Reichenbach's Common Cause Principle: Recent Results and Open Questions

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1 Reichenbach's Common Cause Principle

No correlation without causation. This is, in its most compact and general formulation, the essence of what became called the *Common Cause Principle* (CCP). Further specifications of CCP typically make it either an ontic or an epistemic principle. The ontic version asserts that if two events A and B are correlated, then either there is a causal connection between A and B that brings about the correlation or there is a third event C (common cause) that stands in a causal connection with A and B, and it is this C that causes the correlation. The epistemological CCP reads: on the basis of a correlation between A and B one is justified to infer either the presence of a causal connection between A and B, or to infer the existence of another event C together with the presence of a causal connection between C and A and B.

The ontic CCP is the more fundamental, since the fate of the epistemic CCP rests on the status of the ontic CCP: if the ontic CCP does not hold as a general principle, then it is pointless to formulate a general methodological rule instructing us to safely infer presence of causal relations that might in fact not exist. This is in contrast to the position advocated by Sober [33], who strongly emphasized the need to distinguish between the epistemic and ontic versions of CCP but argued that it does make methodological sense to maintain an epistemic CCP even if the ontic reading of CCP cannot be maintained in its full generality. Sober is forced to take this position because he accepts the standard interpretation of violation of Bell's inequality by quantum mechanics, advocated e.g. by van Fraassen [43] and [3], according to which the ontic CCP is violated by experimentally confirmed quantum correlations. We shall return in Section 6 to the issue of whether quantum correlations do indeed imply violation of CCP, and we shall argue that if CCP is specified in the sense of Reichenbach, then this is either not true or is still an open problem. If, however, quantum correlations did indeed violate CCP, we would *not* consider this "... a very small challenge to the principle construed methodologically..." [33][p. 214] but a major blow. Thus in what follows, we shall interpret CCP in the ontic sense.

Even with the ontic-epistemic specification, the content of CCP, as formulated above, is still very vague, since all the notions involved in it are to be specified: The notion of a causal relation $R_c(A, B)$ between two events A and B is notoriously elusive and the common cause relation $R_{cc}(A, B, C)$ between the three events indicated is also to be made more precise for CCP to have some more specific content. The classic further specification of CCP is due to Reichenbach. In his book [25] Reichenbach interprets A and B as random events in the sense of classical probability theory, he interprets correlation between A and B as (positive) probabilistic correlation and gives an explicit formal definition of the common cause relation $R_{cc}(A, B, C)$ (called by Reichenbach *conjunctive fork*) in terms of the probabilities of the events involved:

Definition 1.1 *Let $(\mathcal{S}, p)$ be a Kolmogorovian probability space with the Boolean algebra $\mathcal{S}$ representing the set of random events and with the probability measure p on $\mathcal{S}$. The triple (A, B, C) of events in $\mathcal{S}$ is called a* conjunctive fork *if the following conditions hold*

$$
\begin{aligned}
p(A \wedge B|C) &= p(A|C)p(B|C) & (1)\\
p(A \wedge B|C^\perp) &= p(A|C^\perp)p(B|C^\perp) & (2)\\
p(A|C) &> p(A|C^\perp) & (3)\\
p(B|C) &> p(B|C^\perp) & (4)
\end{aligned}
$$

where $p(X|Y) = p(X \wedge Y)/p(Y)$ denotes the conditional probability of X on condition Y, $C^\perp$ denotes the complement of C and it is assumed that none of the probabilities $p(X)$, $(X = A, B, C, C^\perp)$ is equal to zero.

It has become standard terminology to call (1) and (2) *screening off* conditions, and to call (1)-(4) *Reichenbachian conditions.*

Reichenbach defines the notion of common cause of a probabilistic correlation in terms of a conjunctive fork:

Definition 1.2 *Given A and B that are correlated*

$$p(A \wedge B) > p(A)p(B) \tag{5}$$

the event C is called common cause *of the correlation (5) if A, B, C form a conjunctive fork.*

Following Reichenbach, we have formulated the notion of common cause of a *positive* correlation. It is clear, however, that a notion of common cause of a negative correlations can also be formulated by a slight and trivial modification of the conditions (3) and (4). To simplify matters and formulas, in what follows we consider only positive correlations; however, every proposition and interpretation we present here is valid for arbitrary correlations, even when we consider a *set* of correlations. Whenever the issue of the sign of the correlation is important, we will indicate it.

Reichenbach considers Definition 1.2 justified by the fact that if A, B, C form a conjunctive fork, then C indeed "causes" the correlation in the sense that the conjunctive fork relation between A, B and C *entails* that there is positive correlation between A and B. To be more precise, Reichenbach proves the following proposition.

Proposition 1.1 *Conditions (1)-(4) imply (5), i.e. if (1)-(4) hold for A, B and C, then A and B are positively correlated.*

Reichenbach's other way of expressing the content of Proposition 1.1 was to say that C *explains* the correlation, by which he meant precisely the *deducibility* of the correlation from the assumption of the conjunctive fork property – an instance of Hempel's concept of explanation [8].

For later purposes we introduce some further terminology in connection with the notion of common cause.

Definition 1.3

1. Let $\Delta(X, Y) \equiv X \setminus Y \cup Y \setminus X$ be the symmetric difference of sets X, Y. A common cause C of the correlation between A, B is called *proper* if $p(\Delta(A, C)) = p(\Delta(B, C)) \neq 0$, i.e. if the common cause differs from the correlated events by more than a measure zero event. Otherwise C is called *improper*.

2. It can happen that, in addition to being a probabilistic common cause, the event C logically implies both A and B, i.e. $C \subseteq A \wedge B$. If this is the case then we call C a *strong* common cause. If C is a common cause such that $C \not\subseteq A$ and $C \not\subseteq B$ then C is called a *genuinely probabilistic* common cause.

3. A common cause C will be called *deterministic* if

$$\mu(A|C) = \quad 1 \quad = \mu(B|C) \tag{6}$$

$$\mu(A|C^{\perp}) = \quad 0 \quad = \mu(B|C^{\perp}) \tag{7}$$

Note that the notions of deterministic and genuinely probabilistic common cause are not negations of each other. There does not seem to exist any straightforward relation between the notions of deterministic, genuinely probabilistic and proper common causes, as we have defined them.

It is easy to see that the following proposition is true ([37]):

Proposition 1.2 *If A and B are maximally correlated, i.e.*

$$p(A|B) = p(B|A) = 1 \tag{8}$$

then the correlation between A and B can have a deterministic common cause only.

With Reichenbach's specification of the notion of common cause one obtains *Reichenbach's Common Cause Principle* (RCCP): If two events A and B are probabilistically correlated in the sense of (5), then this probabilistic correlation is either due to a direct causal connection between A and B, or, if there is no direct causal connection between A and B, then there exists a third event C that is a common cause of the correlation in the sense of Definition 1.2.

RCCP as just formulated is still in need of further clarification in a crucial respect: depending on the particular notion of event and on the related interpretation of probability one obtains different RCCP's, and extreme care must be exercised not to confuse the different notions of event in connection with RCCP. There are two, radically different ways of interpreting events: events can be viewed as singular and individual occurrences that happen only once at a particular time and location, or they can be taken as what became called event *types*. A die landing on one of its sides, say 6, at a specific time and location is a singular event, while "result of the throw is 6" is an event type if it is viewed as the collection of singular events in each of which the die has landed on its side 6 at a particular time and location.

These two interpretations of events give rise to two different kinds of probability: one in which it is singular events that have probabilities (sometimes called "chances"), and the other in which probabilities are assigned to event types only. For instance, in the propensity interpretation it is singular events that have probabilities, while probabilities are assigned only to event types in the frequency interpretation. [1]

[1]In accordance with with our intention to view RCCP as an ontic principle we leave aside a Bayesian, or any other interpretation in which the entities that are assigned probabilities are subjective in the sense of not being accessible to observers as events. For some comments on a Bayesian interpretation of RCCP see [42][p. 194]

The literature on RCCP (and on probabilistic causality in general) is surprisingly rich in misunderstandings, ambiguities and in sheer nonsense related to the confusion about these two interpretations of events. It is not entirely clear, for instance, how Reichenbach himself interpreted events and their probabilities in connection with his notion of common cause. Some of the examples he provides to illustrate the notion of common cause give the impression that he considered meaningful to talk about the probabilities of singular events but he does not say so explicitly, nor does he endorse an explicitly frequentist position in connection with the notion of common cause. Be it as it may, we take the position here that Reichenbach's notion of common cause is to be interpreted in terms of event types and we interpret probability according to the relative frequency view. We are motivated to opt for this choice by the fact that the frequency view, its conceptual problems notwithstanding, is the only serious candidate for an interpretation of probability in sciences.[2]

2 Common cause extendibility of classical events structures

In principle, there are two ways to interpret CCP in general, each determined by how one views the status of the principle with respect to the conditions of its validity: CCP can be viewed as a falsifiable or a non-falsifiable principle. In the falsificationist interpretation, CCP is a claim that can possibly be shown not to hold for some empirically given events and their correlations; in the non-falsificationaist interpretation CCP cannot be false – whatever the actual circumstances. Once CCP is specified by particular definitions of the causal $R_c(A, B)$ and common cause $R_cc(A, B)$, relations, it should be a matter of fact whether the resulting particular CCP is a falsifiable or non-falsifiable principle. Is *Reichenbach's* common cause principle (RCCP) falsifiable or non-falsifiable?

We shall argue that RCCP, as formulated in section 1, is not falsifiable, but the argument cannot be trivial, since RCCP is certainly not trivially non-falsifiable: it is *not* true that *every* classical probability space $(\mathcal{S}, p)$ is provably common close complete in the sense that for any $A, B \in \mathcal{S}$ that are correlated in p there exists a $C \in \mathcal{S}$ which is a (*proper*)[3] common cause of the correlation between A and B. There exists *common cause incomplete* probability spaces. For instance the probability space below is common cause incomplete.

Does existence of common cause incomplete probability spaces entail that RCCP is possibly falsifiable? Such a conclusion would only be justified if one could prove that a certain common cause incomplete probability space $(\mathcal{S}, p)$

[2] For a non-probabilistic specification of Reichenbach's common cause principle in terms of branching spacetime see [2] and [15].

[3] The qualification "proper" is important here: both A and B satisfy Reichenbach conditions, however, we would not consider them proper common causes of the correlation between A and B.

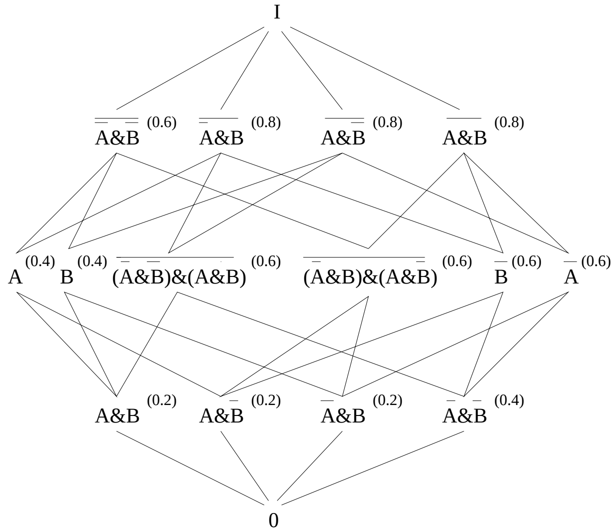

Figure 1: *Common cause incomplete Boolean algebra*

cannot even be made common cause complete with respect to the correlation that does not have a common cause in $\mathcal{S}$. This is because RCCP is *not* the claim that given a correlated pair (A, B) of events in $\mathcal{S}$ there has to exist a common cause C that *belongs to* $\mathcal{S}$: RCCP is a pure existence claim, and it is precisely this pure existence claim character that makes it non-falsifiable. If, however, one wishes to maintain the validity of RCCP despite the threat coming from the existence of common cause incomplete probability spaces, then one has to be able to claim that there might exist an event not accounted for in $\mathcal{S}$ which satisfies Reichenbach's conditions. Furthermore, for such a defense of RCCP to be acceptable, the assumption of the existence of such a "hidden" event as well as the value of its particular probability must be consistent with the events and their probabilities as specified by $(\mathcal{S}, p)$; in short, the probability space $(\mathcal{S}, p)$ must be consistently extendable into a larger probability space $(\mathcal{S}', p')$ that contains an event satisfying Reichenbach's conditions. If this can be done, we call $(\mathcal{S}, p)$ *common cause completable* with respect to the given correlation. It can be shown [11] that every common cause incomplete probability space is common cause completable with respect to any finite set of correlation in it. To present the precise proposition we need some definitions first.

Definition 2.1 *The probability space $(\mathcal{S}', \mu')$ is called an* extension *of $(\mathcal{S}, \mu)$ if there exists a Boolean algebra embedding h of $\mathcal{S}$ into $\mathcal{S}'$ such that*

$$\mu(X) = \mu'(h(X)) \qquad \textit{for all } X \in \mathcal{S} \tag{9}$$

This definition, and in particular the condition (9), implies that if $(\mathcal{S}'\mu')$ is an extension of $(\mathcal{S},\mu)$ (with respect to the embedding h), then every single correlation $\mu(A\wedge B)>\mu(A)\mu(B)$ in $(\mathcal{S},\mu)$ is carried over intact by h into the correlation

$$\begin{aligned}\mu'(h(A)\wedge h(B)) &= \mu'(h(A\wedge B)) \\ &= \mu(A\wedge B) \;>\; \mu(A)\mu(B)=\mu'(h(A))\mu'(h(B))\end{aligned}$$

Hence, it makes sense to ask whether a correlation in $(\mathcal{S},\mu)$ has a Reichenbachian common cause in the extension $(\mathcal{S}',\mu')$.

Given a correlation $p(A\wedge B)>p(A)p(B)$, we call a set of five real numbers r_C, $r_{A|C}$, $r_{B|C}$, $r_{A|C^\perp}$, $r_{B|C^\perp}$ *admissible* if they satisfy conditions (10)-(16) below.

$$\begin{aligned}
0 &\leq r_{A|C},r_{B|C},r_{A|C^\perp},r_{B|C^\perp}\leq 1 &(10)\\
p(A) &= r_{A|C}r_C+r_{A|C^\perp}(1-r_C) &(11)\\
p(B) &= r_{B|C}r_C+r_{B|C^\perp}(1-r_C) &(12)\\
p(A\wedge B) &= r_{A|C}r_{B|C}r_C+r_{A|C^\perp}r_{B|C^\perp}(1-r_C) &(13)\\
0 &< r_C<1 &(14)\\
r_{A|C} &> r_{A|C^\perp} &(15)\\
r_{B|C} &> r_{B|C^\perp} &(16)
\end{aligned}$$

It is easy to see [11] that given a correlation $p(A\wedge B)>p(A)p(B)$ and a common cause C of it the numbers $r_C=p(C)$, $r_{A|C}p(A|C)$, $r_{B|C}p(B|C)$, $r_{A|C^\perp}p(A|C^\perp)$, and $r_{B|C^\perp}p(B|C^\perp)$ are admissible numbers. This motivates the following definition.

Definition 2.2 *A common cause C of a correlation $p(A\wedge B)>p(A)p(B)$ is said to have (be of) the* type $(r_C,r_{A|C},r_{B|C},r_{A|C^\perp},r_{B|C^\perp})$ *if these numbers are equal to the probabilities indicated by the indices, i.e. if the equations (17)-(21) below hold.*

$$\begin{aligned}
p(C) &= r_C &(17)\\
p(A|C) &= r_{A|C} &(18)\\
p(A|C^\perp) &= r_{A|C^\perp} &(19)\\
p(B|C) &= r_{B|C} &(20)\\
p(B|C^\perp) &= r_{B|C^\perp} &(21)
\end{aligned}$$

Elementary algebraic calculations show [11] that the following proposition is true.

Proposition 2.1 *Given any correlation $p(A \wedge B) > p(A)p(B)$ in $(\mathcal{L}, p)$ there exists a non-empty two parameter family of numbers*

$$r_C(t,s), r_{A|C}(t,s), r_{B|C}(t,s), r_{A|C^\perp}(t,s), r_{B|C^\perp}(t,s)$$

that satisfy the relations (10)-(16).

Definition 2.3 *We say that $(\mathcal{S}', p')$ is a type-$(r_C, r_{A|C}, r_{B|C}, r_{A|C^\perp}, r_{B|C^\perp})$* common cause completion of $(\mathcal{S}, p)$ with respect to the correlated events A, B *if $(\mathcal{S}', p')$ is an extension of $(\mathcal{S}, p)$, and there exists a Reichenbachian common cause $C \in \mathcal{S}'$ of type-$(r_C, r_{A|C}, r_{B|C}, r_{A|C^\perp}, r_{B|C^\perp})$ of the correlation $p'(h(A) \wedge h(B)) > p'(h(A))p'(h(B))$.*

We can now give the following definition:

Definition 2.4 *Let $(\mathcal{S}, p)$ be a probability space and $\{(A_i, B_i) \mid i \in I\}$ be a set of pairs of correlated events in $\mathcal{S}$. We say that $(\mathcal{S}, p)$ is* common cause completable with respect to the set $\{(A_i, B_i) \mid i \in I\}$ of correlated events *if, given any set of admissible numbers $(r^i_C, r^i_{A|C}, r^i_{B|C}, r^i_{A|C^\perp}, r^i_{B|C^\perp})$ for every $i \in I$, there exists a probability space $(\mathcal{S}', p')$ such that for every $i \in I$ the space $(\mathcal{S}', p')$ is a type-$(r^i_C, r_{A|C}, r^i_{B|C}, r^i_{A|C^\perp}, r^i_{B|C^\perp})$ common cause extension of $(\mathcal{S}, p)$ with respect to the correlated events A_i, B_i.*

Proposition 2.2 *Every classical probability space $(\mathcal{S}, \mu)$ is common cause completable with respect to any* finite *set of correlated events.*

Proposition 2.2 was proved in [11], the proof is constructive: the probability space $\mathcal{S}', p')$ is constructed from the elements of $(\mathcal{S}, p)$. The following problem is open, we conjecture that the answer to the question formulated in it is positive:

Problem 2.1 *Is Proposition 2.2 true for an arbitrary (i.e. possibly infinite) set of correlated events?*

3 Common cause completability of non-classical probability spaces

Replacing the Boolean algebra $\mathcal{S}$ and p in $(\mathcal{S}, p)$ by a general orthomodular lattice $\mathcal{L}$ and by an additive (σ-additive if $\mathcal{L}$ is a σ-lattice) state on $\mathcal{L}$ one obtains a general non-classical probability space $(\mathcal{L}, p)$. Typical examples of such non-classical probability spaces are the projection lattices $\mathcal{P}(\mathcal{N})$ of von Neumann algebras $\mathcal{N}$ with a normal state p on $\mathcal{N}$. Two elements $A, B \in \mathcal{L}$ are called *compatible* if $A = (A \cap B) \cup (A \cap B^\perp)$. If A, B are compatible and

$$p(A \wedge B) > p(A)p(B) \tag{22}$$

then A and B are called (positively) correlated with respect to the state p.

We wish to raise the problem of whether non-classical probability spaces are also common cause completable. To formulate this question precisely, we need to re-define the notion of common cause in these non-classical probability structures because not all events are compatible in these non-classical structures, consequently, the classical definition of conditional probability does not make sense between arbitrary elements. So one either has to replace the classical conditional probabilities in Reichenbach's definition of common cause (Definition 1.2) by some non-classical analogues, or one has to stipulate that only such C's can be common causes that are compatible with both A and B. We chose the second option here. The reason is that we also wish to consider the common causes in non-classical probability spaces as explanations of the correlation, just like in the classical case. As we discussed in section 1, the explanatory power of common causes in classical probability spaces comes from Proposition 1.1. The validity of this proposition rests on the theorem of total probability, i.e. on the equation $X = (X \wedge C) \vee (X \wedge C^{\perp})$ with $X = A, B$, which holds if and only if A, B and C are compatible. (For a detailed discussion see [9], [10].) So we stipulate

Definition 3.1 *If A and B are positively correlated, then $C \in \mathcal{L}$ is called a* common cause *of the correlation (22) if C is compatible with both A and B and the following conditions hold.*

$$\begin{aligned} p(A \wedge B|C) &= p(A|C)p(B|C) && (23)\\ p(A \wedge B|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) && (24)\\ p(A|C) &> p(A|C^{\perp}) && (25)\\ p(B|C) &> p(B|C^{\perp}) && (26) \end{aligned}$$

where $p(X|Y) = p(X \wedge Y)/p(Y)$ denotes the conditional probability of X on condition Y and it is assumed that none of the probabilities $p(X)$, $(X = A, B, C, C^{\perp})$ is equal to zero.

Just like a classical probability space, a non-classical probability space $(\mathcal{L}, p)$ may contain a correlation without containing a proper common cause of the correlation in the sense of Definition 3.1. If this is the case, then we call the non-classical probability space *common cause incomplete*, and we may ask if the non-classical probability space can be enlarged so that the enlarged space contains a proper common cause. What is meant by "enlargement" is completely analogous to the classical case: The probability space $(\mathcal{L}', p')$ is an *extension* of the probability space $(\mathcal{L}, p)$ if there exists a lattice homomorphism h (preserving also the orthocomplementation) of $\mathcal{L}$ into $\mathcal{L}'$ such that $p'(h(X)) = p(X)$ for all $X \in \mathcal{L}$ and such that $X \neq Y$ implies $h(X) \neq h(Y)$ (h is an embedding).

Having these definitions, we can define the type of the common cause in a non-classical probability space in exactly the same way as in the classical case, we can also speak of admissible numbers, of common cause extension of a given type and of common cause extendability, the latter meaning that common cause extensions of every admissible type exist (see [11] and [12] for details). We can then formulate the following problem.

Problem 3.1 *Is every non-classical probability space* $(\mathcal{L}, p)$ *common cause completable with respect to any set of events that are correlated in* p*?*

The answer to this question is not known; however, it is known to be affirmative in an important special case of non-classical probability spaces: when $\mathcal{L}$ is a von Neumann lattice $\mathcal{P}(\mathcal{N})$ of projections of a von Neumann algebra $\mathcal{N}$:

Proposition 3.1 *Every quantum probability space* $(\mathcal{P}(\mathcal{N}), \phi)$ *is common cause completable with respect to the set of pairs of events that are correlated in the normal state* ϕ.

Note that Propositions 2.2 and 3.1 do *not* imply that probability spaces are common cause closed, where by common cause closedness is meant that the set of events contains a (proper) Reichenbachian common cause of every single correlation in the probability space. In fact, it is not known[4]

Problem 3.2 *Are there common cause closed classical or non-classical probability spaces? In particular, it would be interesting to know if a probability space with a* finite *set of events can be common cause closed.*

Whether or not common cause closed probability spaces exist, it is not reasonable to expect a probabilistic physical theory to be common cause closed, this requirement would be too strong. This is because one does not expect to have a proper common cause explanation of probabilistic correlations that arise as a consequence of a direct physical influence between the correlated events, or which are due to some logical relations between the correlated events. One would want to have a common cause explanation of correlations only between events A, B that are not directly causally related and which do not stand in a straightforward "logical consequence relation" to each other – this is in conformity with the content of RCCP. Thus an explicit notion of the causal dependence relation $R_c(A, B)$, different from the notion of the standard probabilistic independence (correlation), and also a definition of logical independence is needed to formulate a reasonable notion of common cause closedness. We do have a concept of "logical independence" (see [20], [19] and [22]). Two orthocomplemented sub-lattices $\mathcal{L}_1$ and $\mathcal{L}_2$ of an orthomodular lattice $\mathcal{L}$ are called *logically independent* if $A \wedge B \neq 0$ for any $A \in \mathcal{L}_1$ and $B \in \mathcal{L}_2$. This is an independence

[4]Some preliminary investigations seem to indicate that common cause closed probability spaces exist: the probability space with a 2^5 element Boolean algebra can be common cause closed. Interestingly it seems that this is the *only* finite Boolean algebra that can be common cause closed, see [7].

condition that obtains between spacelike separated local systems in the sense of (algebraic) quantum field theory; so this logical independence condition can be viewed as a lattice theoretic formulation of a necessary condition of causal disjointness of events. It seems reasonable then to expect a probabilistic physical theory represented by a probability space $(\mathcal{L}, \mu)$ to be common cause closed with respect to the correlated elements in every two, logically independent, commuting sub-lattices $\mathcal{L}_1, \mathcal{L}_2$. It is not known if this is possible:

Problem 3.3 *Is there a classical $(\mathcal{S}, p)$ probability space such that $(\mathcal{S}, p)$ is common cause closed with respect to all correlations between elements of any two (or two fixed) logically independent sub-Boolean lattices $\mathcal{S}_1, \mathcal{S}_2$ of $(\mathcal{S}, p)$? How about the same problem for non- classical probability spaces?*

4 Common causes are not common common causes

Note that what Propositions 2.2 and 3.1 say is not that there exists a *common* common-cause C of the whole set of correlations between (A_i, B_i) $(i = 1, 2 \ldots n)$; in fact, the common causes C_i constructed explicitly in the proof of Propositions 2.2 and 3.1 in [11] are all different: $C_i \neq C_j$ $(i \neq j)$. This leads to the following question: Let (A_i, B_i) $(i = 1, \ldots n)$ be a finite set of pairs of events in $(\mathcal{S}, p)$ that are correlated $(p(A_i \wedge B_i > p(A_i)p(B_i)$ for every i). We say that C is a *common* common cause of these correlations if C is a Reichenbachian common cause of the correlated pair (A_i, B_i) for every i. Does every set of correlations in a classical probability space have a common common cause? The answer is known to be negative:

Proposition 4.1 *There exists a probability space $(\mathcal{S}, p)$ and two correlated pairs of events (A_1, B_1) and (A_2, B_2) in $(\mathcal{S}, p)$ such that there does not exist in $(\mathcal{S}, p)$ a common common cause of these correlations, and there cannot exists an extension $(\mathcal{S}', p')$ of $(\mathcal{S}, p)$ that contains a common common cause of these two correlations.*

This proposition, proved in [13], shows that Propositions 2.2 and 3.1 cannot be strengthened in the following sense: While it is true that, given a finite set of correlations in a common-cause incomplete probability space, classical or quantum, the probability space can always be enlarged so that the larger one contains a common cause of each correlation in the given set, these common causes differ from correlation to correlation, and, in general, there exists no enlargement that contains a common common cause of even *two* of the correlations in the finite set. One conclusion one can draw from this is that the notions of common cause and of common common cause are radically different; that is to say, if two correlations have a common common cause, then the random events involved stand in a probabilistic relation content of which is not exhausted by the individual relations of the common causes to the correlations explained by them. Formulated differently: The assumption that two correlations have a common common cause is much stronger than the assumption that each of the two correlations has its own common-cause, and, while Propositions 2.2 and 3.1 show that one cannot conclude exclusively on the basis of knowing the probabilities of the events that common causes of correlations do not exist, knowing the probabilities of the events involved one can exclude common common causes. Being aware of this, one should be extremely cautious when requiring that an explanation of a set of correlations should be in terms of a common common cause: such a requirement should always be carefully argued. One has to bear in mind in particular that RCCP has nothing to do with multiple correlations and their (generally nonexistent) common common causes. This simple observation has far-reaching consequences in the case of the so- called "EPR type corrections" (see [21], Chapter 11 in [22], [11] [12]. We return to this issue in section 6.

Note that a necessary condition for two correlations (one possibly being a *negative* correlation) to have a common common cause is known [14], no necessary and sufficient conditions are known for n arbitrary correlations to possess a common common cause. We formulate this as an open problem:

Problem 4.1 *Find necessary and sufficient conditions for a finite set of correlations to have a common cause!*

5 Reichenbachian common cause systems

Confronted with a common cause incomplete probability space $(\mathcal{S}, p)$ in which a direct causal influence between the correlated events is excluded, one can have in principle two strategies in trying to save RCCP: One may try to argue that $\mathcal{S}$ is not "rich enough" to contain a common cause but there might exist a larger $(\mathcal{S}', p')$ that already contains a common cause of the correlation. We have seen in section 2 that this strategy always works in the sense that it is *always* possible to enlarge $(\mathcal{S}, p)$ in such a way that the enlarged space already contains an event C that satisfies the Reichenbachian conditions.

Another natural idea is to suspect that the correlation between A and B is not due to a *single* factor but may be the cumulative result of a (possibly large) number of different "partial common causes", none of which can in and by itself yield a complete common-cause-type explanation of the correlation, all of which, taken together, can however account for the correlation. To see how such a notion of common cause system should be defined, let us recall that it was Proposition 1.1 that showed in what sense a common cause *explains* a correlation: from the assumption that A, B and C satisfy the Reichenbachian conditions one can *derive* that A and B are correlated. Proposition 1.1 is a direct consequence of the following elementary Lemma: (recall that the set of events $C_i \in \mathcal{S}$ $(i \in I)$ is a partition of $\mathcal{S}$ if $\cup_i C_i = \mathcal{S}$ and $C_i \cap C_j =$ if $i \neq j$.)

Lemma 1: *Let C_i $(i \in I)$ be a partition of $\mathcal{S}$ and $A, B \in \mathcal{S}$ arbitrary elements. If $p(A \wedge B|C_i) = p(A|C_i)p(B|C_i)$ for all $i \in I$ then*

$$p(AB) - p(A)p(B) = \frac{1}{2}\sum_{i \neq j} p(C_i)p(C_j)[p(A|C_i) - p(A|C_j)][p(B|C_i) - p(B|C_j)] \tag{27}$$

Applying Lemma 1 with $C_1 = C$ and $C_2 = C^\perp$ one sees that

$$p(AB) - p(A)p(B) = p(C)p(C^\perp)[p(A|C) - p(A|C^\perp)][p(B|C) - p(B|C^\perp)] \tag{28}$$

So the correlation between A and B is indeed positive if (3)-(4) hold. Eq. (28) also shows, however, that for the correlation to be positive (3)-(4) are *not* needed: positivity of the correlation is implied by the positivity of the right hand side of (28); hence, what is decisive from the point of view of the explanatory power of the common cause is that $p(A|C)$, $p(A|C^\perp)$, $p(B|C)$ and $p(B|C^\perp)$ *determine* the correlation between A and B.

Thus we see that the intuitive idea behind explaining a correlation by a Reichenbachian common cause is that one should be able to cut up the statistical ensemble by a pair of events (C and $C^\perp$) into *two* disjoint parts in such a way that (i) the correlation disappears in *both* of the resulting subensembles (this is expressed by the screening off conditions); and (ii) the probabilities conditioned to these subensembles determine the correlation between A and B.

Explaining a correlation by a system of partial common causes could then mean that one partitions the ensemble into more than two subensembles in such a manner that

1. the correlation disappears in *each* of the subensembles
2. the totality of the "partial common causes" explains the correlation in the sense of entailing it.

More precisely, the following definition of a Reichenbachian common cause system seems a natural generalization of Reichenbach's notion of a (single) common cause.

Definition 5.1 *Let $(\mathcal{S}, p)$ be a probability space and A, B be two arbitrary events in $\mathcal{S}$. The partition C_i $(i \in I)$ of $\mathcal{S}$ is a* Reichenbachian common cause system *(RCC system for short) for the pair A, B if the following condition is satisfied*

$$p(A \wedge B|C_i) = p(A|C_i)p(B|C_i) \qquad \textit{for all} \;\; i \in I \tag{29}$$

It is not difficult to see that there exist probability spaces that are common cause incomplete but contain a Reichenbachian common cause system (see [14]). In fact, a Reichenbachian common cause system always exists: if $\mathcal{S}$ is a finite Boolean algebra, then the (finest) partition given by the single atoms is obviously a Reichenbachian common cause system; what is more, this is a single common cause system which is also common for every possible set of correlations. If $\mathcal{S}$ is not finite, hence not necessarily atomic, and (A_i, B_i) $(i = 1, 2, \ldots n$ is a finite set of correlated pairs in $(\mathcal{S}, p)$, then it is easy to see that the atoms of the finite Boolean subalgebra generated by the set $\{A_i, B_i\}$ is also a *common* common cause system for the given set of correlations. This common cause system may not be the finest one, of course.

6 Reichenbach's common cause principle and EPR correlations

Explaining the correlations predicted by quantum mechanics in the case of joint quantum systems of the EPR-Bohm type has proved to be a major challenge for RCCP. The correlations in question have been confirmed by actual experiments, and these correlations are generally viewed as the only ones known to threaten RCCP in the sense that allegedly no common cause type explanation of these correlations seem possible that do not violate relativistic causality (see for instance [43], [3]), for a more careful voice see [30], see also the quotation from [29] at the end of this section.

Note, however, that it is not at all obvious how to link RCCP to quantum mechanics because Reichenbach's notion of common cause was defined in terms of classical probability theory, not in terms of quantum mechanics. Hence, to link RCCP to correlations predicted by quantum theory one has to do one of two things: either to reformulate Reichenbach's notion of common cause in terms of non-classical (quantum) probability spaces, or to represent quantum probabilities and quantum correlations in terms of classical probability theories. We have seen that one can indeed reformulate Reichenbach's notion in terms of non-classical probability theory, and we also have seen that quantum probability spaces more general than those needed to describe the EPR-Bohm scenario *are* common cause completable (Proposition 3.1); so if one chooses this route, we could then conclude that correlations in such spaces cannot falsify RCCP. One can give strong arguments, however, that elements of a non-Boolean (in particular: orthomodular) lattice (such as the lattice of projections of a Hilbert space) cannot at all be interpreted as representatives of events that can occur in a laboratory with definite relative frequency[5] and that therefore the proper representation of the events in the experiments designed to measure EPR correlations should be in terms of classical probability spaces. Such representations are indeed possible: the idea is that "quantum probabilities" $tr(\widehat{A}\widehat{W})$ (with Hilbert space projection $\widehat{A}$ and density matrix $\widehat{W}$) are *classical conditional* probabilities on condition that the event a of measuring the quantity represented by pro-

[5] Nonexistence of Boolean algebra homomorphisms or partial Boolean algebra homomorphisms (Kochen-Specker theorem) from a Hilbert lattice into a Boolean algebra is one such argument; see [39] and [23] for some further discussion of this problem.

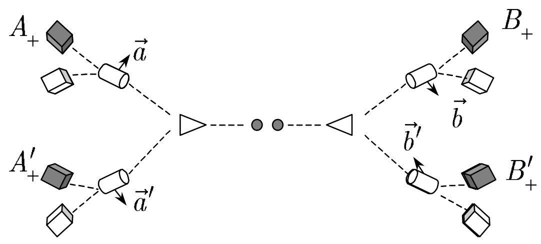

Figure 2: *The EPR experiment with spin-$\frac{1}{2}$ particles*

jection A is performed: $tr(\widehat{A}\widehat{W}) = \frac{p(A \wedge a)}{p(a)}$, where A is the representative in a Boolean algebra $\mathcal{S}$ of the outcome of the a-measurement and p is classical probability measure on $\mathcal{S}$ depending on $\widehat{W}$. (For a proof that such a representation of a quantum probability space $(\mathcal{P}(\mathcal{H}), W)$ always exist see [1] and [39].)

If, however, the EPR-Bohm system and its correlations are represented by a classical probability space, then we can invoke Proposition 2.2 to argue that any correlation *might* have a (possibly hidden) common cause. How can then RCCP be possibly be threatened by these turned-classical EPR correlations? The answer is: it cannot. RCCP, as it was formulated and interpreted with a purely statistical common cause notion in Section 1 cannot be falsified by the EPR correlation. Any attempt to falsify RCCP needs to impose some further restrictions on the principle. Intuitively, those further restrictions should come from linking RCCP to non-probabilistic causal relations between the singular events that comprise the event types A, B and C. While it is not clear how to do this in general, in the case of the EPR correlations we have some specific further information about the non-probabilistic causal structure of the singular events involved, and we can use this information to constrain the possible common cause of the correlations. To formulate these, let us recall the EPR-Bell scenario.

We consider the four 'spin-up' events in the spin-component measurements in directions $\vec{a}, \vec{a}'$ and $\vec{b}, \vec{b}'$ (Fig. 2). There are random switches (independent agents, if you want) choosing between the different possible measurements on both sides. Let $p(a), p(a'), p(b)$ and $p(b')$ be *arbitrary* probabilities with which the different measurements are chosen. The following event types are observed in the experiment:

A_+, A'_+:	*the spin of the left particle is up in direction* $\vec{a}, \vec{a}'$ detector fires
B_+, B'_+:	*the spin of the right particle is up in direction* $\vec{b}, \vec{b}'$ detector fires
a, a':	the left switch chooses the direction $\vec{a}, \vec{a}'$
b, b':	the right switch chooses the direction $\vec{b}, \vec{b}'$

Let $\vec{a}, \vec{a}'$ and $\vec{b}, \vec{b}'$ be coplanar vectors with $\angle\left(\vec{a}, \vec{b}\right) = \angle\left(\vec{a}', \vec{b}'\right) = \angle\left(\vec{a}, \vec{b}'\right) = 120°$, and $\angle\left(\vec{a}', \vec{b}\right) = 0$. We observe the following relative frequencies in the experiment:

$$\begin{array}{lcllcl} p(A_+) & = & \frac{1}{2}p(a) & p(B_+) & = & \frac{1}{2}p(b) \\ p(A'_+) & = & \frac{1}{2}p(a') & p(B'_+) & = & \frac{1}{2}p(b') \\ p(A_+B_+) & = & \frac{3}{8}p(a)p(b) & p(A'_+B_+) & = & 0 \\ p(A_+B'_+) & = & \frac{3}{8}p(a)p(b') & p(A'_+B'_+) & = & \frac{3}{8}p(a')p(b') \end{array} \tag{30}$$

As we can see, there are correlations among the outcome events A_+, A'_+ and B_+, B'_+:

$$\begin{array}{rcl} p(A \wedge B) - p(A)p(B) & = & \frac{1}{8}p(a)p(b) \\ p(A \wedge B') - p(A)p(B') & = & \frac{1}{8}p(a)p(b') \\ p(A' \wedge B') - p(A')p(B') & = & \frac{1}{8}p(a')p(b') \\ p(A' \wedge B) - p(A')p(B) & = & -\frac{1}{4}p(a')p(b) \end{array} \tag{31}$$

These are the notorious EPR correlations and the question is whether they falsify RCCP.

According to RCCP, either there is a causal connection between the above correlated events, or there is a common cause of the correlations. Let us see whether common causes can exist under the following further requirements imposed on the events and on the hypothetical common cause (we shall interpret these requirements once they are written down).

$$\begin{array}{c} p(X) = p(X|x)\,p(x) = tr\left(\widehat{W}\widehat{X}\right)p(x) \\ p(Y) = p(Y|y)\,p(y) = tr\left(\widehat{W}\widehat{Y}\right)p(y) \end{array} \tag{32}$$

$$p(x \wedge y) = p(x)p(y) \tag{33}$$

$$\begin{array}{c} p(X \wedge y) = p(X)\,p(y) \\ p(x \wedge Y) = p(x)\,p(Y) \end{array} \tag{34}$$

$$\begin{array}{c} p(x \wedge V) = p(x)p(V) \\ p(U \wedge y) = p(U)\,p(y) \end{array} \tag{35}$$

$$C_{XY} \neq X, Y \tag{36}$$

$$\begin{array}{c} \text{if } C_{XY} \subset X \text{ or } C_{XY} \supset X \text{ then } p(C_{XY}) \neq p(X) \\ \text{if } C_{XY} \subset Y \text{ or } C_{XY} \supset Y \text{ then } p(C_{XY}) \neq p(Y) \end{array} \tag{37}$$

$$\begin{array}{c} p(X \wedge Y|C_{XY}) = p(X|C_{XY})\,p(Y|C_{XY}) \\ p\left(X \wedge Y|C_{XY}^{\perp}\right) = p\left(X|C_{XY}^{\perp}\right)p\left(Y|C_{XY}^{\perp}\right) \end{array} \tag{38}$$

$$\begin{array}{l} \left\{\begin{array}{l} \delta(XC_{XY}) > 0 \\ \delta(YC_{XY}) > 0 \end{array}\right\} \text{ or } \left\{\begin{array}{l} \delta(XC_{XY}) < 0 \\ \delta(YC_{XY}) < 0 \end{array}\right\} \text{ if } \delta(XY) > 0 \\ \left\{\begin{array}{l} \delta(XC_{XY}) > 0 \\ \delta(YC_{XY}) < 0 \end{array}\right\} \text{ or } \left\{\begin{array}{l} \delta(XC_{XY}) < 0 \\ \delta(YC_{XY}) > 0 \end{array}\right\} \text{ if } \delta(XY) < 0 \end{array} \tag{39}$$

where C_{XY} denotes the hypothetical common cause of correlation $\delta(XY) = p(X \wedge Y) - p(X)p(Y)$, $X = A, A'$; $Y = B, B'$; $x = a, a'$; $y = b, b'$, U is any event in the backward light-cone of X, like C_{XY}, $C_{XY}^{\perp}$, $\ldots C_{AB} \wedge \left(C_{AB'}^{\perp} \vee a\right)$, $\ldots$ and V is any similar event in the backward light-cone of Y.

Interpretation of these conditions is the following.

1. Condition (32) expresses that quantum "probabilities" are now interpreted as classical conditional probabilities.
2. The standard interpretation of (33) is that the event types "settings of the instruments" in the left and right wings are statistically independent.
3. Condition (34) means that event type "outcome in the left (right) wing" is statistically independent of event type "measurement setting in the right (left) wing".
4. Condition (35) is interpreted as expressing the intuition that all combinations of the hypothetical common causes (as event types) should be statistically independent of the measurement settings. Occasionally the terminology of "excluding any mysterious conspiracy between the things that determine the choices of the measurement setting and those that determine the outcomes" is used to describe the content of this condition.
5. Condition (36) and (37) are just to exclude trivial common causes.
6. Conditions (38) and (39) are just repetitions of the familiar screening off and statistical relevance conditions required of the common cause.

Also note that the probabilities $p(a), p(a'), p(b)$ and $p(b')$ are arbitrary, since, apparently, the number of times a certain measurement settings is made is not constrained in any way.

Problem 6.1 *Do common causes C_{XY} satisfying the conditions (32)-(39) exist for any choice of the probabilities $p(a), p(a'), p(b)$ and $p(b')$?*

While the exact solution of this problem is not known, computer calculations indicate [40] that such common causes do not exist.

7 Closing comments on the status of Reichenbach's Common Cause Principle

There exist both classical and quantum probability spaces that contain correlations without containing proper common causes of the correlations; hence, if one wants to maintain Reichenbach's Common Cause Principle, one must be able to claim that there exist "hidden" events ("hidden" in the sense of not being accounted for in the given event structure which is thus common cause incomplete) that can be interpreted as the common causes of the correlations. If such "hidden" common cause events exist, then there must exist an extension of the original probability space, an extension that accommodates the common causes, since otherwise there would be an irreconcilable tension between the observable and unobservable world. Propositions 2.2 and 3.1 tell us that such extensions are always possible. In other words, Propositions 2.2 and 3.1 show that a necessary condition for a common cause explanation of correlations in both classical and quantum event structures can *always* be satisfied. To put this negatively: one cannot disprove Reichenbach's Common Cause Principle by proving that the necessary condition (common cause completability) for its validity cannot be satisfied. To formulate this conclusion even more sharply, we can say that if Reichenbach's Common Cause Principle does not mean more than formulated in Section 1, then this principle is not falsifiable by displaying a common cause incomplete probability space.

It is generally accepted that the Reichenbachian conditions (1)-(4) are just necessary conditions for an event to be accepted as a common cause. If an event C must satisfy also some Supplementary Conditions (in addition to the Reichenbachian conditions) to qualify as a common cause, then a disproof of Reichenbach's Common Cause Principle requires establishing that there exists no event whatsoever that satisfies *both* the Reichenbachian conditions *and* the Supplementary Conditions. It goes without saying that such a disproof requires first the specification of the Supplementary Conditions. Propositions 2.2 and 3.1 impose very strong restrictions on the possible mathematical specifications of the Supplementary Conditions: these conditions cannot be formulated in terms of the probabilities $p(C)$, $p(A|C)$, $p(B|C)$, $p(A|C^{\perp})$ and $p(B|C^{\perp})$. This is because the assumptions in Propositions 2.2 and 3.1 contain no restrictions whatsoever on these probabilities – beyond the Reichenbach conditions. To put this sharply we can say the following. Reichenbach's Common Cause Principle cannot be made falsifiable by amending it by conditions formulated in terms of probabilities of the events involved, as long as these additional probabilistic conditions are consistent with the Reichenbachian conditions.

The only way to make RCCP susceptible to a possible falsificationist attack is to amend Reichenbach's notion of common cause, for instance by relating this notion to some non-probabilistic causal structure between singular events. Specification of common causes of the particular EPR type correlations proceeds along these lines. We have argued, however, that with these amendments the problem of existence of common causes of the EPR correlations is still an open problem.

References

[1] XXXXX, G. Bana: Proof of Kolmogorovian Censorship

Foundations of Physics

[2] N. Belnap, and L.E. Szabó: Branching Space-time Analysis of the GHZ Theorem

Foundations of Physics **26** (1996) 989-1002

[3] J. Butterfield: A space-time approach to the Bell inequality

in [5] pp. 114-144

[4] N. Cartwright: How to tell a common cause: Generalization of the conjunctive fork criterion

in [6] pp. 181-188

[5] J. Cushing and E. McMullin (*eds.*): *Philosophical Consequences of Quantum Theory*, Notre Dame: University of Notre Dame Press, 1989

[6] J. H. Fetzer (*ed.*): *Probability and Causality*, Boston: Reidel Pub. Co., 1988

[7] B. Gyenis: unpublished manuscript

[8] C.G. Hempel: *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science* New York, Free Press, 1965

[9] G. Hofer: The formal existence and uniqueness of the Reichenbachian common cause on Hilbert lattices

International Journal of Theoretical Physics **36** (1997) 1973-1980

[10] G. Hofer: Reichenbach's common cause definition on Hilbert lattices

International Journal of Theoretical Physics **37** (1998) 435-443

[11] G. Hofer-Szabó, M. Rédei and L.E. Szabó: On Reichenbach's common cause principle and Reichenbach's notion of common cause

The British Journal for the Philosophy of Science

[12] G. Hofer-Szabó, M. Rédei and L.E. Szabó: Common cause completability of classical and quantum probability spaces

International Journal of Theoretical Physics **39** (2000) 913-919

[13] G. Hofer-Szabó, M. Rédei and L.E. Szabó: Common-causes are not common common-causes

Manuscript, to be submitted

[14] G. Hofer-Szabó, M. Rédei and L.E. Szabó: Reichenbachian common cause systems

Manuscript, to be submitted

[15] T. Kowalski, T. Placek: Outcomes in branching space-time and GHZ-Bell theorems

British Journal for the Philosophy of Science **50** (1999) 349-375

[16] R. McLaughlin (*ed.*): *What? Where? When? Why?*, Boston: D. Reidel Pub. Co., 1982

[17]

[18] R. Penrose, I.C. Percival: The direction of time

Proceedings of the Physical Society **79** (1962) 605-616

[19] M. Rédei: Logical independence in quantum logic

Foundations of Physics **25** (1995) 411-422

[20] M. Rédei: Logically independent von Neumann lattices

International Journal of Theoretical Physics **34** (1995) 1711-1718

[21] M. Rédei: Reichenbach's Common Cause Principle and quantum field theory

Foundations of Physics **27** (1997) 1309-1321

[22] M. Rédei: *Quantum Logic in Algebraic Approach*, Dordrecht: Kluwer Academic Publishers, 1998

[23] M. Rédei: John von Neumann's concept of quantum logic and quantum probability

in [24]

[24] M. Rédei: *John von Neumann and the Foundations of quantum mechanics*, M. Rédei, M. Stöltzner (eds.), Kluwer Academic Publsihers, forthcoming

[25] H. Reichenbach: *The Direction of Time*, Los Angeles: University of California Press, 1956

[26] M.H. Salmon (et al.): *Introduction to the Philosophy of Science*, Englewood Cliffs: Prentice Hall Inc., 1992

[27] W.C. Salmon: Why ask "Why?"?

Proceedings and Addresses of the American Philosophical Association **51** (1978) 683-705

[28] W.C. Salmon: Probabilistic causality

Pacific Philosophical Quarterly **61** (1989) 50-74

[29] W.C. Salmon: Further reflections
in [16]

[30] W.C. Salmon: *Scientific Explanation and the Causal Structure of the World*, Princeton: Princeton University Press, 1984

[31] W. Salmon and G. Wolters (*eds.*): *Logic, Language and the Structure of Scientific Theories*, Pittsburgh: University of Pittsburgh Press, 1991

[32] E. Sober: Common cause explanation
Philosophy of Science **51** (1984) 212-241

[33] E. Sober: The principle of the common cause
in [6] 211-228

[34] W. Spohn: On Reichenbach's Principle of the Common Cause
in [31] pp. 211-235

[35] P. Suppes: Probabilistic causality in quantum mechanics
Journal of Statistical Planning and Inference **25** (1990) 293-302

[36] P. Suppes: *A Probabilistic Theory of Causality*, Amsterdam: North-Holland, 1970

[37] P. Suppes and M. Zanotti: On the determinism of hidden variables with strict correlation and conditional statistical independence
in [36]

[38] P. Suppes and M. Zanotti: When are probabilistic explanations possible?
Synthese **48** (1981) 191-199

[39] L.E. Szabó: Critical reflections on quantum probability theory,
Forthcoming in M. Rédei, M. Stoeltzner (eds.), *John von Neumann and the Foundations of Physics*, Kluwer, Dordrecht, 2000.

[40] L.E. Szabó: On an attempt to resolve the EPR-Bell paradox via Reichenbachian concept of common cause
http://arXiv.org/abs/quant-ph/9806074

[41] B.C. Van Fraassen: The pragmatics of explanation
American Philosophical Quarterly **14** (1977) 143-150

[42] B.C. Van Fraassen: Rational belief and the common cause principle
in [16] pp. 193-209

[43] B.C. Van Fraassen: The Charybdis of Realism: Epistemological Implications of Bell's Inequality
in [5] pp. 97-113

[44] J. Uffink: The Principle of the Common Cause faces the Bernstein paradox
Philosophy of Science **66** (Supplement) (1999) 512-525