Does special relativity theory tell us anything new about space and time?

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Abstract

It will be shown that, in comparison with the pre-relativistic Galileo-invariant conceptions, special relativity tells us nothing new about the geometry of space-time. It simply calls something else "space-time", and this something else has different properties. All statements of special relativity about those features of reality that correspond to the original meaning of the terms "space" and "time" are identical with the corresponding traditional pre-relativistic statements. It will be also argued that special relativity and Lorentz theory are completely identical in both senses, as theories about space-time and as theories about the behaviour of moving physical objects.

Abstract

Key words: Lorentz theory, special relativity, space, time, relativity principle, Lorentz's principle, operationalism, conventionalism
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Prolog

Consider the following definitions of electrodynamical quantities:

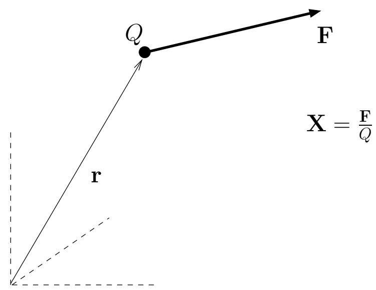

Figure 1: **X** is defined as the force felt by the unit test charge

$\mathbf{X}(\mathbf{r})$ Locate a test charge Q at point $\mathbf{r}$ and measure the force $\mathbf{F}$ felt by the charge. $\mathbf{X}(\mathbf{r}) = \frac{\mathbf{F}}{Q}$ (Fig 1).

$\mathbf{Y}(\mathbf{r})$ Locate two contacting metal plates of area A at point $\mathbf{r}$. Separate them and measure the influence charge Q on one of the plates. $Y(\mathbf{r}) = \frac{Q}{A}$. The direction of $\mathbf{Y}(\mathbf{r})$ is determined by the normal vector of the plates, when the charge separation is maximal (Fig 2).

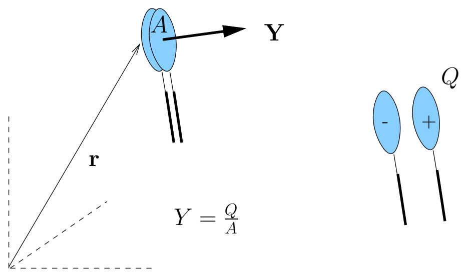

Figure 2: **Y** is defined by means of the influence charge divided by the surface

It is a well known empirical fact that these quantities are not independent of each other. For the sake of simplicity, assume the simplest material equation

$$\mathbf{Y} = \varepsilon \mathbf{X} \tag{1}$$

where ε, called dielectric constant, is a scalar field characterising the medium.

Traditionally, in phenomenological electrodynamics, physical quantity $\mathbf{X}$ is called 'electric field strength' and denoted by $\mathbf{E}$, and $\mathbf{Y}$ is called 'electric displacement' and denoted by $\mathbf{D}$. Due to the material equation (1) one can eliminate one of the field variables.

Imagine a text book (I shall refer to it as the "old" one), which only uses $\mathbf{E}$. The equations of electrostatics are written as follows:

$$\text{div}\ \varepsilon \mathbf{E} = \rho \tag{2}$$

$$\text{rot}\ \mathbf{E} = 0 \tag{3}$$

For example, the book contains the following exercise and solution:

> **Exercise** Consider the static electric field around a point charge q located at the border of two materials of dielectric constant ε_1 and ε_2. Is the electric field strength spherically symmetric, or not?
>
> **Solution** (see Fig 3)
>
> $$\mathbf{E}_1 = \frac{1}{2\pi(\varepsilon_1 + \varepsilon_2)} \frac{q}{r^3} \mathbf{r} \tag{4}$$
>
> $$\mathbf{E}_2 = \frac{1}{2\pi(\varepsilon_1 + \varepsilon_2)} \frac{q}{r^3} \mathbf{r} \tag{5}$$
>
> Consequently,
>
> The electric field strength is spherically symmetric. (6)

Now, imagine a new electrodynamics text book which is non-traditional in the following sense: it uses only field variable $\mathbf{Y}$ (traditionally called 'electric displacement' and denoted by $\mathbf{D}$), but it systematically calls $\mathbf{Y}$ 'electric field strength' and denotes it by $\mathbf{E}$. Accordingly, the equations of electrostatics are written as follows:

$$\text{div } \mathbf{E} = \rho \tag{7}$$

$$\text{rot } \frac{\mathbf{E}}{\varepsilon} = 0 \tag{8}$$

This new book also contains the above exercise, but with the following solution:

> **Solution** (see Fig 4)
>
> $$\mathbf{E}_1 = \frac{\varepsilon_1}{2\pi(\varepsilon_1 + \varepsilon_2)} \frac{q}{r^3} \mathbf{r} \tag{9}$$
>
> $$\mathbf{E}_2 = \frac{\varepsilon_2}{2\pi(\varepsilon_1 + \varepsilon_2)} \frac{q}{r^3} \mathbf{r} \tag{10}$$
>
> Consequently,
>
> The electric field strength is not spherically symmetric. (11)

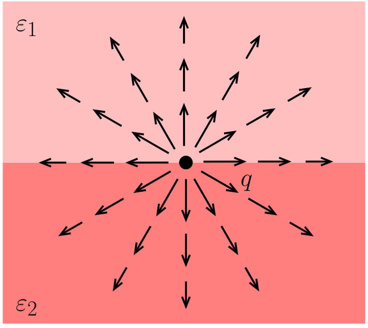

Figure 3: The 'electric field strength' of the static electric field around a point charge q located at the border of two materials of dielectric constants ε_1 and ε_2

Now, does sentence (11) of the new book contradict to sentence (6) of the old book? Is it true that the theory described in the new book is a *new theory* of electromagnetism? Of course, not. Seemingly the two sentences contradict to each other, on the level of the words. However, in order to clarify the meaning of sentence (11) and (6), one has to go back to the first pages of the corresponding book and clarify the definition of the physical quantity called 'electric field strength'. And it will be clear that the term 'electric field strength' stands for two different physical quantities in the two books. Moreover, both text books provide complete descriptions of electromagnetic phenomena. Therefore, although the theory in the old book does not use the field variable $\mathbf{Y}$, it is capable to account for the physical phenomena by which physical quantity $\mathbf{Y}$ is empirically defined. It is capable to determine the influence charge on the separated plates (by calculating εEA). In other words, it is capable to determine the value of $\mathbf{Y}$, that is, the value of what the new book calls 'electric field strength'. And vice versa, on the basis of the theory described in the new book one can calculate the force felt by a unit test charge (by calculating $\frac{\mathbf{E}}{\varepsilon}$), that is, one can predict the value of $\mathbf{X}$, what the old book calls 'electric field strength'. And both, the theory in the old book and the theory in the new book have the same predictions for both, $\mathbf{X}$ and $\mathbf{Y}$. That is to say, although they use different terminology, the two text books contain the same electrodynamics, they provide the same description of physical reality.

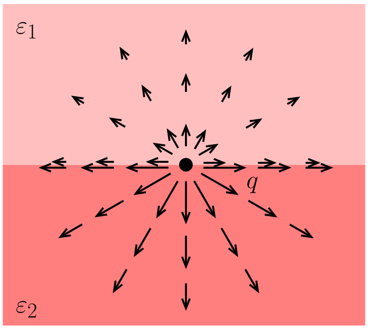

Figure 4: The 'electric field strength' of the static electric field around a point charge q located at the border of two materials of dielectric constants ε_1 and ε_2

1 Introduction

I have for long thought that if I had the opportunity to teach this subject, I would emphasize the continuity with earlier ideas. Usually it is the discontinuity which is stressed, the radical break with more primitive notions of space and time. Often the result is to destroy completely the confidence of the student in perfectly sound and useful concepts already acquired. (From J. S. Bell: "How to teach special relativity", Bell 1987, p. 67.)

It is widely believed that the principal difference between Einstein's special relativity and its contemporary rival Lorentz theory was that while the Lorentz theory was also capable of "explaining away" the null result of the Michelson–Morley experiment and other experimental findings by means of the distortions of moving measuring-rods and moving clocks, special relativity revealed more fundamental new facts about the geometry of space-time behind these phenomena. (I use the term "Lorentz theory" as classification to refer to the similar approaches of Lorentz, FitzGerald, and Poincaré, that save the classical Galilei covariant conceptions of space and time by explaining the null result of the Michelson–Morley experiment and other similar experimental findings through the physical distortions of moving objects, first of all of moving measuring-rods and clocks, no matter whether these physical distortions are simply hypothesised in the theory, or prescribed by some "principle" like Lorentz's principle, or they are constructively derived from the behaviour of the molecular forces. From the point of view of my recent concerns what is important is the logical possibility of such an alternative theory. Although, Lorentz's 1904 paper is very close to be a good historic example.) According to this widespread view, special relativity theory has radically changed our conceptions about space and time by claiming that space-time is not like an $\mathbb{E}^3 \times \mathbb{E}^1$ space, as was believed in classical physics, but it is a four dimensional Minkowski space $\mathbb{M}^4$. One can express this revolutionary change by the following logical schema: Earlier we believed in $G_1(M)$, where M stands for space-time and G_1 denotes some predicate (like $\mathbb{E}^3 \times \mathbb{E}^1$). Then we discovered that $\neg G_1(M)$ but $G_2(M)$, where G_2 denotes a predicate different from G_1 (something like $\mathbb{M}^4$).

Contrary to this common view, the first main thesis in this paper is the following:

Thesis 1. *In comparison with the pre-relativistic Galileo-invariant conceptions, special relativity tells us nothing new about the geometry of space-time. It simply calls something else "space-time", and this something else has different properties. All statements of special relativity about those features of reality that correspond to the original meaning of the terms "space" and "time" are identical with the corresponding traditional pre-relativistic statements.*

Thus the only new factor in the special relativistic account of space-time is the decision to designate something else "space-time". In other words: Earlier we believed in $G_1(M)$. Then we discovered for some $\widetilde{M} \neq M$ that $\neg G_1\left(\widetilde{M}\right)$ but $G_2\left(\widetilde{M}\right)$. Consequently, it still holds that $G_1(M)$.

So the real novelty in special relativity is some $G_2\left(\widetilde{M}\right)$. As we will see, this is nothing but the description of the physical behaviour of moving measuring-rods and clocks. It will be also argued, however, that $G_2\left(\widetilde{M}\right)$ does not contradict to what Lorentz theory claims. More exactly, as my second main thesis asserts, both theories claim that $G_1(M)\,\&G_2\left(\widetilde{M}\right)$:

Thesis 2. *Special relativity and Lorentz theory are completely identical in both senses, as theories about space-time and as theories about the behaviour of moving physical objects.*

2 On the meaning of the question "What is space-time like?"

A theory *about* space-time describes a certain group of objective features of physical reality, which we call (the structure of) space-time. According to classical physics, the geometry of space-time $\mathbb{E}^3 \times \mathbb{E}^1$, where $\mathbb{E}^3$ is a three-dimensional Euclidean space for space, and $\mathbb{E}^1$ is a one-dimensional Euclidean space for time, with two independent invariant metrics corresponding to the space and time intervals. In contrast, special relativity claims that the geometry of space-time—understood as the same objective features of physical reality—is different: it is a Minkowski geometry.

Physics describes objective features of reality by means of physical quantities. Our scrutiny will therefore start by clarifying how classical physics and relativity theory define the space and time tags assigned to an arbitrary event. It will be seen that these empirical definitions are different.

The empirical definition of a physical quantity requires an *etalon* measuring equipment and a precise description of the operation how the quantity to be defined is measured. For example, assume we choose, as the *etalon* measuring-rod, the meter stick that is lying in the International Bureau of Weights and Measures (BIPM) in Paris. Also assume—this is another convention—that "time" is defined as a physical quantity measured by the standard clock also sitting in the BIPM. When I use the word "convention" here, I mean the semantical freedom we have in the use of the uncommitted signs "distance" and "time"—a freedom what Grünbaum (1974, p. 27) calls "trivial semantical conventionalism".

Now we are going to describe the empirical definitions of the space and time tags[1] of an arbitrary event A, relative to the reference frame K in which the the *etalons* are at rest, and to another reference fame K' which is moving (at constant velocity v) relative to K. For the sake of simplicity consider only one space dimension and assume that the origin of both K and K' is at the BIPM at the initial moment of time.

[1] I call them "space and time tags" rather than "space and time coordinates". By this terminology I would like to distinguish a particural kind of space and time coordinates (tags) which are provided with direct empirical meaning from space and time coordinates in general, the empirical meaning of which can be deduced from the empirical meaning of the space and time tags. (Once we have space and time tags defined, we can introduce arbitrary other coordinates in the space/manifold of space-time tags. The physical/empirical meaning of a point of the manifold is however derived from the empirical meaning of the space-time tags. In this way we can confirm or falsify, empirically, a spatio-temporal physical statement.)

(D1) Time tag in K according to classical physics

Take a synchronised copy of the standard clock at rest in the BIPM, and slowly move it to the locus of event A. The time tag $\hat{t}^K(A)$ is the reading of the transfered clock when A occurs. ("Slowly" means that we move the clock from one place to the other over a long period of time, according to the reading of the clock itself. The reason is to avoid the loss of phase accumulated by the clock during its journey. With this definition we actually use the standard "$\varepsilon = \frac{1}{2}$-synchronisation". I do not want to enter now into the question of the conventionality of simultaneity, which is a hotly debated problem, in itself. See Reichenbach 1956; Grünbaum 1974; Salmon 1977; Malament 1977; Friedman 1983.)

(D2) Space tag in K according to classical physics

The space tag $\hat{x}^K(A)$ of event A is is the distance from the origin of K of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K. (The straight line is defined by a light beam.)

(D3) Time tag in K according to special relativity

Take a synchronised copy of the standard clock at rest in the BIPM, and slowly move it to the locus of event A. The time tag $\widetilde{t}^K(A)$ is the reading of the transfered clock when A occurs.

(D4) Space tag in K according to special relativity

The space tag $\widetilde{x}^K(A)$ of event A is the distance from the origin of K of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K.

(D5) Space and time tags of an event in K' according to classical physics

The space tag of event A relative to the frame K' is

$$\hat{x}^{K'}(A) := \hat{x}^K(A) - v\hat{t}^K(A) \tag{12}$$

where $v = \hat{v}^K(K')$ is the velocity of K' relative to K in the sense of definition (D8).
The time tag of event A relative to the frame K' is

$$\hat{t}^{K'}(A) := \hat{t}^K(A) \tag{13}$$

(D6) Time tag in K' according to special relativity

Take a synchronised copy of the standard clock at rest in the BIPM, gently accelerate it from K to K' and set it to show 0 when the origins of K and K' coincide. Then slowly (relative to K') move it to the locus of event A. The time tag $\widetilde{t}^{K'}(A)$ is the reading of the transfered clock when A occurs.

(D7) Space tag in K' according to special relativity

The space tag $\widetilde{x}^{K'}(A)$ of event A is the distance from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest.

(D8) Velocities in the different cases

Velocity is a quantity derived from the above defined space and time tags:

$$\begin{aligned} \hat{v}^K &= \frac{\Delta \hat{x}^K}{\Delta \hat{t}^K} \\ \widetilde{v}^K &= \frac{\Delta \widetilde{x}^K}{\Delta \widetilde{t}^K} \\ \hat{v}^{K'} &= \frac{\Delta \hat{x}^{K'}}{\Delta \hat{t}^{K'}} \\ \widetilde{v}^{K'} &= \frac{\Delta \widetilde{x}^{K'}}{\Delta \widetilde{t}^{K'}} \end{aligned}$$

With these empirical definitions, in every inertial frame we define four different quantities for each event, such that:

$$\hat{x}^K(A) \equiv \widetilde{x}^K(A) \qquad (14)$$
$$\hat{t}^K(A) \equiv \widetilde{t}^K(A) \qquad (15)$$
$$\hat{x}^{K'}(A) \not\equiv \widetilde{x}^{K'}(A) \qquad (16)$$
$$\hat{t}^{K'}(A) \not\equiv \widetilde{t}^{K'}(A) \qquad (17)$$

where $\equiv$ denotes the identical empirical definition.

In spite of the different empirical definitions, it could be a *contingent* fact of nature that $\hat{x}^{K'}(A) = \widetilde{x}^{K'}(A)$ and/or $\hat{t}^{K'}(A) = \widetilde{t}^{K'}(A)$ for every event A. Let me illustrate this with an example. The inertial mass m_i and gravitational mass m_g are two quantities having different experimental definitions. But, it is a contingent fact of nature (experimentally proved by Eötvös around 1900) that, for any object, the two masses are equal, $m_i = m_g$. But a little reflection reveals that this is not the case here. It follows from special relativity that $\widetilde{x}^K(A), \widetilde{t}^K(A)$ are related with $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ through the Lorentz transformation, while $\hat{x}^K(A), \hat{t}^K(A)$ are related with $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ through the corresponding Galilean transformation, therefore, taking into account identities (14)–(15), $\hat{x}^{K'}(A) \neq \widetilde{x}^{K'}(A)$ and $\hat{t}^{K'}(A) \neq \widetilde{t}^{K'}(A)$, if $v \neq 0$.

Thus, our first partial conclusion is that *different physical quantities are called "space" tag, and similarly, different physical quantities are called "time" tag in special relativity and in classical physics.* (This was first recognised by Bridgeman (1927, p. 12), although he did not investigate the further consequences of this fact.) In order to avoid further confusion, from now on $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags will mean the physical quantities defined in (D1), (D2) and (D5)—according to the usage of the terms in classical physics—, and "space" and "time" in the sense of the relativistic definitions (D3), (D4), (D6) and (D7) will be called $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$.

Special relativity theory makes *different* assertions about somethings which are *different* from $\widehat{\text{space}}$ and $\widehat{\text{time}}$. In our symbolic notation, classical physics claims $G_1\left(\hat{M}\right)$ about $\hat{M}$ and relativity theory claims $G_2\left(\widetilde{M}\right)$ about some other features of reality $\widetilde{M}$. The question is what special relativity and classical physics say when they are making assertions about the same things.

3 Special relativity does not tell us anything new about space and time

Classical physics calls "space" and "time" what we denoted by $\widehat{\text{space}}$ and $\widehat{\text{time}}$. So relativity theory would tell us something new if it accounted for physical quantities $\hat{x}$ and $\hat{t}$ differently. If there were any event A and any inertial frame of reference $K^\star$ in which the $\widehat{\text{space}}$ or $\widehat{\text{time}}$ tag assigned to the event by special relativity, $\left[\hat{x}^{K^\star}(A)\right]_{relativity}$, $\left[\hat{t}^{K^\star}(A)\right]_{relativity}$, were different from the similar tags assigned by classical physics, $\left[\hat{x}^{K^\star}(A)\right]_{classical}$, $\left[\hat{t}^{K^\star}(A)\right]_{classical}$. If, for example, there were any two events $\widehat{\text{simultaneous}}$ in relativity theory which were not $\widehat{\text{simultaneous}}$ according to classical physics, or vice versa—to touch on a sore point. But a little reflection shows that this is not the case. Taking into account empirical identities (14)–(15), one can calculate the relativity theoretic prediction for the outcomes of the measurements described in (D1), (D2) and (D5), that is, the relativity theoretic prediction for $\hat{x}^{K'}(A)$:

$$\left[\hat{x}^{K'}(A)\right]_{relativity} = \widetilde{x}^{K}(A) - \widetilde{v}^{K}(K')\widetilde{t}^{K}(A) \tag{18}$$

the value of which is equal to

$$\hat{x}^{K}(A) - \hat{v}^{K}(K')\hat{t}^{K}(A) = \left[\hat{x}^{K'}(A)\right]_{classical} \tag{19}$$

Similarly,

$$\left[\hat{t}^{K'}(A)\right]_{relativity} = \widetilde{t}^{K}(A) = \hat{t}^{K}(A) = \left[\hat{t}^{K'}(A)\right]_{classical} \tag{20}$$

This completes the proof of Thesis 1.

4 Lorentz theory and special relativity are completely identical theories

Since Lorentz theory adopts the classical conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$, it does not differ from special relativity in its assertions about $\widehat{\text{space}}$ and $\widehat{\text{time}}$. What about the other claim—$G_2\left(\widetilde{\mathcal{M}}\right)$—about $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$? In order to prove what Thesis 2 asserts, that is to say the complete identity of Lorentz theory and of special relativity, we also have to show that the two theories have identical assertions about $\widetilde{x}$ and $\widetilde{t}$, that is,

$$\begin{aligned}\left[\widetilde{x}^{K'}(A)\right]_{relativity} &= \left[\widetilde{x}^{K'}(A)\right]_{LT} \\ \left[\widetilde{t}^{K'}(A)\right]_{relativity} &= \left[\widetilde{t}^{K'}(A)\right]_{LT}\end{aligned}$$

According to relativity theory, the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags in K' and in K are related through the Lorentz transformations. From (14)–(15) we have

$$\left[\widetilde{t}^{K'}(A)\right]_{relativity} = \frac{\hat{t}^K(A) - \frac{v\,\hat{x}^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{21}$$

$$\left[\widetilde{x}^{K'}(A)\right]_{relativity} = \frac{\hat{x}^K(A) - v\,\hat{t}^K(A)}{\sqrt{1-\frac{v^2}{c^2}}} \tag{22}$$

On the other hand, taking the assumptions of Lorentz theory that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K', one can directly calculate the $\widetilde{\text{space}}$ tag $\widetilde{x}^{K'}(A)$ and the $\widetilde{\text{time}}$ tag $\widetilde{t}^{K'}(A)$, following the descriptions of operations in (D6) and (D7).

First, let us calculate the reading of the clock slowly transported in K' from the origin to the locus of an event A. The clock is moving with a varying velocity (For the sake of simplicity we continue to restrict our calculation to one space dimension. For the general calculation of the phase shift suffered by moving clocks, see Jánossy 1971, pp. 142–147.):

$$\hat{v}_C^K(\hat{t}^K) = v + \hat{w}^K(\hat{t}^K)$$

where $\hat{w}^K(\hat{t}^K)$ is the velocity of the clock relative to K', that is, $\hat{w}^K(0) = 0$ when it starts at $\hat{x}_C^K(0) = 0$ (as we assumed, $\hat{t}^K = 0$ and the transported clock shows 0 when the origins of K and K' coincide) and $\hat{w}^K(\hat{t}_1^K) = 0$ when the clock arrives at the place of A. The reading of the clock at the time $\hat{t}_1^K$ will be

$$T = \int_0^{\hat{t}_1^K} \sqrt{1 - \frac{\left(v + \hat{w}^K(\hat{t})\right)^2}{c^2}}\, d\hat{t} \tag{23}$$

Since $\hat{w}^K$ is small we may develop in powers of $\hat{w}^K$, and we find from (23) when neglecting terms of second and higher order

$$T = \frac{\hat{t}_1^K - \frac{\left(\hat{t}_1^K v + \int_0^{\hat{t}_1^K} \hat{w}^K(\hat{t})\, d\hat{t}\right) v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\hat{t}^K(A) - \frac{\hat{x}^K(A) v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{24}$$

(where, without loss of generality, we take $\hat{t}_1^K = \hat{t}^K(A)$). Thus, according to the definition of $\widetilde{t}$, we have

$$\left[\widetilde{t}^{K'}(A)\right]_{LT} = \frac{\hat{t}^K(A) - \frac{v\,\hat{x}^K(A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{25}$$

which is equal to $\left[\widetilde{t}^{K'}(A)\right]_{relativity}$ in (21).

Now, taking into account that the length of the co-moving meter stick is only $\sqrt{1-\frac{v^2}{c^2}}$, the distance of event A from the origin of K is the following:

$$\hat{x}^K(A) = \hat{t}^K(A) v + \widetilde{x}^{K'}(A) \sqrt{1-\frac{v^2}{c^2}} \tag{26}$$

and thus

$$\left[\widetilde{x}^{K'}(A)\right]_{LT} = \frac{\hat{x}^{K}(A) - v\,\hat{t}^{K}(A)}{\sqrt{1-\frac{v^2}{c^2}}} = \left[\widetilde{x}^{K'}(A)\right]_{relativity}$$

This completes the proof. The two theories make completely identical assertions not only about the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}, \hat{t}$ but also about the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\tilde{x}, \tilde{t}$.

Consequently, there is full agreement between the Lorentz theory and special relativity theory in the following statements:

(a) $\widetilde{\text{Velocity}}$—which is called "velocity" by relativity theory—is not an additive quantity,

$$\widetilde{v}^{K'}(K''') = \frac{\widetilde{v}^{K'}(K'') + \widetilde{v}^{K''}(K''')}{1 + \frac{\widetilde{v}^{K'}(K'')\widetilde{v}^{K''}(K''')}{c^2}}$$

while $\widehat{\text{velocity}}$—that is, what we traditionally call "velocity"—is an additive quantity,

$$\hat{v}^{K'}(K''') = \hat{v}^{K'}(K'') + \hat{v}^{K''}(K''')$$

where K', K'', K''' are arbitrary three frames. For example,

$$\hat{v}^{K'}(light\,signal) = \hat{v}^{K'}(K'') + \hat{v}^{K''}(light\,signal)$$

(b) The $\left(\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}\right)$-map of the world can be conveniently described through a Minkowski geometry, such that the $\widetilde{t}$-simultaneity can be described through the orthogonality with respect to the 4-metric of the Minkowski space, etc.

(c) The $\left(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}\right)$-map of the world, can be conveniently described through a traditional "space-time geometry" like $\mathbb{E}^3 \times \mathbb{E}^1$.

(d) The $\widehat{\text{velocity}}$ of light is not the same in all inertial frames of reference.

(e) The $\widetilde{\text{velocity}}$ of light is the same in all inertial frames of reference.

(f) $\widehat{\text{Time}}$ and $\widehat{\text{distance}}$ are invariant, the reference frame independent concepts, $\widetilde{\text{time}}$ and $\widetilde{\text{distance}}$ are not.

(g) $\hat{t}$-simultaneity is an invariant, frame-independent concept, while $\widetilde{t}$-simultaneity is not.

(h) For arbitrary K' and K'', $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ can be expressed by $\hat{x}^{K''}(A), \hat{t}^{K''}(A)$ through a suitable Galilean transformation

(i) For arbitrary K' and K'', $\widetilde{x}^{K'}(A), \widetilde{t}^{K'}(A)$ can be expressed by $\widetilde{x}^{K''}(A), \widetilde{t}^{K''}(A)$ through a suitable Lorentz transformation.

⋮

Moreover, they agree in the following observation (Relativity Principle):

(j) The behaviour of similar systems co-moving as a whole with different inertial frames, expressed in terms of the results of measurements obtainable by means of co-moving measuring-rods and clocks, that is, in terms of quantities $\widetilde{x}$ and $\widetilde{t}$, is the same in every inertial frame of reference.

Combining this with (i),

(k) The laws of physics, expressed in terms of $\widetilde{x}$ and $\widetilde{t}$, must be given by means of Lorentz covariant equations.

Finally, they agree that

(l) All facts about $\widetilde{x}$ and $\widetilde{t}$ (and, consequently, all facts about $\hat{x}$ and $\hat{t}$) can be derived *backward* from (e) and (j).

To sum up symbolically, Lorentz theory and and special relativity theory have identical assertions about both $\hat{M}$ and $\widetilde{M}$: they unanimously claim that $G_1\left(\hat{M}\right)\&G_2\left(\widetilde{M}\right)$.

Finally, note that in an arbitrary inertial frame K' for every event A the tags $\hat{x}_1^{K'}(A)$, $\hat{x}_2^{K'}(A)$, $\hat{x}_3^{K'}(A)$, $\hat{t}^{K'}(A)$ can be expressed in terms of $\widetilde{x}_1^{K'}(A)$, $\widetilde{x}_2^{K'}(A)$, $\widetilde{x}_3^{K'}(A)$, $\widetilde{t}^{K'}(A)$ and *vice versa*. Consequently, we can express the laws of physics—as is done in special relativity—equally well in terms of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ instead of the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$. On the other hand, we should emphasise that the one-to-one correspondence between $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$ and $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ also entails that the laws of physics (so called "relativistic" laws included) can be equally well expressed in terms of the (traditional) $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ instead of the variables $\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3, \widetilde{t}$. In brief, physics could manage equally well with the classical Galileo-invariant conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$.

With these remarks I have completed the argumentation for my two theses.

5 Comments

5.1 Are relativistic deformations real physical changes?

Many believe that it is an essential difference between the two theories that relativistic deformations like the Lorentz–FitzGerald contraction and the time dilatation are real physical changes in Lorentz theory, but there are no similar physical effects in special relativity. Let us examine two typical argumentations.

According to the first argument the "Lorentz contraction/dilatation" of a rod cannot be an objective physical deformation in relativity theory, because it is a frame-dependent fact whether "the rod is shrinking or expanding". Consider a rod accelerated from the sate of rest in reference frame K' to the state of rest in reference frame K''. According to relativity theory, "the rod shrinks in frame K' and, at the same time, expands in frame K''". But this is a contradiction, the argument says, if the deformation was a real physical change. (In contrast, the argument says, Lorentz's theory claims that "the length of a rod" is a frame-independent concept. Consequently, in Lorentz's theory, "the contraction/dilatation of a rod" can indeed be an objective physical change.)

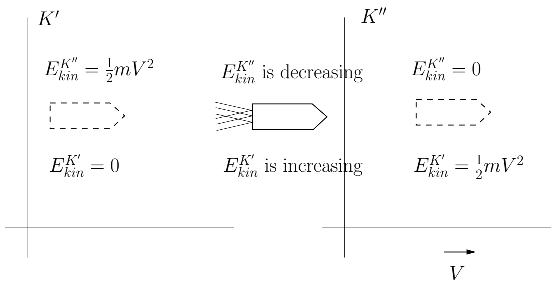

Figure 5: One and the same objective physical process is traced in the increase of kinetic energy of the spaceship relative to frame K', while it is traced in the decrease of kinetic energy relative to frame K''

However, we have already clarified, that the terms "distance" and "time" have different meanings in relativity theory and Lorentz's theory. Due to the difference between $\widehat{\text{length}}$ and $\widetilde{\text{length}}$, we must also differentiate $\widehat{\text{dilatation}}$ from $\widetilde{\text{dilatation}}$, $\widehat{\text{contraction}}$ from $\widetilde{\text{contraction}}$, and so on. For example, consider the reference frame of the *etalons* K and another frame K' moving relative to K. The following statements are true about the "length" of a rod accelerated from the sate of rest in reference frame K ($state_1$) to the state of rest in reference frame K' ($state_2$):

$$\hat{l}^K(state_1) > \hat{l}^K(state_2) \qquad \widehat{\text{contraction}} \text{ in } K \tag{27}$$

$$\hat{l}^{K'}(state_1) > \hat{l}^{K'}(state_2) \qquad \widehat{\text{contraction}} \text{ in } K' \tag{28}$$

$$\tilde{l}^K(state_1) > \tilde{l}^K(state_2) \qquad \widetilde{\text{contraction}} \text{ in } K \tag{29}$$

$$\tilde{l}^{K'}(state_1) < \tilde{l}^{K'}(state_2) \qquad \widetilde{\text{dilatation}} \text{ in } K' \tag{30}$$

And there is no difference between relativity theory and Lorentz's theory: *all* of the four statements (27)–(30) are true *in both theories*. If, in Lorentz's theory, facts (27)–(28) provide enough reason to say that there is a real physical change, then the same facts provide enough reason to say the same thing in relativity theory. And *vice versa*, if (29)–(30) contradicted to the existence of real physical change of the rod in relativity theory, then the same holds for Lorentz's theory.

It should be mentioned, however, that there is no contradiction between (29)–(30) and the existence of real physical change of the rod. Relativity theory and Lorentz's theory unanimously claim that $\widetilde{\text{length}}$ is a relative physical quantity. It is entirely possible that one and the same objective physical change is traced in the increase of the value of a relative quantity relative to one reference frame, while it is traced in the decrease of the same quantity relative to another reference frame (Fig 5). (What is more, both, the value relative to one frame and the value relative to the other frame, reflect objective features of the objective physical process in question.)

According to the other wide-spread argument the relativistic deformations cannot be real physical effects since they can be observed by an observer also if the object is at rest but the observer is in motion at constant velocity. And these "relativistic deformations" cannot be explained as real physical deformations of the object at rest—the argument says.

There is, however, a triple misunderstanding behind such an argument:

- Of course, no real distortion is suffered by an object which is continuously at rest relative to a reference frame K', and, consequently, which is continuously in motion at a constant velocity relative to another frame K''. None of the observers can observe such a distortion. For example,

$$\begin{aligned} \widetilde{l}^{K'}\left(\text{distortion free rod at } \tilde{t}_1\right) &= \widetilde{l}^{K'}\left(\text{distortion free rod at } \tilde{t}_2\right) \\ \widetilde{l}^{K''}\left(\text{distortion free rod at } \tilde{t}_1\right) &= \widetilde{l}^{K''}\left(\text{distortion free rod at } \tilde{t}_2\right) \end{aligned}$$

- It is surely true,

$$\widetilde{l}^{K'}\,(\text{distortion free rod}) \neq \widetilde{l}^{K''}\,(\text{distortion free rod}) \tag{31}$$

  This fact, however, does not express a $\widetilde{\text{contraction}}$ of the rod—neither a real nor an apparent $\widetilde{\text{contraction}}$.

- On the other hand, inequality (31) is a *consequence* of the real physical distortions suffered by the measuring equipments—with which the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags are empirically defined—when they are transfered from the BIPM to the other reference frame in question. (For further details of what a moving observer can observe by means of his or her distorted measuring equipments, see Bell 1983, pp. 75–76.)

Thus, relativistic deformations are real physical deformations also in special relativity theory. One has to emphasise this fact because it is an important part of the physical content of relativity theory. It must be clear, however, that this conclusion is independent of our main concern. What is important is the following: Lorentz's theory and special relativity have identical assertions about $\widehat{\text{length}}$ and $\widetilde{\text{length}}$, $\widehat{\text{duration}}$ and $\widetilde{\text{duration}}$, $\widehat{\text{shrinking}}$ and $\widetilde{\text{shrinking}}$, etc. Consequently, whether or not these facts provide enough reason to say that the deformations are real physical changes, the conclusion is common to both theories.

5.2 The intuition behind the definitions

Before entering into the discussion of the intuitions behind definitions (D1)–(D8), I would like to emphasise that, from the point of view of our main concern, it is not important how the different definitions are justified and whether these justifications are correct or not. What is important is the terminological confusion caused by the mere fact that the "space" and "time" tags mean *different* physical quantities in classical physics and relativity theory.

The basic difference between the intuitions behind the classical and relativistic definitions is the following. As we have seen, both Lorentz theory and special relativity "know" about the distortions of measuring-rods and clocks when they are transfered from the BIPM to the moving (relative to the BIPM) reference frame K'. In the relativistic definitions, (D6) and (D7), we *ignore* this fact and define the space and time tags as they are measured by means of the distorted equipments. In contrast, as it follows from the whole tradition of classical physics, in definition (D5) we *take into account* the distortions of the measuring equipments. That is why the space and time tags in K' are defined through the original space and time data, measured by the original distortion free measuring-rod and clock, which are at rest relative to the BIPM.

In order to see this "compensatory view" of the classical definition in a more explicit form, it worth while to mention a possible alternative definition instead of (D5). We know that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from K to K'. Therefore, according to the compensatory view, if we measure a distance and the result is X, then the "real distance" is $X\sqrt{1-\frac{v^2}{c^2}}$. Similarly, taking into account the phase shift suffered by a moving clock, we know from (24) that if the reading of the clock is T then the "real time" is

$$\frac{T+X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Accordingly, the alternative definition is the following:

(D5') Space and time tags of an event in K' according to classical physics

> Let X be the "distance" from the origin of K' of the locus of A along the x-axis measured by superposing the standard measuring-rod, being always at rest relative to K', in just the same way as if all were at rest. The space tag $\check{x}^{K'}(A)$ of event A is
>
> $$\check{x}^{K'}(A) := X\sqrt{1-\frac{v^2}{c^2}} \tag{32}$$
>
> Take a synchronised copy of the standard clock at rest in the BIPM, gently accelerate it from K to K' and set it to show 0 when the origins of K and K' coincide. Then slowly (relative to K') move it to the locus of event A. Let T be the reading of the transfered clock when A occurs. The time tag $\widetilde{t}^{K'}(A)$ is
>
> $$\check{t}^{K'}(A) := \frac{T+X\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{33}$$

Since X and T are nothing but $\widetilde{x}^{K'}(A)$ and $\widetilde{t}^{K'}(A)$, it follows from (25) and (26) that

$$\begin{aligned}\check{x}^{K'}(A) &= \hat{x}^{K'}(A)\\ \check{t}^{K'}(A) &= \hat{t}^{K'}(A)\end{aligned}$$

5.3 On the null result of the Michelson–Morley experiment

Consider the following passage from Einstein:

> A ray of light requires a perfectly definite time T to pass from one mirror to the other and back again, if the whole system be at rest with respect to the aether. It is found by calculation, however, that a slightly different time T^1 is required for this process, if the body, together with the mirrors, be moving relatively to the aether. And yet another point: it is shown by calculation that for a given velocity v with reference to the aether, this time T^1 is different when the body is moving perpendicularly to the planes of the mirrors from that resulting when the motion is parallel to these planes. Although the estimated difference between these two times is exceedingly small, Michelson and Morley performed an experiment involving interference in which this difference should have been clearly detectable. But the experiment gave a negative result — a fact very perplexing to physicists. (Einstein 1920, p. 49)

The "calculation" that Einstein refers to is based on the Galilean "kinematics", that is, on the invariance of "time" and "simultaneity", on the invariance of "distance", on the classical addition rule of "velocities", etc. That is to say, "distance", "time", and "velocity" in the above passage mean the classical $\widehat{\text{distance}}$, $\widehat{\text{time}}$, and $\widehat{\text{velocity}}$ defined in (D1), (D2) and (D5). The negative result was "very perplexing to physicists" because their expectations were based on traditional concepts of $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and they could not imagine other that if the $\widehat{\text{speed}}$ of light is c relative to one inertial frame then the $\widehat{\text{speed}}$ of the same light signal cannot be the same c relative to another reference frame.

Now, Einstein continues this passage in the following way:

> Lorentz and FitzGerald rescued the theory from this difficulty by assuming that the motion of the body relative to the aether produces a contraction of the body in the direction of motion, the amount of contraction being just sufficient to compensate for the difference in time mentioned above. Comparison with the discussion in Section 11 shows that also from the standpoint of the theory of relativity this solution of the difficulty was the right one. But on the basis of the theory of relativity the method of interpretation is incomparably more satisfactory. According to this theory there is no such thing as a "specially favoured" (unique) co-ordinate system to occasion the introduction of the aether-idea, and hence there can be no aether-drift, nor any experiment with which to demonstrate it. Here the contraction of moving bodies follows from the two fundamental principles of the theory, without the introduction of particular hypotheses; and as the prime factor involved in this contraction we find, not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point. Thus for a co-ordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a co-ordinate system which is at rest relatively to the sun.(Einstein 1920, p. 49)

What "rescued" means here is that—within the framework of the classical $\widehat{\text{space-time}}$ theory and Galilean $\widehat{\text{kinematics}}$—Lorentz and FitzGerald proved that if the assumed deformations of moving bodies exist then the expected result of the Michelson–Morley experiment is the null effect. On the other hand, we have already clarified, what Einstein also confirms in the above quoted passage, that these deformations also derive from the two basic postulates of special relativity. We can put these facts together in the following schema:

$$
\underbrace{[\text{deformations}]\ \&\ \left[\begin{array}{c}\text{Galilean kinematics}\\ \text{for } \hat{x},\hat{t} \text{ (the speed}\\ \text{of light is NOT}\\ \text{the same in all}\\ \text{inertial frame)}\end{array}\right]}
\quad\Rightarrow\quad
\left[\begin{array}{c}\text{the result of the}\\ \text{Michelson-Morley}\\ \text{experiment must}\\ \text{be the null effect}\end{array}\right]
$$

$$
\Updownarrow
$$

$$
\overbrace{[\text{deformations}]\ \Leftarrow\ \left[\begin{array}{c}\text{Lorentz kinematics}\\ \text{for } \tilde{x},\tilde{t} \text{ (the speed}\\ \text{of light IS the same}\\ \text{in all inertial frame)}\end{array}\right]}
\quad\Rightarrow\quad
\left[\begin{array}{c}\text{the result of the}\\ \text{Michelson-Morley}\\ \text{experiment must}\\ \text{be the null effect}\end{array}\right]
$$

That is to say the null result of the Michelson–Morley experiment simultaneously confirms *both*, the classical rules of Galilean kinematics for $\hat{x}$ and $\hat{t}$, and the violation of these rules (that is the Lorentzian kinematics) for the space and time tags $\tilde{x}, \tilde{t}$. It confirms the classical addition rule of velocities, on the one hand, and, on the other hand, it also confirms that velocity of light is the same in all frames of reference.

This actually holds for all other experimental confirmations of special relativity. That is why the only difference Einstein can mention in the quoted passage is that special relativity does not refers to the aether. (As a historical fact, this difference is true. Although the concept of aether can be entirely removed from the recent logical reconstruction of the Lorentz theory. See sections 5.6 and 5.8.)

Finally, it is no surprise that the deformations can be "derived" from the Lorentz kinematics. The *physical* information about the deformations suffered by objects accelerated from one state of motion to another, say from the state of rest relative to K' to the state of rest relative to K'', is inbuilt into the relationship between the tags $\tilde{x}^{K'}(A), \tilde{t}^{K'}(A)$ and $\tilde{x}^{K''}(A), \tilde{t}^{K''}(A)$. For these relations are determined by the physical behaviour of measuring rods and clocks during the acceleration and relaxation process. As Einstein warns us, the Lorentz transformations, relating the space and time tags in different reference frames, are nothing but physical laws governing the *physical behaviour* of the measuring-rods and clocks:

> A Priori it is quite clear that we must be able to learn something about the physical behaviour of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. (Einstein 1920, p. 35)

5.4 The conventionalist approach

According to the conventionalist thesis (see Friedman 1983, p. 293; Einstein 1983, p. 35.), Lorentz's theory and Einstein's special relativity are two alternative scientific theories which are equivalent on empirical level. Due to the empirical underdeterminacy, the choice between these alternative theories is based on external aspects. (Cf. Zahar 1973; Grünbaum 1974; Friedman 1983; Brush 1999; Janssen 2002.) Following Poincaré's similar argument about the relationship between geometry, physics, and the empirical facts, the conventionalist thesis asserts the following relationship between Lorentz theory and special relativity:$\mathbb{E}^3 \times \mathbb{E}^1$

$$\begin{array}{ccccc} \left[\begin{array}{c}\text{classical}\\ \mathbb{E}^3\times\mathbb{E}^1\text{-theory}\\ \text{of space-time}\end{array}\right] & + & \left[\begin{array}{c}\text{physical content}\\ \text{of Lorentz theory}\end{array}\right] & = & \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right] \\ \left[\begin{array}{c}\text{relativistic}\\ \mathbb{M}^4\text{-theory}\\ \text{of space-time}\end{array}\right] & + & \left[\begin{array}{c}\text{special relativistic}\\ \text{physics}\end{array}\right] & = & \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right] \end{array} \tag{34}$$

Continuing the symbolic notations we used in the Introduction, denote Z those objective features of physical reality that are described by the alternative physical theories P_1 and P_2 in question. With these notations, the logical schema of the conventionalist thesis can be described in the following way: We cannot distinguish by means of the available experiments whether $G_1(M)\,\&P_1(Z)$ is true about the objective features of physical reality $M\cup Z$, or $G_2(M)\,\&P_2(Z)$ is true about the *same* objective features $M\cup Z$. Schematically,

$$[G_1(M)]+[P_1(Z)] = \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right]$$

$$[G_2(M)]+[P_2(Z)] = \left[\begin{array}{c}\text{empirical}\\ \text{facts}\end{array}\right]$$

However, it is clear from the previous sections that the terms "space" and "time" have different meanings in the two theories. Lorentz theory claims $G_1\left(\hat{M}\right)$ about $\hat{M}$ and relativity theory claims $G_2\left(\widetilde{M}\right)$ about some other features of reality $\widetilde{M}$. Of course, this terminological confusion also appears in the physical assertions. Let us symbolise with $\hat{Z}$ the objective features of physical reality, such as the $\widehat{\text{length}}$ of a rod, etc., described by physical theory P_1. And let $\widetilde{Z}$ denote some (partly) different features of reality described by P_2, such as the $\widetilde{\text{length}}$ of a rod, etc. Now, as we have seen, both theories actually claim that $G_1\left(\hat{M}\right)\&G_2\left(\widetilde{M}\right)$. It is also clear that, for example, within Lorentz's theory, we can legitimately query the $\widetilde{\text{length}}$ of a rod. For Lorentz's theory has complete description of the behaviour of a moving rigid rod, as well as the behaviour of a moving clock and measuring-rod. Therefore, it is no problem in Lorentz's theory to predict the result of a measurement of the "length" of the rod, if the measurement is performed with a co-moving measuring equipments, according to empirical definition (D7). This prediction will be exactly the same as the prediction of special relativity. And vice versa, special relativity would have the same prediction for the $\widehat{\text{length}}$ of the rod as the prediction of the Lorentz theory. That is to say, the physical contents of Lorentz's theory and special relativity also are identical: both claim that $P_1\left(\hat{Z}\right) \& P_2\left(\widetilde{Z}\right)$. So we have the following:

$$\left[G_1\left(\hat{M}\right) \& G_2\left(\widetilde{M}\right)\right] + \left[P_1\left(\hat{Z}\right) \& P_2\left(\widetilde{Z}\right)\right] = \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix}$$

$$\left[G_1\left(\hat{M}\right) \& G_2\left(\widetilde{M}\right)\right] + \left[P_1\left(\hat{Z}\right) \& P_2\left(\widetilde{Z}\right)\right] = \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix}$$

In other words, since there are no two different theories, there is *no choice*, based neither on internal nor on external aspects.

5.5 Methodological remarks

1. It worth while emphasising that my argument is based on the following very weak "operationalist" premise: physical terms, assigned to measurable physical quantities, have different meanings if they have different empirical definitions. This premise is one of the fundamental pre-assumptions of Einstein's 1905 paper and is widely accepted among physicists. Without clear empirical definition of the measurable physical quantities a physical theory cannot be empirically confirmable or disconfirmable. In itself, this premise is not yet equivalent to operationalism or verificationalism. It does not generally imply that a statement is necessarily meaningless if it is neither analytic nor empirically verifiable. However, when the physicist assigns time and space tags to an event, relative to a reference frame, (s)he is already after all kinds of metaphysical considerations about "What is space and what is time?" and means definite physical quantities with already settled empirical meanings.

2. In saying that the meanings of the words "space" and "time" are different in relativity theory and in classical physics, it is necessary to be careful of a possible misunderstanding. I am talking about something entirely different from the incommensurability thesis of the relativist philosophy of science. (See Kuhn 1970, Chapter X; Feyerabend 1970.) How is it that relativity makes any assertion about classical $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and vice versa, how can Lorentz's theory make assertions about quantities which are not even defined in the theory? As we have seen, each of the two theories is sufficiently complete account of physical reality to make predictions about those features of reality that correspond—according to the empirical definitions—to the variables used by the other theory, and we can *compare* these predictions. For example, within Lorentz's theory, we can legitimately query the reading of a clock slowly transported in K' from one place to another. That exactly is what we calculated in section 4. Similarly, in relativity theory, we can legitimately query the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags of an event in the reference frame of the *etalons* and then apply formulas (12)–(13). This is a fair calculation, in spite of the fact that the result so obtained is not explicitly mentioned and named in the theory. This is what we actually did. And the conclusion was that not only are the two theories commensurable, but they provide completely identical accounts of the same physical reality.

5.6 Privileged reference frame

Due to the popular/textbook literature on relativity theory, there is a widespread aversion to a privileged reference frame. However, like it or not, there is a privileged reference frame in both special relativity and classical physics. It is the frame of reference in which the *etalons* are at rest. This privileged reference frame, however, has nothing to do with the concepts of "absolute rest" or the aether, and it is not privileged by nature, but it is privileged by the trivial semantical convention providing meanings for the terms "distance" and "time", by the fact that of all possible measuring-rod-like and clock-like objects floating in the universe, we have chosen the ones floating together with the International Bureau of Weights and Measures in Paris. In Bridgman's words:

> It cannot be too strongly emphasised that there is no getting away from preferred operations and unique standpoint in physics; the unique physical operations in terms of which interval has its meaning afford one example, and there are many others also. (Bridgman 1936, p. 83)

Many believe that one can avoid a reference to the *etalons* sitting in a privileged reference frame by defining, for example, the unit of $\widetilde{\text{time}}$ for an arbitrary (moving) frame of reference K' through a cesium clock, or the like, co-moving with K'. In this way, one needs not to refer to a standard clock accelerated from the reference frame of the *etalons* into reference frame K'. But further thought reveals that such a definition has several difficulties. For if this operation is regarded as a convenient way of *measuring* $\widetilde{\text{time}}$, then we still have $\widetilde{\text{time}}$ in the theory, together with the privileged reference frame of the *etalons*. If, however, this operation is regarded as the empirical *definition* of a physical quantity, then it must be clear that this quantity is not $\widetilde{\text{time}}$ but a new physical quantity, say $\widetilde{\widetilde{\text{time}}}$. In order to establish any relationship between $\widetilde{\widetilde{\text{time}}}$ tags belonging to different reference frames, it is a must to use an "*etalon* cesium clock" as well as to refer to its behaviour when accelerated from one inertial frame into the other.

5.7 The relativity principle and the physics of moving objects

Although special relativity does not tell us anything new about space and time, both special relativity and Lorentz theory enrich our knowledge of the physical world with *the physics of objects moving at constant velocities*—in accordance with the title of Einstein's original 1905 paper. The essential physical content of their discoveries is that physical objects suffer distortions when they are accelerated from one inertial frame to the other, and that these distortions satisfy some uniform laws.

FitzGerald, Lorentz and Poincaré derived these laws from the requirement that the deformations must explain the null result of the Michelson–Morley experiment. (FitzGerald and Lorentz also made an attempt to understand how these deformations actually come about from the molecular forces. See Bell 1992; Brown and Pooley 2001; Brown 2001; 2003.) They arrived to the conclusion that the standard clock slows down by factor $\sqrt{1-\frac{v^2}{c^2}}$ and that a rigid rod suffers a contraction by factor $\sqrt{1-\frac{v^2}{c^2}}$ when they are gently accelerated from

K to K'. As we have shown in section 4, this claim is equivalent with the assertion that the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\tilde{x}^{K''}(A), \tilde{t}^{K''}(A)$ measured by the co-moving distorted equipments can be expressed from the similar tags $\tilde{x}^{K'}(A), \tilde{t}^{K'}(A)$ by a suitable Lorentz transformation.

The general laws of deformations apply to both the measuring-equipment and the object to be measured. Therefore, it is no surprise that the "length" of a moving, consequently distorted, rod measured by co-moving, consequently distorted, measuring-rod and clock, that is the $\widetilde{\text{length}}$ of the rod, is the same as the $\widetilde{\text{length}}$ of the corresponding stationary rod measured with stationary measuring-rod and clock. The $\widetilde{\text{duration}}$ of a slowed down process in a moving object measured with a co-moving, consequently slowed down, clock will be the same as the $\widetilde{\text{duration}}$ of the same process in a similar object at rest, measured with the original distortion free clock at rest. These and similar observations lead Lorentz and Poincaré to conclude with the general validity of the relativity principle. (Whether or not relativity principle generally holds in relativistic physics is a more complex question. See Szabó 2004.) In his 1905 paper Einstein showed how to derive the same rules from the assumption that relativity principle generally holds and (or consequently) the $\widetilde{\text{velocity}}$ of a light signal is the same in all inertial reference frames. These historic differences are, however, not important from the point of view of our main concern. What is important is that in both ways one can derive exactly the same laws of deformations, exactly the same rules for $\hat{x}$ and $\hat{t}$, and exactly the same rules for $\tilde{x}$ and $\tilde{t}$.

The relativity principle together with the Lorentz transformation of $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ provide the general description of the behaviour of moving physical systems: Let $\mathcal{E}'$ be a set of differential equations describing the behaviour of the system in question in an arbitrary reference frame K'. Let ψ'_0 denote a set of (initial) conditions, such that the solution determined by ψ'_0 describes the behaviour of the system when it is, as a whole, at rest relative to K'. Let $\psi'_{\widetilde{v}}$ be a set of conditions which corresponds to the solution describing the same system in uniform motion at velocity $\widetilde{v}$ relative to K'. To be more exact, $\psi'_{\widetilde{v}}$ corresponds to a solution of $\mathcal{E}'$ that describes the same behaviour of the system as ψ'_0 but in superposition with a collective translation at velocity $\widetilde{v}$. Denote $\mathcal{E}''$ and ψ''_0 the equations and conditions obtained from $\mathcal{E}'$ and ψ'_0 by substituting every $\widetilde{x}^{K'}$ with $\widetilde{x}^{K''}$ and $\widetilde{t}^{K'}$ with $\widetilde{t}^{K''}$. Denote $\Lambda_{\widetilde{v}}(\mathcal{E}'), \Lambda_{\widetilde{v}}(\psi'_{\widetilde{v}})$ the set of equations and conditions expressed in terms of the double-primed variables, applying the Lorentz transformations. Now, what the relativity principle (statement (j) in Section 4) states is that the laws of physics describing the behaviour of moving objects are such that they satisfy the following relationships:

$$\Lambda_{\widetilde{v}}(\mathcal{E}') = \mathcal{E}'' \tag{35}$$

$$\Lambda_{\widetilde{v}}(\psi'_{\widetilde{v}}) = \psi''_0 \tag{36}$$

To make more explicit how this principle provides a useful method in the description of the deformations of physical systems when they are accelerated from one inertial frame K' into some other K'', consider the following situation: Assume we know the relevant physical equations and know the solution of the equations describing the physical properties of the object in question when it is at rest in K': $\mathcal{E}', \psi'_0$. We now inquire as to the same description of the object when it is moving at a given constant $\widetilde{\text{velocity}}$ relative to K'. If (35)–(36) is true, then we can solve the problem in the following way. Simply take $\mathcal{E}'', \psi''_0$—by putting one more prime on each variable—and express $\psi'_{\widetilde{v}}$ from (36) by means of the inverse Lorentz transformation: $\psi'_{\widetilde{v}} = \Lambda^{-1}_{\widetilde{v}}\left(\psi''_0\right)$. (Actually, the situation is much more complex. Whether or not the solution thus obtained is correct depends on the details of the relaxation process after the acceleration of the system. See Szabó 2004.) This is the way we usually solve problems such as the electromagnetic field of a moving point charge, the Lorentz contraction of a rigid body, the loss of phase suffered by a moving clock, the dilatation of the mean life of a cosmic ray μ-meson, etc.

5.8 The aether

As it is obvious from the previous sections, we did not make any reference to the aether in the logical reconstruction of Lorentz's theory. It is however a historic fact that Lorentz did. In this section, I want to clarify that the concept of aether is merely a verbal decoration in Lorentz theory, which can be interesting for the historians, but negligible from the point of view of recent logical reconstructions.

One can find various verbal formulations of the relativity principle and Lorentz-covariance. In order to compare these formulations, let us introduce the following notations:

$A\left(K', K''\right) :=$ The laws of physics in inertial frame K' are such that the laws describing a physical system co-moving with frame K'' are obtainable by solving the problem for the similar physical system at rest relative to K' and perform the following substitutions:

$$\begin{aligned} \widetilde{x}_1^{K'} &\mapsto \alpha_1 = \widetilde{x}_1^{K'} \\ \widetilde{x}_2^{K'} &\mapsto \alpha_2 = \widetilde{x}_2^{K'} \\ \widetilde{x}_3^{K'} &\mapsto \alpha_3 = \frac{\widetilde{x}_3^{K'} - \widetilde{v}\widetilde{t}^{K'}}{\sqrt{1-\frac{\widetilde{v}^2}{c^2}}} \\ \widetilde{t}^{K'} &\mapsto \tau = \frac{\widetilde{t}^{K'} - \frac{\widetilde{v}}{c^2}\widetilde{x}_3^{K'}}{\sqrt{1-\frac{\widetilde{v}^2}{c^2}}} \end{aligned} \tag{37}$$

$B\left(K', K''\right) :=$ The laws of physics in K' are such that the mathematically introduced variables $\alpha_1, \alpha_2, \alpha_3, \tau$ in (37) are equal to $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, that is, the "space" and "time" tags obtained by means of measurements in K'', performed with the same measuring-rods and clocks we used in K' after that they were transfered from K' into K'', ignoring the fact that the equipments undergo deformations during the transmission.

$C\left(K', K''\right) :=$ The laws of physics in K' are such that the laws of physics empirically ascertained by an observer in K'', describing the behaviour of physical objects co-moving with K'', expressed in variables $\widetilde{x}_1^{K''}, \widetilde{x}_2^{K''}, \widetilde{x}_3^{K''}, \widetilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in in K', describing the similar physical objects co-moving with K', expressed in variables $\widetilde{x}_1^{K'}, \widetilde{x}_2^{K'}, \widetilde{x}_3^{K'}, \widetilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transfered from K' to K'', ignoring the fact that the equipments undergo deformations during the transmission.

It is obvious that

$$A(K', K'') \,\&\, B(K', K'') \Rightarrow C(K', K'')$$

So, let us restrict our considerations on the more fundamental

$$A(K', K'') \,\&\, B(K', K'')$$

Taking this statement, the usual Einsteinian formulation of the relativity principle is the following:

$$\text{Einstein's Relativity Principle} = (\forall K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right]$$

Many believe that this version of relativity principle is essentially different from the similar principle of Lorentz, since Lorentz's principle makes explicit reference to the motion relative to the aether. Using the above introduced notations, it says the following:

$$\text{Lorentz's Principle} = (\forall K'')\left[A(\text{aether}, K'') \,\&\, B(\text{aether}, K'')\right]$$

It must be clearly seen, however, that Lorentz's aether hypothesis is logically independent from the actual physical content of his theory. In fact, as a little reflection reveals, *Lorentz's principle and Einstein's relativity principle are logically equivalent to each other.* It is trivially true that

$$\begin{aligned} \text{Einstein's Relativity Principle} &= (\forall K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right] \\ &\Rightarrow (\forall K'')\left[A(\text{aether}, K'') \,\&\, B(\text{aether}, K'')\right] \\ &= \text{Lorentz's Principle} \end{aligned}$$

It follows from the meaning of $A(K', K'')$ and $B(K', K'')$ that

$$\begin{aligned} &(\exists K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right] \\ \Rightarrow\; &(\forall K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right] \end{aligned}$$

Consequently,

$$\begin{aligned} \text{Lorentz's Principle} &= (\forall K'')\left[A(\text{aether}, K'') \,\&\, B(\text{aether}, K'')\right] \\ &\Rightarrow (\exists K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right] \\ &\Rightarrow (\forall K')(\forall K'')\left[A(K', K'') \,\&\, B(K', K'')\right] \\ &= \text{Einstein's Relativity Principle} \end{aligned}$$

Thus, it is Lorentz's principle itself—the verbal formulation of which refers to the aether—that renders any claim about the aether a logically separated hypothesis outside of the scope of the factual content of both Lorentz theory and special relativity. The role of the aether could be played by anything else. As both theories claim, it follows from the empirically confirmed laws of physics that physical systems undergo deformations when they are transferred from one inertial frame K' to another frame K''. One could say, these deformations are caused by the transmission of the system from K' to K''. You could say they are caused by the "wind of aether". By the same token you could say, however, that they are caused by "the wind of *anything*", since if the physical system is transfered from K' to K'' then its state of motion changes relative to an arbitrary third frame of reference.

On the other hand, it must be mentioned that special relativity does not exclude the existence of the aether. (Not to mention that already in 1920 Einstein himself argues for the existence of some kind of aether. See Reignier 2000.) Neither does the Michelson–Morley experiment. If special relativity/Lorentz theory is true then there must be no indication of the motion of the interferometer relative to the aether. Consequently, the fact that we do not observe indication of this motion is not a challenge for the aether theorist. Thus, the hypothesis about the existence of aether is logically independent of both Lorentz theory and special relativity.

5.9 Symmetry principle and heuristic value

Finally, it worth while mentioning that Lorentz's theory and special relativity, as completely identical theories, offer the same symmetry principles and heuristic power. As we have seen, both theories claim that quantities $\widetilde{x}^{K'}, \widetilde{t}^{K'}$ in an arbitrary K' and the similar quantities $\widetilde{x}^{K''}, \widetilde{t}^{K''}$ in another arbitrary K'' are related through a suitable Lorentz transformation. This fact in conjunction with the relativity principle implies that laws of physics are to be described by Lorentz covariant equations, if they are expressed in terms of variables $\widetilde{x}$ and $\widetilde{t}$, that is, in terms of the results of measurements obtainable by means of the corresponding co-moving equipments—which are distorted relative to the *etalons*. There is no difference between the two theories that this $\widetilde{\text{space-time}}$ symmetry provides a valuable heuristic aid in the search for new laws of nature.

6 Conclusion

With these comments I have completed the argumentation for my basic claim that special relativity and Lorentz theory are completely identical in both senses, as theories about space-time and as theories about the behaviour of moving physical objects. Consequently, in comparison with the classical Galileo-invariant conceptions, special relativity theory does not tell us anything new about space and time. As we have seen, the longstanding belief that it does is the result of a simple but subversive terminological confusion.

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