Lorentz's theory and relativity theory are completely identical

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Talk presented on JÁNOS BOLYAI CONFERENCE ON HYPERBOLIC GEOMETRY

8-12 July, 2002, Budapest, Hungary.

Dialog 1

Physicist ...So, we are living in a 4-dimensional world, the geometry of which, the geometry of space-time is a Minkowski geometry ... The distance and the duration between two events are frame-dependent concepts ... The metric of the Minkowski space-time is... so, this is a non-Euclidean geometry ... The Lagrange functions of the various physical theories must be invariant with respect to the ...

Bob What is space, and what is time? These are very fascinating questions! ... But, isn't all that you claimed completely absurd ... which is contrary to our intuition?

Physicist Well, perhaps it sounds absurd ... But that's why we love physics! The physicist is not stuck in metaphysical pre-conceptions. Instead, he is inquiring NATURE, that is, performing measurements. And the experimental facts show that our world is, contrary to our previous belief, as it is described by relativity theory.

Alice If it is an empirical fact that the geometry of our world is the 4-dimensional Minkowski geometry, how is it possible that human beings did not recognize this fact before?

Physicist The reason is very simple: the whole evolution was going on in a non-relativistic domain of the physical reality. Only the development of experimental physics provided us to see what's going on in the relativistic domain.

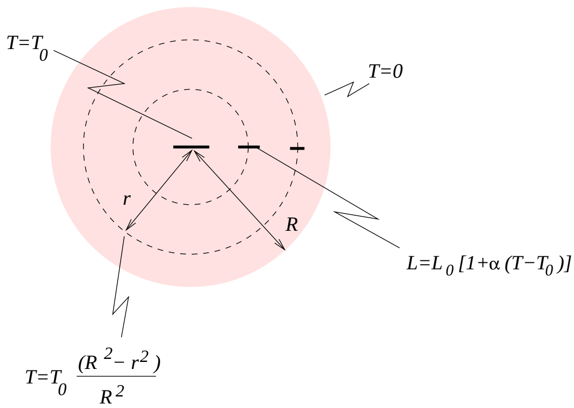
Dialog 2

Philosopher The scientific theories are empirically under-determined. That is, empirical facts do not uniquely determine a theory. It is often the case that the same empirical data can be explained by different theories.

Let me tell you an example: As Poincare pointed out, it is impossible to tell the "real" geometry of the world, independently of the physical theories based on that geometry.

$$\begin{array}{lcl} (\text{spacetime geometry})_1 + (\text{physics})_1 & = & (\text{empirical facts}) \\ (\text{spacetime geometry})_2 + (\text{physics})_2 & = & (\text{empirical facts}) \\ & \vdots & \end{array}$$

We are free in the demarcation between geometry and physics. Here is Poincare's example:



Poincare showed that if the 2-dimensional beings do not recognize the expansion of their measuring rods, they conclude that the geometry of their 2-dimensional world is an infinite, Bolyai-Lobachevsky plane of constant negative curvature.

$$\begin{pmatrix} \text{Bolyai-} \\ \text{Lobachevsky} \\ \text{geometry} \end{pmatrix} + \left(T = \text{const} \right) = \begin{pmatrix} \text{empirical} \\ \text{facts} \end{pmatrix}$$

$$\begin{pmatrix} \text{Euclidean} \\ \text{geometry} \end{pmatrix} + \left(T(r) = T_0 \frac{(R^2 - r^2)}{R^2} \right) = \begin{pmatrix} \text{empirical} \\ \text{facts} \end{pmatrix}$$

Alice Could you give us a more realistic example, please?

Philosopher Sure! Consider the case of the Lorentz theory and special relativity:

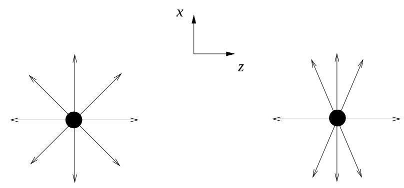
$$\begin{array}{lcl} (\text{Newton's spacetime}) + (\text{Lorentz theory}) & = & (\text{empirical facts}) \\ (\text{Minkowski's spacetime}) + (\text{relativistic physics}) & = & (\text{empirical facts}) \end{array}$$

Bob What is Lorentz theory?

Philosopher Here are two references:

- Bell, J. S. (1987): How to teach special relativity?, In *Speakable and unspeakable in quantum mechanics*, Cambridge University Press, Cambridge.
- Jánossy, L. (1971): *Theory of relativity based on physical reality,* Akadémiai Kiadó, Budapest.

A brief summary is this:



Electric field of source at rest

and of source moving in z direction

$$\vdots$$

Finally, we conclude that

$$\begin{pmatrix}\text{Newton's}\\ \text{spacetime}\end{pmatrix} \quad + \quad \begin{pmatrix}\text{contraction of rods}\\ \text{slowing down of clocks}\\ \text{and other deformations}\\ \text{of moving objects}\end{pmatrix} \quad = \quad (\text{empirical facts})$$

$$\begin{pmatrix}\text{Minkowski's}\\ \text{spacetime}\end{pmatrix} \quad + \quad (\text{relativistic physics}) \quad = \quad (\text{empirical facts})$$

Dialog 3

Alice We've both solved the exercise *What is the electric field of a point charge Q moving with constant velocity* $\vec{v}$ *?*, and have got the same result. How is it possible, then, that your solution is only one page while my calculation is more than 12 pages?

Bob I don't know, Alice. I found a very simple solution in the Landau-Lifshitz Vol. 2: Consider the Coulomb field of a charged point particle at rest. It follows from the covariance principle that the electric field of the moving source in the co-moving reference frame is also the Coulomb field. Finally, you can perform a Lorentz transformation from the co-moving frame back to the reference frame at rest.

Alice My God! I followed Feynman's Volume 6. He derives the same result directly from the Maxwell equations: First we solve the Maxwell equations for arbitrary time-depending sources. Then, from the retarded potentials such obtained, we derive the Lienart-Wiechert potentials, from which we can determine the electric field. This way is much more complicated, indeed!

Dialog 4

Bob It is not entirely clear to me from your lectures, whether the Lorentz contraction and the time dilatation are real physical processes ... or they are just obtained somehow from the comparison of quantities defined in different reference frames. I tried to find a definite answer to this question in many different books, ...

Physicist but you read many different, contradictory claims! I am sorry for being not clear enough in my lessons.

Now, we can ask the following questions:

- What kind of physical processes are going on when we set in motion a charged particle?
- What kind of physical processes are going on when we set in motion a rod, as a real physical object?
- What kind of physical processes are going on when we set in motion a clock, as a real physical object?

Relativistic physics provides crystal clear answers to these questions:

- The electric field of the particle changes.
- The rod becomes shorter.
- The clock slows down.

These are real physical deformations! For if Lorentz covariance is true, then the length of the moving rod in the co-moving frame is equal to the length of the rod at rest in the original reference frame. Since the two reference frames are related through the Lorentz transformation, the length of the moving rod in the original frame cannot be equal to the length of the rod at rest in the same original frame!

So, it is true in any reference frame, that the length of the rod after we set it in motion is different from its length before we set it in motion. This is nothing but a real, frame-independent deformation of the rod! We cannot argue that there is no real deformation only because there exists some other reference frame for which we have

$$\left[\begin{array}{c}\text{length of the}\\ \text{deformed rod}\end{array}\right]_{\text{in another frame}} = \left[\begin{array}{c}\text{length of the}\\ \text{original rod}\end{array}\right]_{\text{in this frame}}$$

Dialog 5

Alice ... I don't really understand Poincare's conventionalism in the case of relativity and the Lorentz theory.

$$\begin{pmatrix}\text{Newton's}\\ \text{spacetime}\end{pmatrix} + \begin{pmatrix}\text{contraction of rods}\\ \text{slowing down of clocks}\\ \text{and other deformations}\\ \text{of moving objects}\end{pmatrix} = (\text{empirical facts})$$

$$\begin{pmatrix}\text{Minkowski's}\\ \text{spacetime}\end{pmatrix} + (\text{relativistic physics}) = (\text{empirical facts})$$

All the deformations described by the Lorentz theory also exist in relativistic physics. ... What is then the difference between Lorentz's theory and Einstein's relativity?

Bob Sure! We could put it this way:

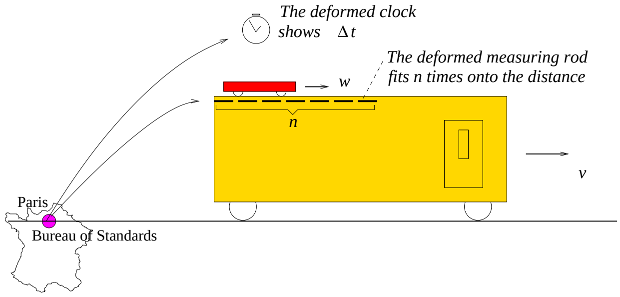
$$\begin{pmatrix}\text{Newton's}\\ \text{spacetime}\end{pmatrix} + \begin{pmatrix}\text{contraction of rods}\\ \text{slowing down of clocks}\\ \text{and other deformations}\\ \text{of moving objects}\end{pmatrix} = (\text{empirical facts})$$

$$\begin{pmatrix}\text{Minkowski's}\\ \text{spacetime}\end{pmatrix} + \begin{pmatrix}\text{contraction of rods}\\ \text{slowing down of clocks}\\ \text{and other deformations}\\ \text{of moving objects}\end{pmatrix} = (\text{empirical facts})$$

What's then the difference between the two theories? We must ask this question tomorrow!

Dialog 6

Philosopher To be sure, the two theories are identical in the sense that they describe the same observable physical phenomena. But, they are different THEORIES! For they account for the geometry of spacetime differently. That is, the same phenomena are described by them in two different ways. For example, velocity is an additive quantity in the Lorentz theory, but this is not true in relativity theory:



According to Lorentz's theory:

$$w_L = \frac{nl_0\sqrt{1-v^2/c^2}}{\frac{\Delta t}{\sqrt{1-v^2/c^2}}}$$

$$V = w_L + v$$

According to relativity:

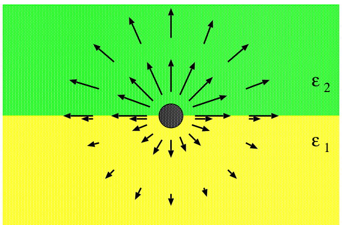
$$w_E = \frac{nl_0}{\Delta t}$$

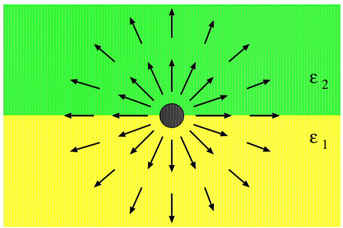
$$V \neq w_E + v$$

Dialog 7

Bob Did you solve *what the electric field of a charged particle is on the border of two dielectric materials?*

Alice Yes, I did. Let's compare the results!





Bob That's strange. They are completely different. One of them is spherically symmetric, the other is not.

Physicist Notice that you and Alice used different physical quantities for the characterization of the electrostatic field. Alice used the $\vec{E}$-field, while your solution is about the $\vec{D}$-field.

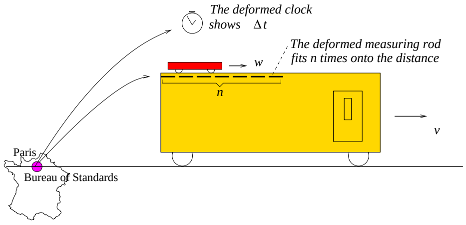
Physicists describe the world in the language of precisely defined physical quantities. If you have any confusion with respect to the precise meaning of a term, you must clarify the experimental/operational definition.

Alice May I ask a question: Is that also true for "space" and "time"?

Physicist Of course! To the physicist, space and time coordinates, just like any other quantities are defined experimentally.

Dialog 8

Alice If space and time coordinates are notions which are defined in experimental/operational terms, then the words "space" and "time" coordinates in the velocity addition example have different meaning for the Lorentz-type physicist and the Einstein-type physicist. Consequently, "the velocity of the carriage relative to the train" does not mean the same physical quantity for them. They both agree that the *etalon* measuring rod as well as the *etalon* clock undergo a deformation when we transport them to the train. The Lorentz-type physicist takes into account this deformation in the operational definition of space and time coordinates, the Einstein-type physicist does not.



According to Lorentz's theory:

$$w_L = \frac{nl_0\sqrt{1-v^2/c^2}}{\frac{\Delta t}{\sqrt{1-v^2/c^2}}}$$

$$V = w_L + v$$

According to relativity:

$$w_E = \frac{nl_0}{\Delta t}$$

$$V \neq w_E + v$$

Bob That's right! w_L and w_E are different quantities defined by different experimental operations. On the margin, one of them is additive, the other is not. Unfortunately, both are called "relative velocity". But this is only a linguistic problem! On the other hand, all statements about w_L and w_E are the same in both theories!

Alice Taking into account the precise experimental meaning of the terms, the Lorentz theory (pre-relativistic physics) and special relativity are identical theories. We can introduce two pairs of physical quantities in a reference frame moving relative to the *etalons*:

x	:	the deformed rod fits n times onto the distance	$\Rightarrow$	$x = n\sqrt{1-\frac{v^2}{c^2}}$
t	:	the slowed down clock shows n second	$\Rightarrow$	$t = \frac{n}{\sqrt{1-\frac{v^2}{c^2}}}$

And,

x'	:	the deformed rod fits n times onto the distance	$\Rightarrow$	$x = n$
t'	:	the slowed down clock shows n second	$\Rightarrow$	$t = n$

Bob And this resolves your problem with Poincare's conventionalism, too. Because the two theories describe the same geometry of spacetime:

$$(\text{geometry of spacetime})_{classical} = (\text{geometry of spacetime})_{Minkowski}$$

expressed in different physical variables.

Alice That' right! Neither "physics" nor "spacetime geometry" are different in pre-relativistic and relativistic theories. The only difference is of linguistic nature: they use the words "space" and "time" differently.

Epilogue

Alice I've found something very interesting in the library:

> Bridgman, P. (1927): *The Logic of Modern Physics*, MacMillan, New York.

His claim about relativity theory was completely the same as our conclusions.

Bob In 1927! My God!