

Metalogic

2026 Fall

Lecturer: Zalán Molnár

Code of the course: BMA-LOTD-305.03, BMA-LOTD-305E.03

Aim of the course: This course provides an introduction to metalogic. Metalogic is the study of the properties and foundations of formal systems. Rather than using these to prove statements about a particular objects, metalogic examines logic itself. We use and develop results from computability and model theory in order to study its limits, expressiveness and properties reflecting to the question of what can and cannot be formally proved.

The course assumes basic familiarity with logic, including the Completeness and Compactness Theorems, as well as basic knowledge of computability theory universal (partial) recursive function, Halting problem, Rice's Theorem and related concepts. The course can be seen as a continuation of the Computability Theory course.

After a brief review of the main concepts and results from logic and computability theory we will use, the following topics will be explored.

Content of the course: We elaborate on topics including:

- Gödel's incompleteness theorems: Encoding syntax, Representation of recursive functions, Constructing the Gödel sentence, Tarski's undefinability of truth, arithmetic hierarchy,
- Introduction to the models of Peano Arithmetic, Tennenbaum's theorem, PA is not finitely axiomatizable
- Tarski's semantic embedding, undecidability of set theory, graph theory, group theory, ring theory.
- Trakhtenbrot's theorem, basics of finite model theory
- undecidability of tiling problems and applications.

Grading criteria: Oral exam

References

- [1] N. Cutland *Computability Theory: An introduction to recursive functions*, Cambridge. 1984.
- [2] Jaap van Oosten. *Introduction to Peano Arithmetic*. Lecture notes. 1999.
- [3] László Csirmaz, Zalán Gyenis *Mathematical logic. Exercises and solutions*. Springer. 2022.
- [4] Lecture Notes on Computability