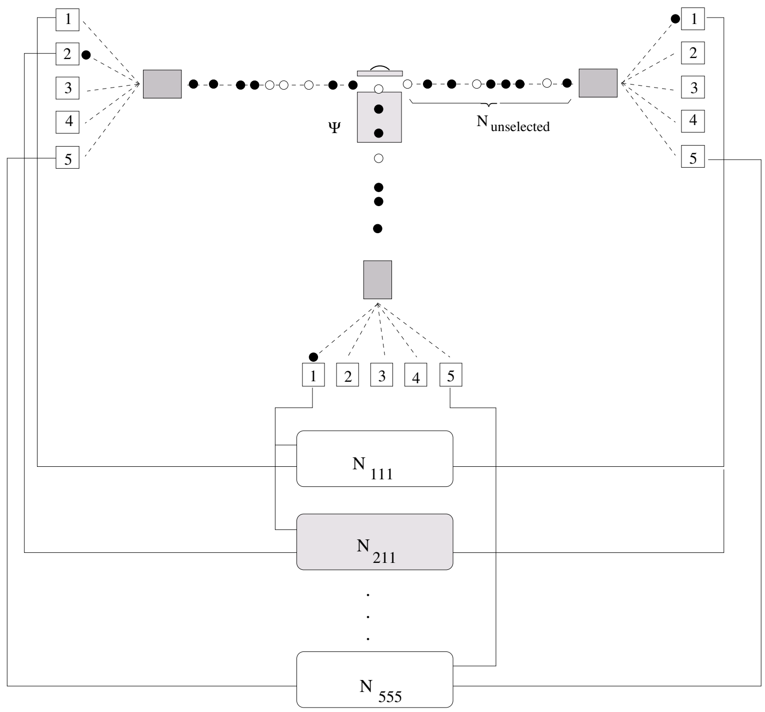
Theory ⇒ Experiment

As in the case of the Bell inequalities, however, the de Barros and Suppes derivation is based on the assumption that the variables $\sigma_{1x}, \sigma_{1y}, \ldots \sigma_{3y}$ are two valued (either $+1$ or -1).

Consider, however, a typical configuration of a quantum measurement shown in Figure 1. We have no information about the original ensemble of emitted particles. Quantum mechanical "probabilities" are equal to the relative frequencies taken on a *sub-ensemble of* objects producing any outcome (passing the analyzer).

In case when the conjunction of three properties are measured (Fig. 2), like the GHZ experiment, quantum mechanical "probabilities" are experimentally identified with the relative frequencies calculated on *sub-ensemble* of the complex systems that produce triple detection coincidences.

Fine's prism model reflects the above experimental scenario. The variables can take on a third value,"D", corresponding to an inherent "no show" or defectiveness. Consequently, the space Λ of hidden variables is a subset of $\{+, -, D\}^6$. In Szabó and Fine (arXiv:quant-ph/000102 v4, 2001) we gave explicit prism models for a GHZ experiment with perfect detector efficiency and with zero dark-photon detection probability. Each element of Λ is a 6-tuple that corre-



$$N_{\text{unselected}} \neq \sum_{i,j,k} N_{ijk}$$

$$tr\left(W\left(P_i \otimes I \otimes I\right)\left(I \otimes P_j \otimes I\right)\left(I \otimes I \otimes P_k\right)\right) = \frac{N_{ijk}}{\sum_{i,j,k} N_{ijk}}$$

Figure 2: *Quantum mechanical "probabilities" are experimentally identified with the relative frequencies calculated on* sub-ensemble *of the complex systems that produce triple detection coincidences*

sponds to combinations like

$$\sigma_{1x}, \sigma_{1y}, \sigma_{2x}, \sigma_{2y}, \sigma_{3x}, \sigma_{3y} = (+ - D - ++)$$

which, for example, stands for the case when particle 1 is predetermined to produce the outcome $+1$ if x-measurement is performed, -1 if the setup is y in the measurement, particle 2 is x-defective, i.e., it gives no outcome if for an x-measurement, but produces an outcome -1 for y, particle 3 produces outcome $+1$ for both cases. Some of these combinations have probability zero, which rule out a large number of 6-tuples. One can show that we achieve the best efficiency if we take for Λ the subset, listed in Table 1, and simply omit all the others. Each atomic element has probability $\frac{1}{48}$. Each GHZ event is represented as a subset $U \subseteq \Lambda$. For instance, $U_{x^+y^-y^-}$ stands for the triple outcome $x^+y^-y^-$ with probability

$$p\left(U_{x^+y^-y^-}\right) = p\left(\{\lambda_{32}, \lambda_{34}, \lambda_{37}, \lambda_{39}, \lambda_{41}, \lambda_{44}\}\right) = \frac{6}{48}$$

The probability of a triple detection for the measurement setups x, y, y:

$$p\left(U_{x^{\neq D}y^{\neq D}y^{\neq D}}\right) = p\left(\left\{\underbrace{\lambda_1, \lambda_4, \lambda_5, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{10}, \ldots \lambda_{44}, \lambda_{45}, \lambda_{48}}_{24}\right\}\right) = \frac{24}{48}$$

Quantum probabilities are reproduced as conditional probabilities:

$$p_{QM}\left(x^+y^-y^-\right) = \underbrace{\frac{1}{8}\left(1 + \sin\left(\frac{\pi}{2} + 0 + 0\right)\right)}_{\frac{1}{4}}$$

$$= p\left(U_{x^+y^-y^-} \left| U_{x^{\neq D}y^{\neq D}y^{\neq D}} \right.\right) = \frac{\frac{6}{48}}{\frac{24}{48}} = \frac{1}{4}$$

etc. All quantum probabilities and the GHZ correlations are correctly reproduced in the model. The triple detection efficiency $= \frac{1}{2}$!

Experiment ⇒ Theory

The question is what is the triple detection/emission ratio in the realistic GHZ experiments. Although the reported triple detection probability is very low ($\approx 10^{-4}$), this question is, actually, irrelevant in case of the Innsbruck experiment. The reason is that **the preparation of GHZ entangled states is performed on selected sub-ensembles conditioned by the triple coincidence detections. Therefore, all of these experimental observations will be treated by our local hidden variable model.**

References

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