Codes:	BMA-LOTD-308, BMI-LOTD-308E
Course:	Logic of Time and Spacetime
Teacher:	Attila Molnár
Location and time:	I/224, Tue 16:00-17:30
Consultation:	Tue, 17:30 – 18:00 and online, molnar.attila@szlgbp.hu, https://molnatt.github.io/temporallogic
First occasion:	February 11, 2025, 12:00 – 14:00

Courses required:	Logic seminar and lecture
Requirements:	One of the following three options: Homeworks, oral exam, participation at a workshop at the end of the year. (The workshop participants present articles or own research. To ensure the success of this, regular consultation will be mandatory. The articles can be chosen from the papers suggested for further readings.)

Description: Basic temporal logics are modal logics with two modalities:

$\mathbf{F}\varphi$: It will at some time be the case that φ", and
$\mathbf{P}\varphi$: It has at some time been the case that φ".

In models of temporal logic, possible worlds are *moments*, and the accessibility relation that connects the worlds is a *linear ordering* of these moments. Such a linear structure is often called a "flow of time."
Due to this linearity, the truth values of statements like:

There will be a sea battle tomorrow.

are already determined. To create a temporal logic that allows for *indeterminism*, we need to consider more general structures, such as trees. In a tree, different branches represent alternative continuations of the present – on some branches, there will be a sea battle, while on others, there will not.

However, even this model poses challenges. Over a hundred years ago, we learned that time and simultaneity are relative to observers. For example, Alice may be certain (based on her measurements) that two events are simultaneous, while Bob, using a different frame of reference, might disagree. This relativity fundamentally arises from the fact that the speed of light is constant for all observers – a principle that forms the cornerstone of the *special theory of relativity*.[a]

To adapt temporal logic to account for relativistic phenomena, we must relativize it to observers: Alice's flow of time, Bob's flow of time, and so on. But this seems paradoxical. Even if Alice and Bob disagree on which event happened first, shouldn't the universe itself have an objective view? Such disagreements, which are *epistemic*, should not affect the *ontological* status of events. Is there any room for objective indeterminism within the framework of special relativity?

When examining the works of physicists, we find that spacetime models are generally deterministic: there are no alternative possibilities, no *branching spacetimes* in standard physics.

Our goals in this course are:

- To provide an introduction to **Propositional Temporal Logic**, with an emphasis on **completeness proofs**. We will explore both deterministic and indeterministic frameworks (Ockhamist and Peircean approaches to temporal semantics) and discuss the representation of **choice and agency** (stit-semantics) within these systems.
- To introduce **special relativity** to those with no prior knowledge of the subject, we will follow the approach of the Andréka-Németi school.
- To provide a basic introduction to the concept of branching spacetime.

Course materials (slides, PDFs) will be available online throughout the course.

[a] This principle is continually confirmed by our GPS systems. If GPS systems were calibrated using Newtonian physics (where time and simultaneity are absolute), the resulting errors would accumulate to approximately 10 km/day.

Bibliography:

[1] H. Andréka, J. X. Madarász és I. Németi. “Logic of space-time and relativity theory”. *Handbook of spatial logics*. Szerk. M. Aiello, I. Pratt-Hartmann és J. van Benthem. Dordrecht: Springer-Verlag, 2007, 607–711. old.

[2] N. Belnap, T. Müller és T. Placek. “New Foundations for Branching Space-Times”. *Studia Logica* 109.2 (2020), 239–284. old. DOI: 10.1007/s11225-020-09905-2.

[3] Nuel Belnap. “Branching Space-Time”. *Synthese* 92.3 (1992), 385–434. old.

[4] P. Blackburn, M. de Rijke és Y. Venema. *Modal Logic*. Cambridge: Cambridge University Press, 2001. ISBN: 0521527147, 9780521527149.

[5] Roberto Ciuni és Alberto Zanardo. “Completeness of a Branching-Time Logic with Possible Choices”. *Studia Logica* 96 (2010), 393–420. old.

[6] Mark Reynolds. “Axioms for Branching Time”. *Journal of Logic and Computation* 12.4 (2002), 679–697. old. DOI: 10.1093/logcom/12.4.679. URL: https://www.researchgate.net/publication/220388427_Axioms_for_Branching_Time.

[7] Antje Rumberg. “Transition Semantics for Branching Time”. *Journal of Logic, Language and Information* 25.1 (2016), 77–108. old. DOI: 10.1007/s10849-015-9231-6.

[8] Richmond H. Thomason. “Indeterminist Time and Truth-Value Gaps”. *Theoria* 36.3 (1970), 264–281. old.

[9] Alberto Zanardo. “A Finite Axiomatization of the Set of Strongly Valid Ockhamist Formulas”. *Journal of Philosophical Logic* 14.4 (1985), 447–468. old. DOI: 10.1007/bf00649485.

Further readings:

[1] Ferenc Altrichter. *Észérvek az európai filozófiai hagyományban*. Budapest: Atlantisz, 1993, 289–327. old. ISBN: 963-79-7835-6.

[2] H. Andréka és tsai. “A logic road from special relativity to general relativity”. *Synthese* 186.3 (2012), 633–649. old. ISSN: 0039-7857. URL: http://dx.doi.org/10.1007/s11229-011-9914-8.

[3] H. Andréka és tsai. “Axiomatizing relativistic dynamics without conservation postulates”. *Studia Logica* 89.2 (2008), 163–186. old. ISSN: 0039-3215.

[4] Nuel Belnap, Thomas Müller és Tomasz Placek. *Branching Space-Times: Theory and Applications*. New York: Oxford University Press, 2022. ISBN: 978-0-19-088431-4. DOI: 10.1093/oso/9780190884314.001.0001. URL: https://global.oup.com/academic/product/branching-space-times-9780190884314.

[5] Robert Goldblatt. “Diodorean Modality in Minkowski Spacetime”. *Studia Logica* 39.2-3 (1980), 219–236. old.

[6] J. X. Madarász és G. Székely. “Comparing Relativistic And Newtonian Dynamics In First Order Logic”. *Wiener Kreis und Ungarn*. Szerk. F. Stadler. To appear. Veröffentlishungen des Instituts Wiener Kreis, Vienna, 2009.

[7] Axel Rumberg és Alberto Zanardo. “First-Order Definability of Transition Structures”. *Journal of Logic, Language and Information* 28.3 (2019), 459–488. old.

[8] Imre Ruzsa. *Logikai szintaxis és szemantika*. Budapest: Akadémiai Kiadó, 1988. ISBN: 963-05-5313-9.

[9] László E. Szabó. *A nyitott jövő problémája*. Budapest: Typotex, 2002.

[10] Alberto Zanardo, Bruno Barcellan és Mark Reynolds. “Non-Definability of the Class of Complete Bundled Trees”. *Logic Journal of the IGPL* 7.1 (1999). Special Issue on Temporal Logic, 125–136. old.