# Ontology of Logic*†

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**Abstract**

According to the formalist doctrine mathematical objects have no meanings; we have symbols and rules governing how these symbols can be combined. That's all.

This paper goes further by formulating a more radical thesis: The signs of a formal system of mathematics should be considered as physical objects, and the formal operations as physical processes. The rules of the formal operations are (or can be, in principle) expressed in terms of the laws of physics, governing these processes. In accordance with the physicalist understanding of mind, this is true even if the operations in question are executed in head. A truth obtained through (mathematical) reasoning is, therefore, an observed outcome of a neuro-physiological (or other physical) experiment. Consequently, deduction is nothing but a particular case of induction; the certainty available in inductive generalization is the best of all possible certainties.

## 1.

The central question of the philosophy of mathematics is what is mathematical truth, that is, what makes a mathematical proposition true.

***Mathematical realism*** is the view that mathematical propositions are true insofar as they correspond with our physical environment. In other words, mathematics is an empirical science: mathematical propositions express the most general features of physical reality. Although it played an important role in the history of mathematical sciences, this view cannot be taken seriously

*[*] **Dedication to my friend István Németi on his 60th birthday:** István, we make strenuous efforts to employ logic in physics. Why don't we try to employ physics in logic, instead?*

[†] Talk presented on *Logic, Algebra, Relativity - 2002,* November 4 - 8, 2002, Budapest.

in time of modern mathematics. For there is no such a direct correspondence between the mathematical notions and the elements of physical reality. For example, nothing in the external world (outside of mathematics) corresponds to the notion of infinity. So, we reject that "mathematics is an empirical science", as this thesis is usually understood, although, according to our final conclusion, we will see that it is an empirical science in another sense. As such, it does not express, however, the most general features of the physical world. On the contrary, it reflects some particular and not necessary important features of it.

According to ***mathematical Platonism***, substantive existence can be attributed to the classical concepts of mathematics, independently of whether or not anybody has these concepts in mind. A truth about a mathematical concept can be, like any other truth about any other existing thing, *discovered.* The particular way of discovery in which a true mathematical proposition can be obtained is the logical analysis of these concepts. For a Platonist, because of the independent existence of the mathematical concepts, it does not mean a problem to *define* a number as follows: "$l$ is the largest prime number such that $l-2$ is also a prime, or $l=1$ if there exists no number satisfying this condition." For the Platonist Gödel it does not mean a problem to talk about "the set of non-provable formulas" and compare it with "the set of the true formulas".

***Intuitionists*** do not ascribe any existence of mathematical objects independent of their (evidently finite) construction by the basic intuition. Instead, they believe in the existence of "their own god" (Curry's expression), Intuition, something which is *a priori* given to the universal human apprehension, something which, in this way, guarantees the *objectivity* and *usefulness* of mathematics.

***Realists, Platonists and intuitionists*** jointly believe, however, that mathematical concepts and propositions have *meanings*, and when we formalize the language of mathematics, in accordance with Hilbert's program, these meanings are meant to be reflected in a more precise and more concise form.

The rest of the story is known: Realism died when we cut the navel-string between mathematics and the physical world. Platonism suffers from the Gödel theorems. Intuitionism is already about to amputate a big part of the body of mathematics.

## 2.

According to the ***formalist*** understanding of mathematics (at least, according to the radical version of formalism I am proposing here) *the truth, on the contrary, is that a mathematical object has no meaning.* Mathematics is the science of formal systems: we have *symbols* and *rules* governing how these symbols can be combined. That's all. Mathematics has nothing to do with the metaphysical concept of infinity and it is totally indifferent to our intuition about space, time, probability or continuity. Mathematics does not produce and does not solve Zeno paradoxes. As Dieudonné pointed out: "I can write down a sign, say $\alpha$, and call it the cardinal number of the integers. After that I can fix rules for its manipulation." (See Heyting, 1956.) Once the notion of infinity plays a part, obscurity and confusion penetrate into the reasoning. The whole finitist struggle is unnecessary. Such a sign as $10^{10^{10}}$ has no other meaning than as a figure on the paper with which we operate according to certain rules, just like with any other symbols.

Just like chess. We are given the symbols and the rules of game. Each party is a proof of a theorem. So, deduction is a combinatorial, mechanical game.

### 3.

The ***ontology of formal systems*** is crystal clear: signs, say ink-molecules diffused among paper-molecules, more exactly, their interaction with the lightening electromagnetic field, or something like that. The rules according to which these signs are written on the paper form a strict mechanism which is, or easily can be, encoded in the regularities of real physical processes. Of course, a derivation on paper is rather similar to a production line in a factory applying low-paid manpower: at certain points of the technological process human hands (and brains) transpose the workpiece from one conveyor belt to the other. This is, however, an unimportant technical problem. Since each step of manipulation is governed by strict rules, human beings can be replaced by trained animals, robots, etc. Also the signs can be of entirely different nature, like, for instance, the cybernetic states of a computer, supervening on the underlying physical processes.

One can execute formal operations also *in head*: If the signs and the rules of a formal system can be embodied in various physical states/processes, why not let them be embodied in the neuro-physiological, biochemical, biophysical brain configurations – more exactly, in the physical processes of human brain? If this is the case, that one of the paths – as some rationalists believe, the only path – to the trustworthy knowledge, the deductive/logical thinking can be construed as a mere complex of physical phenomena, without any reference to the notions of "meaning" and "intentionality", or the vague and untenable concept of the acausal "global supervenience on the physical" (Cf. Chalmers, 1996), then it is a very strong argument for physicalism.

***"Representation"***, ***"translation"*** **and** ***"understanding"*** are three words to be avoided. As a complex of particular physical phenomena, a formal system is complete, operable, useful and metaphysically intelligible. We do not need to suppose that it "represents" anything. That would mean an unnecessary return to the realistic, Platonistic or intuitionistic philosophy of mathematics. Since formal systems do not represent anything, it cannot be the case that different formal systems would be different representations of some one common thing. Consequently, there is no "translation" between them. One formal system cannot "understand" the other.

***Interaction*** is the proper term instead of representation, translation and understanding. That is what is going on in reality: *a physical interaction* between two formal systems as two particular *physical systems*. Wittgenstein would probably call the usage of language the interaction of a brain with another formal system, first of all the interaction with another brain, which is usually realized through a third intermediate formal system. It is not our nominalism but rather an ontological clarity what keeps us from imagining "translation" or "understanding" of a "represented meaning" behind these interactions. For, from an ontological point of view it makes no difference between two situations, when a mathematician is reading a proof of a theorem written by another mathematician, and when I am reading out Tolstoy's "War and Peace" to my dog.

## 4.

It is a widespread opinion, that one cannot justify a general statement about the world by ***induction***. For the empirical fact that a statement turned out to be true in $n$ cases *does not imply* that it is also true in the $(n+1)$th case. According to this opinion, deduction, contrary to induction, provides secure confidence because it is based on pure reasoning, without referring to empirical facts.

This is the key idea of ***rationalism***. Cognition is an independent source of trustworthy knowledge. More over, it is the only secure source of knowledge, the rationalists say, because cognition is the only source of *necessary* truth. Experience cannot deliver to us necessary truths; truths completely demonstrated by reason.

Let us leave aside the epistemological valuation of knowledge we obtain through inductive inference and consider in more detail the problem of deduction. The empiricist encounters difficulties in connection with the truth of formal logic and mathematics, as Ayer writes:

> "For whereas a scientific generalization is readily admitted to be fallible, the truths of mathematics and logic appear to everyone to be necessary and certain. But if empiricism is correct no proposition which has a factual content can be necessary or certain. Accordingly the empiricist must deal with the truths of logic and mathematics in one of the following ways: he must say either that they are not necessary truths, in which case he must account for the universal conviction that they are; or he must say that they have no factual content, and then he must explain how a proposition which is empty of all factual content can be true and useful and surprising. ...
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> If neither of these courses proves satisfactory, we shall be obliged to give way to rationalism. We shall be obliged to admit that there are some truths about the world which we can know independently of experience; ..." (Ayer, 1952, p. 72.)

According to the ***mathematical realist Mill***, mathematical and logical truths are not certain and not necessary, since they are nothing but generalizations of our fundamental experiences about the physical world, and, as such, they are admitted to be fallible.

***Logical empiricists***, on the contrary, did not reject the necessity and certainty of mathematical and logical truths. According to their solution, analytical truths do not refer to the facts of reality. For we cannot obtain more information through deductive inference than that already contained in the premises. In other words, according to the logical empiricism, there are no synthetic a priori statements.

***Popper's falsification principle*** also accepts the necessity and certainty of mathematical and logical truths. This is the basis of the principal distinction between induction and deduction. Similarly, this principal distinction between the "trustworthy deductive inference" and the "always uncertain inductive generalization" is the fundamental tenet upon which the widely accepted hypothetico-deductive and Bayesian theories of science are built up, seemingly eliminating the problem of induction.

## 5.

From the standpoint of ***radical formalism***, one can arrive at the following conclusion: The mathematical and logical truths are *not necessary and not certain, but they do have factual content referring to the real world.* It must be emphasized, however, that this reference to the physical world is of nature completely different from that assumed by Mill in his realistic philosophy of mathematics. The point is that "deduction" is a concept which is meaningful only in a given formal system. On the other hand, as it has been already pointed out, a formal system is nothing but a complex of signs and rules, that is, real physical objects and real physical mechanisms. Therefore, a derivation is the *observation* of a real process going on in a concrete physical system. Consequently, the certainty of mathematics, that is the degree of certainty with which one can know the outcome of a deductive process, is the same as the degree of certainty of the knowledge about the outcomes of any other physical processes. In order to explain the universal conviction that mathematical and logical truths are necessary and certain, notice that there are many elements of our knowledge about the world which *seem* to be necessary and certain, although they are obtained from inductive generalization. If we need a shorter stick, we break a long one. We are "sure" about the outcome of such an operation: the result is a shorter stick. This regularity of the physical world is known to us from the experiences. It can be known also for a chimp, from its own experiences obtained by trying to use a long stick. The certainty of this knowledge is, however, not less than the certainty of the inference from the Euclidean axioms to the height theorem. *Deduction is a particular case of induction.* Mathematical and logical truths are nothing but knowledge obtained through inductive generalization from the experiences with a particular physical system: this system is the formal system itself. The formal system itself is that part of physical reality the mathematical and logical truths refer to. The formal system itself is that part of the physical world the properties of which are reflected by the formal propositions. The reason why the truth of the height theorem is uncertain is not that our knowledge about the properties of the "real triangles" is uncertain, as Mill takes it, but rather that our knowledge about the deductive (physical) process, the outcome of which is the height theorem, is uncertain, no matter how many times we repeat the observation of this process.

Mathematics and logic are, in this sense, empirical sciences. Reasoning is, if you like, a neuro-physiological experiment. So, contrary to Leibniz's position that

> "There are two kinds of truths: those of reasoning and those of fact. The truths of reasoning are necessary and their opposite is impossible; the truths of fact are contingent and their opposites are possible." (Monadology)

we must draw the following epistemological conclusion: *The certainty available in inductive generalization is the best of all possible certainties!*

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