# Contextuality without contextuality: Fine's Interpretation of Quantum Mechanics*

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**Abstract**

The aim of this paper is to provide an introduction to Arthur Fine's local hidden variable interpretation of quantum mechanics and to show how it can resolve the non-locality problem articulated in the EPR-Bell theorem.

## The problem of non-locality in quantum mechanics

A typical example for the violation of locality is the EPR experiment (Fig. 1): We consider the four 'spin-up' events in the spin-component measurements in directions $\mathbf{a}, \mathbf{a}'$ and $\mathbf{b}, \mathbf{b}'$. There are random switches (independent agents, if you want) choosing between the different possible measurements on both sides. Let $p(a), p(a'), p(b)$ and $p(b')$ be *arbitrary* probabilities with which the different measurements are chosen. We observe the following events in the experiment:

$A_+, A'_+$: *the spin of the left particle is up in direction* $\vec{a}, \vec{a}'$ detector fires

$B_+, B'_+$: *the spin of the right particle is up in direction* $\vec{b}, \vec{b}'$ detector fires

$a, a'$: the *left* switch chooses the direction $\vec{a}, \vec{a}'$

$b, b'$: the *right* switch chooses the direction $\vec{b}, \vec{b}'$

Let $\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}'$ be coplanar vectors with $\angle(\mathbf{a}, \mathbf{a}') = \angle(\mathbf{a}', \mathbf{b}') = \angle(\mathbf{a}, \mathbf{b}') = 120°$, and $\angle(\mathbf{a}', \mathbf{b}) = 0$. We observe the following relative frequencies in the experiment:

$$
\begin{array}{lcllcl}
p(A_+) & = & \frac{1}{2}p(a) & p(B_+) & = & \frac{1}{2}p(b) \\
p(A'_+) & = & \frac{1}{2}p(a') & p(B'_+) & = & \frac{1}{2}p(b') \\
p(A_+B_+) & = & \frac{3}{8}p(a)p(b) & p(A'_+B_+) & = & 0 \\
p(A_+B'_+) & = & \frac{3}{8}p(a)p(b') & p(A'_+B'_+) & = & \frac{3}{8}p(a')p(b')
\end{array}
\tag{1}
$$

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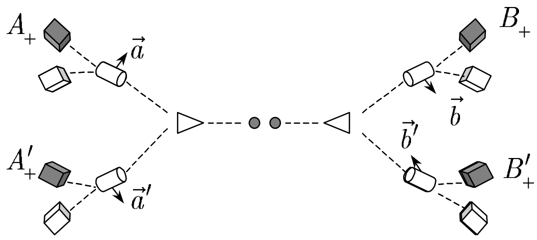


Figure 1: *The EPR experiment with spin-$\frac{1}{2}$ particles*

Now, the original EPR–Bell problem, in my view, consists in the following simple question: Can the EPR experiment be accommodated in a *relativistic* and *deterministic* universe? Figure 2 shows the space-time diagram of a single run of the EPR experiment. In a relativistic and deterministic universe, the Cauchy data along a hypersurface $S$ predetermine everything going on in the future domain of dependence $D^+(S)$. In particular, all future EPR events in the domain of dependence $D^+(S)$ are predetermined by the (partly "hidden") Cauchy data $\{\mu, \lambda, \nu\}$ defined on the three spatially separated regions of hypersurface $S$. To express this determination we assign a function to each event $X$ in $D^+(S)$:

$$u_X(\mu, \lambda, \nu) = \begin{cases} 1 & \text{if } X \text{ is the case} \\ 0 & \text{otherwise} \end{cases}$$

Because of the spatial separation, the occurrence of event $A$ is independent of the value of parameter $\nu$. Taking into account the similar separations, the following conditions must be satisfied:

$$\begin{array}{rclrcl} u_A(\mu, \lambda, \nu) & = & u_A(\mu, \lambda) & u_B(\mu, \lambda, \nu) & = & u_B(\nu, \lambda) \\ u_{A'}(\mu, \lambda, \nu) & = & u_{A'}(\mu, \lambda) & u_{B'}(\mu, \lambda, \nu) & = & u_{B'}(\nu, \lambda) \\ u_a(\mu, \lambda, \nu) & = & u_a(\mu) & u_b(\mu, \lambda, \nu) & = & u_b(\nu) \\ u_{a'}(\mu, \lambda, \nu) & = & u_{a'}(\mu) & u_{b'}(\mu, \lambda, \nu) & = & u_{b'}(\nu) \end{array} \tag{2}$$

## The probabilistic description

Quantum mechanics provides a probabilistic description of the EPR experiment. The statistical ensemble consists of similar space-time patterns corresponding to subsequent repetitions of the experiment (Fig. 3). All probabilities are understood as relative frequencies measured on this ensemble. We also assume that

$$p(\mu \wedge \lambda \wedge \nu) = p(\mu)\, p(\lambda)\, p(\nu) \tag{3}$$

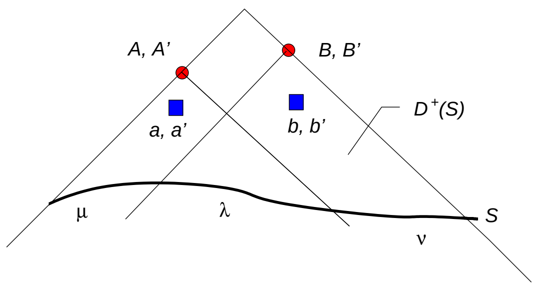


Figure 2: *The space-time diagram of the EPR scenario. All events in the future domain of dependence $D^+(S)$ are predetermined by the Cauchy data along the hypersurface $S$*

This is typically the case if $\mu$ and $\nu$ are nothing but the independent measurement choices in the two wings, or, equivalently, some data on the hypersurface $S$ predetermining the selections between $a$, $a'$, and $b, b'$, respectively.

Now,

$$\begin{aligned}
p(A) &= \sum_{\mu,\lambda} u_A(\mu,\lambda)\, p(\mu)\, p(\lambda) \\
p(B) &= \sum_{\lambda,\nu} u_B(\lambda,\nu)\, p(\lambda)\, p(\nu) \\
p(a) &= \sum_{\mu} u_a(\mu)\, p(\mu) \\
p(b) &= \sum_{\nu} u_b(\nu)\, p(\nu) \\
p(A\wedge B) &= \sum_{\mu,\lambda,\nu} u_A(\mu,\lambda)\, u_B(\lambda,\nu)\, p(\mu)\, p(\lambda) p(\nu) \\
p(a\wedge b) &= \sum_{\mu,\nu} u_a(\mu)\, u_b(\nu)\, p(\mu)\, p(\nu) = p(a)\, p(b)
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
p(A\wedge B|a\wedge b\wedge\lambda) &= \frac{\sum_{\mu,\nu} u_A(\mu,\lambda)\, u_B(\lambda,\nu)\, p(\mu)\, p(\nu)}{\sum_{\mu,\nu} u_a(\mu)\, u_b(\nu)\, p(\mu)\, p(\nu)} \\
&= \frac{\sum_{\mu} u_A(\mu,\lambda)\, p(\mu) \sum_{\nu} u_B(\lambda,\nu)\, p(\nu)}{\sum_{\mu} u_a(\mu)\, p(\mu) \sum_{\nu} u_b(\nu)\, p(\nu)} \\
&= p(A|a\wedge\lambda)\, p(B|b\wedge\lambda)
\end{aligned} \tag{5}$$

So, $\lambda$ is a stochastic hidden parameter satisfying the *screening off* condition, from which, together with (4), the well known *Clauser-Horne inequalities* follows immediately:

$$-1 \leq \underbrace{\left\{ \begin{array}{c} p\,(A \wedge B|a \wedge b) + p\,(A \wedge B'|a \wedge b') - p\,(A' \wedge B|a' \wedge b) \\ +p\,(A' \wedge B'|a' \wedge b') - p\,(A'|a') - p\,(B|b) \end{array} \right\}}_{CH} \leq 0 \qquad (6)$$

However, the Clauser–Horne inequalities are violated by the probabilities (1). ($CH = \frac{1}{8}$.) Consequently, as the standard conclusion says, the EPR experiment cannot be accommodated in a relativistic and deterministic universe.

## Loophole in the real EPR experiments

There is, however, a serious loophole in the real experiments: Compare the original apparatus configuration used for Bell's 1971 proof with the one used in the real Aspect experiment (Fig. 5 and 6). The original configuration contains two 'event-ready' detectors, which signal both arms that a pair of particles has been emitted. So, the statistics is assumed to be taken on the ensemble of particle pairs emitted by the source. It is obviously impossible, however, to realize an 'event-ready' detection and to perform the measurement on the unselected ensemble. In the real experiments instead of the 'event-ready' detectors, a four-coincidence circuit detects the 'emitted particle-pairs'. This method yields a *selected* statistical ensemble: only those pairs are taken into account, which coincidentally fire one of the left and one of the right detectors. Denote $[A]$ the event that there is a detection in the left wing with analyzer set-up $a$, that is, either the "up" detector or the "down" detector fires. Similarly, $[A] \wedge [B]$ denotes the corresponding double detection. So, what we actually observe is the violation of the following inequality:

$$-1 \leq \left\{ \begin{array}{c} p\,(A \wedge B|a \wedge b \wedge [A] \wedge [B]) + p\,(A \wedge B'|a \wedge b' \wedge [A] \wedge [B']) \\ -p\,(A' \wedge B|a' \wedge b \wedge [A'] \wedge [B]) + p\,(A' \wedge B'|a' \wedge b' \wedge [A'] \wedge [B']) \\ -p\,(A'|a' \wedge [A']) - p\,(B|b \wedge [B]) \end{array} \right\} \leq 0 \qquad (7)$$

If the selection procedure were *completely random* then the observed relative frequencies on the selected ensemble would be equal to the ones taken on the original ensemble, that is,

$$\begin{array}{rcl} p\,(A \wedge B|a \wedge b \wedge [A] \wedge [B]) & = & p\,(A \wedge B|a \wedge b) \\ p\,(A \wedge B'|a \wedge b' \wedge [A] \wedge [B']) & = & p\,(A \wedge B'|a \wedge b') \\ & \text{etc.} & \end{array}$$

(*enhancement hypothesis*) and the violation of inequality (7) would imply the violation of (6).

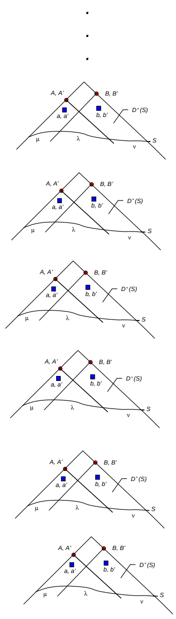

Figure 3: *The statistical ensemble consists of similar space-time patterns corresponding to the subsequent repetitions of the experiment. The values of $\mu, \lambda$ and $\nu$ predetermine all events in the corresponding run of the experiment*

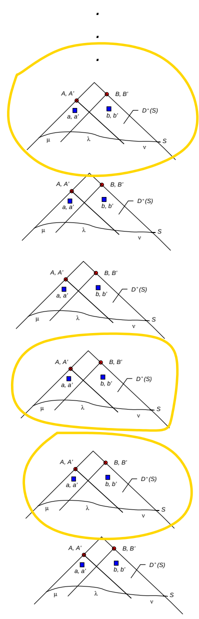

Figure 4: *In a real EPR experiment the statistics is taken on a selected sub-ensemble: only those elements of the original ensemble are taken into account which produce double detections in the two wings*

However, this is not necessarily the case. The Cauchy data $\{\mu, \lambda, \nu\}$ predetermine the whole future behavior of the particle pair (in $D^{+}(S)$), in particular, they predetermine whether the particles can or cannot go through the analyzers. And it is quite plausible to assume that the shared part of these data $\lambda$ influences both particles' behavior in the analyzers. If this is the case, then the actually observed ensemble *is not randomly selected.* Thus, the widespread conclusion that the observed violation of Bell-type inequalities implies incompatibility between the EPR experiments and relativistic determinism is mistaken.

It is to be emphasized that this claim is based on the logical schema of the experiment, independently of the detectors' inefficiency problem; in the above consideration the detector efficiency was taken 100%. Distinction must be made between the low detection/emission efficiency caused by the random errors in the *analyzer + detector* equipment and the low efficiency that is a *systematic*

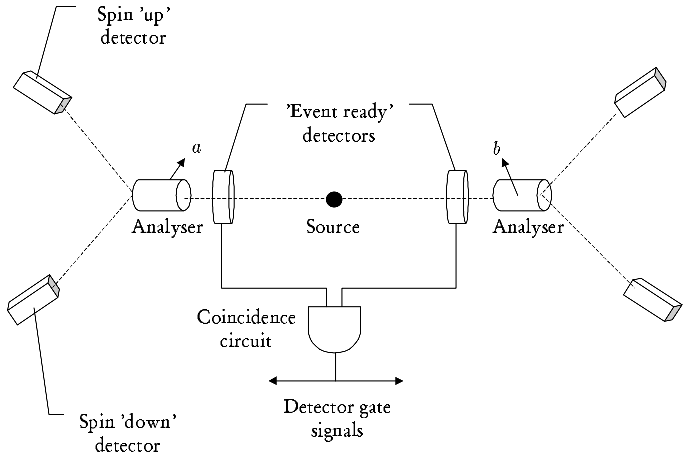


Figure 5: *Apparatus configuration used for Bell's 1971 proof. 'Event-ready' detectors signal both arms that a pair of particles has been emitted*

*manifestation of certain (hidden) properties of the emitted particles.* So, those who agree with Bell that

> ... it is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups and is yet going to fail badly when sufficient refinements are made. (Bell 1987, p. 154.)

are missing the point. We do not expect that quantum mechanics will fail badly when sufficient refinements are made, but rather that there are principal limits in achieving such refinements.

It is worthy of note another possible objection to the above explanation of the EPR experiment. One could argue that the we can ignore the concepts related with the original statistical ensemble of the emitted particles. We can restrict our considerations only to the ensemble of the detected particle pairs. On this ensemble we do observe correlations among spatially separated events and the probabilities defined on this ensemble do violate the Clauser-Horne inequalities. Therefore, one might argue, what we actually observe is incompatible with a deterministic and relativistic world.

This objection is, however, flawed. The reason is that the assumption (3) plays a key role in the derivation of the Clauser-Horne inequality. Again, if the selection went on *completely randomly* then for all $X$, $p_{selected}(X) = p(X)$, and the derivation of the Clauser–Horne inequalities would still be valid, and the EPR experiment would still contradict to relativistic determinism. The

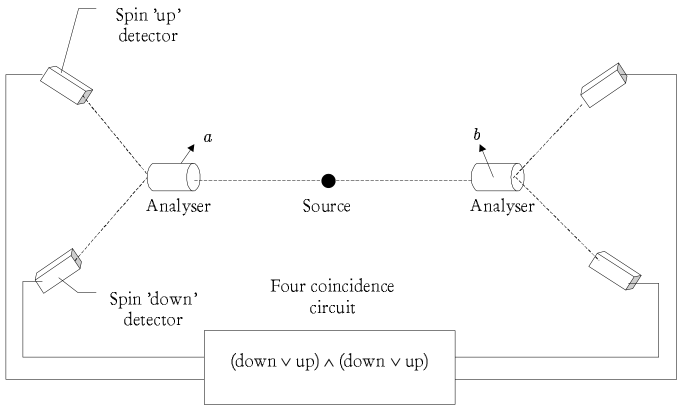


Figure 6: *In the real experiments, instead of the 'event-ready' detectors, a four-coincidence circuit detects the 'emitted particle-pairs'*

actually observed ensemble is not, however, randomly selected: it depends on the properties of an element of the ensemble whether it is selected or not. Therefore, in general, not all combinations of $\mu$, $\nu$ and $\lambda$ are selected with equal probability, that is, contrary to (3), the selective local physical process itself generates the following "contextuality":

$$p_{selected}\left(\mu \wedge \nu \wedge \lambda\right) \neq p_{selected}\left(\mu \wedge \nu\right) p_{selected}\left(\lambda\right)$$

# Fine's interpretation of quantum mechanics

A natural question is whether it is possible to realize an 'event-ready' detection and to perform the measurement on the unselected ensemble. It seems, however, no way to solve this problem: in practice all conceivable 'event-ready' detectors depolarize or destroy the particles. Moreover, this seems to be a uniform feature of all quantum measurements.

Consider a typical configuration of a quantum measurement depicted in Figure 7. We have no information about the content of the original unselected ensemble of objects. It is always assumed that the total number of objects is $N = \sum_i N_i$, where $N_i$ denotes the number how many times the $i$-th outcome occurred. The theoretical "probabilities" predicted by quantum mechanics are compared with the experimental results in the sense of $tr\left(WP_i\right) = \frac{N_i}{N}$, where $P_i$ denotes the projector belonging to the $i$-th outcome. That is, quantum mechanical "probabilities" are equal to the relative frequencies taken on a *selected* ensemble, namely, on the ensemble of objects producing an outcome (passing

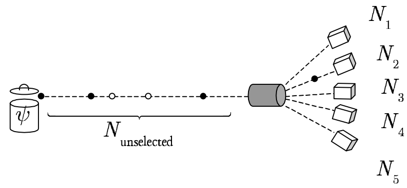


Figure 7: *We have no information about the content of the original unselected ensemble of objects. It is always assumed that each object on which the measurement is performed produces one of the possible measurement outcomes*

the analyzer). In order to understand why the measured conditional probabilities are systematically equal to quantum probabilities, consider a simple case of a measurement $a$ testing the value of a two-valued ($\{A_+, A_-\}$) observable $A$ (Fig. 8). Let $n^A$ denote the number of elements, which are predetermined (by the hidden properties) to produce an outcome at all. $n_+^A$ is the number of those elements which are predetermined to produce outcome $A_+$. Subset $a$ contains the randomly chosen $N$ elements on which the measurement is performed. Among the measured objects, $N_+^A$ is the number of those which produce outcome $A_+$. Now, because of the random choice of the measured elements, the conditional probability, for instance, of the outcome $A_+$, given that the measurement $a$ has been performed, $p\,(A_+|a)$, must be equal to the relative frequency of elements having property $A_+$ among those which are capable to produce an outcome of such a measurement, $\frac{n_+^A}{n^A}$. According to the interpretation we are developing here, this relative frequency is equal to the quantum probability:

$$p\,(A_+|a) = \frac{N_+^A}{N} = \frac{n_+^A}{n^A} = tr\left(WP_{A_+}\right)$$

Similarly, in case of two simultaneous measurements (Fig. 9) we have

$$p\,(A_+ \wedge B_+|a \wedge b) = \frac{N_{++}^{AB}}{N} = \frac{n_{++}^{AB}}{n^{AB}} = tr\left(WP_{A_+ \wedge B_+}\right)$$

The above developed interpretation of quantum mechanics is nothing else but Arthur Fine's (1982) "Prisms Model", and, in some respects, the key idea goes back to Einstein's understanding of the statistical interpretation of quantum states (see Fine (1986), p. 52.).

As an example, consider a prism model reproducing the statistics (1). Figure 10 shows a parameter space $\Lambda \ni \lambda$ consisting of disjoint blocks of measure $\frac{3}{32}$ and $\frac{1}{32}$ respectively. A point in $\Lambda$ (a value of the parameter $\lambda$) predetermines all events in question. Therefore, each EPR event can be represented as a subset

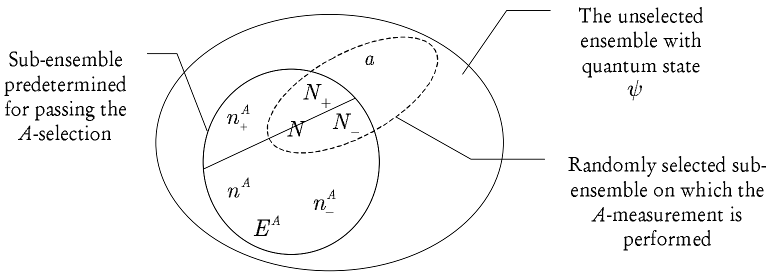


Figure 8: *Subset a consists of elements of the ensemble on which the value of a two-valued observable A has been measured. Let* $n^A$ *denote the number of elements* $E^A$*, which are predetermined to produce an outcome at all.* $n_+^A$ *is the number of those elements, which are predetermined to produce outcome* $A_+$*. Subset a contains the randomly chosen N elements on which the measurement is performed. Among the measured objects,* $N_+^A$ *is the number of those which produce outcome* $A_+$

of $\Lambda$. Assume for instance that $\lambda = \lambda_{\text{example}}$ (see Fig. 10): An $a$-measurement on the left particle produces neither event "up" nor event "down", while if an $a'$-measurement is performed then the outcome is "down". In the right wing, if we perform a $b$-measurement then the outcome is "up", and if the $b'$-measurement is performed, the outcome is "down". Consequently, if we perform, for example, an $a$-measurement on the left particle and a $b$-measurement on the right one, then there is no coincidence registered, and the particle pair in question does not appear in the statistics of the measurement. On the other hand, if we perform an $a'$-measurement on the left particle and a $b$-measurement on the right one, then a double detection coincidence is registered and the counter of the total number of events as well as the $B$-counter count.

Thus, the hidden variable governs the whole process and reproduces the probabilities measured in the experiment:

$$
\begin{aligned}
\frac{\mu(A)}{\mu([A])} &= \frac{\mu(A')}{\mu([A'])} = \frac{\mu(B)}{\mu([B])} = \frac{\mu(B')}{\mu([B'])} = \frac{\frac{12}{32}}{\frac{24}{32}} = \frac{1}{2} \\
\frac{\mu(A \cap B)}{\mu([A] \cap [B])} &= \frac{\mu(A \cap B')}{\mu([A] \cap [B'])} = \frac{\mu(A' \cap B')}{\mu([A'] \cap [B'])} = \frac{\frac{6}{32}}{\frac{16}{32}} = \frac{3}{8} \\
\frac{\mu(A' \cap B)}{\mu([A'] \cap [B])} &= \frac{0}{\frac{16}{32}} = 0
\end{aligned}
$$

In Fine's above prism model for the $2 \times 2$ spin-correlation experiment the double detection/emission efficiency is 50%. A natural question is what the similar rate is in the actual experiments. In one of the best experiments of the

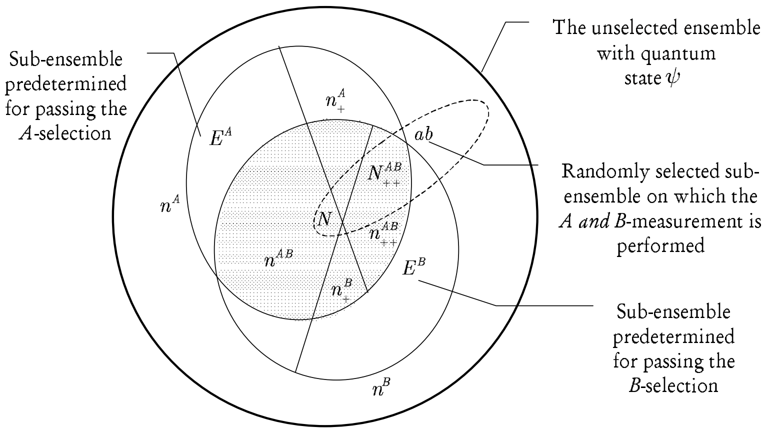


Figure 9: *Subset ab consists of elements of the ensemble on which the value of the two-valued observables $A$ and $B$ have been measured. Let $n^A$ denote the number of elements in $E^A$, which are predetermined to produce an outcome at all. $n^A_+$ is the number of those elements, which are predetermined to produce outcome $A_+$. We use the same notation for observable $B$. Subset ab contains the randomly chosen $N$ elements on which the two simultaneous measurements are performed. Among the measured objects, $N^{AB}_{++}$ is the number of those which produce outcome $A_+$ and $B_+$*

last years (Weihs *et al.* 1998) the similar rate is about 0.25%. That is, Fine's model is in complete accordance with the known experimental results. There appeared, however, a theoretical demand to embed the $2 \times 2$ prism models into a large $n \times n$ prism model reproducing all potential $2 \times 2$ experiments. This demand was motivated by the idea that the real physical process does not know which directions are chosen in an experiment. On the other hand, it seemed that in the known prism models of the $n \times n$ spin-correlation experiment the efficiencies tended to zero, if $n \to \infty$, which contradicts what we expect of actual experiments (Sharp and Shank 1985; Fine 1991; Maudlin 1994). This problem was recently solved in (Szabó 2000), which shows that there is a wide class of physically plausible prism models for the $n \times n$ spin-correlation experiment, where the efficiencies do not tend to zero if $n \to \infty$. This new result has completed Fine's resolution of the contradiction between the violation of Bell-type inequalities and the assumption that the EPR experiment can be accommodated in a relativistic and deterministic universe.

A similar local hidden variable model was recently constructed in (Szabó and Fine 2000) for the Greenberger-Horne-Zeilinger experiment. It was also shown that the model is completely compatible with the most recent experimental results (cf. Bouwmeester *et al.* 1999).

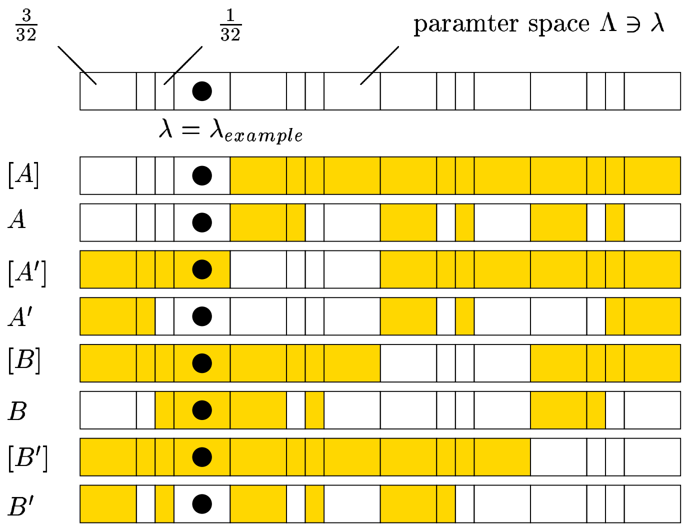


Figure 10: *Parameter $\lambda \in \Lambda$ completely predetermines all future events for all possible combinations of the freely chosen measurements. Each EPR event is represented as a (shaded) subset of $\Lambda$.*

To sum up, Fine's approach seems to be the only tenable interpretation of quantum mechanics. It is free from the difficulties and contradictions, which the other rival interpretations suffer from: a) As a statistical interpretation, it is devoid of the measurement paradox and the likes. b) It is obviously out of the scope of the Kochen-Specker theorem (the Fine interpretation escapes such preliminary assumptions of the Kochen-Specker theorem as "The Spectrum Rule" and the likes, (cf. Redhead 1989), although it is a realistic interpretation admitting that the individual quantum systems have pre-settled intrinsic properties, prior to, independent of and revealed by the measurements. c) All probabilities can be interpreted as relative frequencies in a well-defined ordinary statistical ensemble. The "quantum probabilities", too, obtain a meaningful explanation inside of the classical Kolmogorovian theory of probability. d) Fine's interpretation admits, without contradiction, a local deterministic hidden variable theory.

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