Models of arithmetic

2025 Fall

Lecturer: Zalán Molnár

Code of the course: BMA-LOTD-305.03, BMA-LOTD-305E.03

Aim of the course: This is a half-way course between a reading seminar and a lecture introducing participants to models of arithmetic. The course is planned to be an introduction to the field, however we presuppose some knowledge from model theory and recursion theory/Gödel's incompleteness theorems. Thus, it can be seen as a continuation of these courses.

Content of the course: We elaborate on topics including:

- Overspill-underspill principles, least number principle, collection principle
- Theory of PA^-, Parikh's Theorem
- End-extensions, conservative extensions, cofinal extensions
- Gaifman's Splitting Theorem
- Atomic models of PA, MacDowell-Specker Theorem
- Partial truth predicates, PA is not finitely axiomatizable (Ryll-Nardewski)
- Theory of coded sets, Tennenbaum's Theorem
- Bounded recursive saturation, Friedman's Embedding Theorem
- Decidability of Presburger Arithmetic, definabile subsets of Presburger Arithmetic

Grading criteria: Students have to work on and elaborate certain parts of the material by themselves. The grading will be based on these presentations.

References

[1] Pavel Pudlák, Petr Hájek. *Metamathematics of first-order arithmetic* Cambridge. 1993

[2] László Csirmaz, Zalán Gyenis. *Mathematical logic. Exercises and solutions.* Springer. 2022.

[3] Richard Kaye. *Models of Peano Arithmetic.* Oxford University Press. 1991.

[4] Jaap van Oosten. *Introduction to Peano Arithmetic.* Lecture notes. 1999.