Absolute Theory of Space and Time

69. Faithfully reflecting how "space" and "time" tags are understood in classical physics and relativity theory, definitions (D1)–(D8) in Point **38** answered the purpose of demonstrating that Einstein's special relativity has exactly the same claims about space and time as classical physics and Lorentz's theory. However, neither the classical nor the relativistic definitions are trouble free. They are based on several pre-assumptions about contingent facts of nature which cannot be known or even formulated prior to the definitions of space and time tags.

Let us focus on what is common to both the classical and relativistic approaches, definitions (D1)–(D4). The first difficulty is caused by the usage of measuring rod for the definition of distance. The problem I mean is different from the one proposed by Reichenbach (1958), namely that the length of the rod may be altered by some universal forces when the rod is transported from one place to the another. This—known, or unkwon—behavior of the *etalon* is no problem from logical/operational point of view, as long as the operational procedure provides an unambiguous definition. (For example, we are completely aware of the Lorentz contraction of the measuring rod. But this is no problem; procedure (D8) in Point **38** provides an unambiguous definition of $\widetilde{\text{space}}$ tags $\widetilde{x}^{K'}(A)$.) In accordance with Reichenbach's final conclusion, I believe that the Newtonian concept of "absolute length" (see Point **73**) of the rod, independent of operational definition of "distance", is meaningless or at least is outside of the scope of physics. If space tags are defined according to (D2) then the length of the measuring rod is—by definition—constant, no matter what is our metaphysical pre-assumption about the length of the rod *ansich*.

There are, however, real circularities here that appear at the very operational level. The operations described in (D2) and (D4) rest on the concept of a measuring rod at rest relative to a given reference frame. However, we encounter the following difficulties:

(a) We have seen in Point **24** that the concept of a rod "at rest" relative to a reference frame is problematic in itself.

(b) One might think that this is no problem if the measuring rod is always in equilibrium state when we are measuring with it. It must be clear however that the equilibrium state of a rod cannot be ascertained prior to the definition of its length, that is, prior to the definition of distance.

(c) The concept of rest relative to a reference frame is problematic not only for the measuring rod as a whole but even for one single particle of the rod. The reason is that we are missing a prior definition of velocity relative to a given reference frame.

(d) Throughout definitions (D1)–(D9) we nonchalantly used the term "reference frame". Of course, it is no problem to give the usual meaning to this term *after* having defined space and time tags of events; when we already have the concepts of simultaneity, the distance of simultaneous events, dimensions, straight lines, etc. But the term "reference frame" has no meaning prior to the space and time tags. We encounter this wrong circularity in definitions (D2) and (D4): we ought to superpose the measuring-rod along a straight line, such that the rod is always at rest relative to the reference frame.

(e) We also used the term "inertial" frame of reference. This is another term that has no meaning without a previous definition of space and time tags.

70. Another source of circularities is the "slow transportation" of the standard clock in definitions (D1) and (D3). The reason why the transportation must be slow is that the clock may accumulate a loss of phase during its journey. From (56) we can

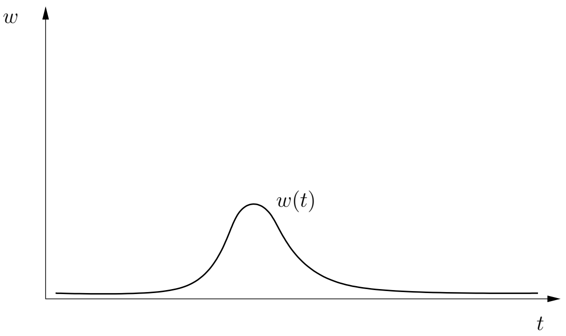

Figure 13: *Velocity may vary such that the clock's journey takes very long time, nevertheless the integral in (88) is less than t*

express this phase shift:

$$\Delta T = t - \int_0^t \sqrt{1 - \frac{w(\tau)^2}{c^2}}\, d\tau \tag{88}$$

where $w(t)$ is the clock's velocity during its journey. Of course, $\Delta T \to 0$ if $w(t)$ tends to zero in some uniform sense, for instance if $max\ |w(t)| \to 0$. One might think that this condition can be provided without any vicious circularity by moving the standard clock from its original place to the locus of the event in question over a very long period of time, according to the reading of the clock itself. This is however not the case. If function $w(t)$ is something like as shown in Fig. 13 then the clock's journey takes very long time, nevertheless the loss of phase in (88) does not vanish. Yet one might also think that this does not cause a vicious circularity in the operational definition of time tags, because we can include the loss of phase into the definition, just like in the relativistic definition (D6). (In definition (D6), the time tag is simply defined by the reading of the clock, disregarding the loss of phase accumulated during its journey. This phase shift—calculated in Point **42**—is, for example, the origin of the difference between $\hat{t}$-simultaneity and $\widetilde{t}$-simultaneity.) However, this operation could not provide an unambiguous definition of time tags. The reason is that the phase shift (consequently, the reading) of the clock depends on the concrete shape of function $w(t)$. To keep $w(t)$ controlled—either in order to avoid ambiguity, or in order to provide the condition $max\ |w(t)| \rightarrow 0$—we must be able to ascertain the clock's instantaneous velocity relative to reference frame K, throughout its journey. And this leads to the same vicious circularities we mentioned in Point **69** (c) and (d).

71. One has to recognize that some of the circularity problems are independent of the relativistic effects and they are already there in classical physics. Let me illustrate this with one example. Assume, the time tags of events are somehow defined by transported clocks. So we have the concept of "space" S_t, that is the set of simultaneous events at a given time t. The congruence of space intervals in S_t is traditionally defined by means of transportation of rigid bodies. There has been a long discussion about the conventionality of the concept of congruence so obtained (Poincaré 1952; Einstein 1969b; Reichenbach 1958; Grünbaum 1974; Friedman 1983). But nobody contested that the operational definition in itself is meaningful and applicable for the coordination of (classical) space-time. In fact, as a little reflection reveals, this is not the case; the definition of congruence by means of transportation of rigid bodies contains an operational circularity. For, assume that a rigid body indeed "retains its size" during the transportation; its size before the transportation, at t_0, is equal to its size after the transportation, at time t_1. (In some objective sense or/and by convention—it does not matter now.) This only means, however, that its size in S_{t_0} is "congruent" with its size in S_{t_1} (Fig. 14). So, in order to establish, in this way, the concept of congruence in space S_{t_0}, we need a previous definition of "rest", that is, a previous concept of identity of two locuses of space at two different times. But, in this construction, there seems no way to define the concept of

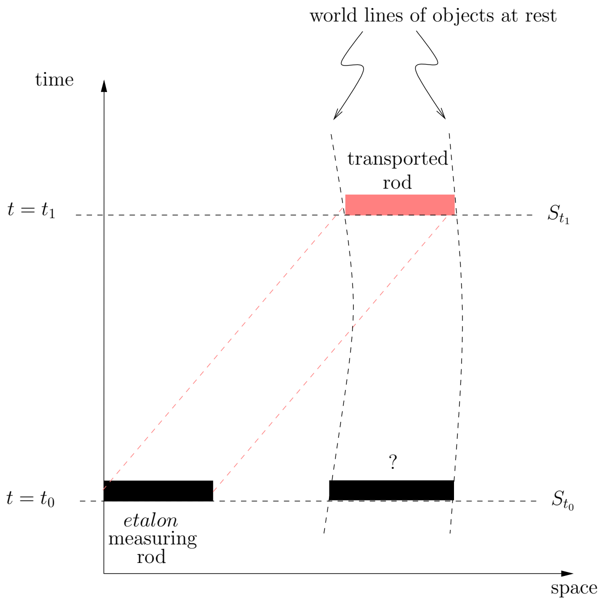

Figure 14: The definition of spatial congruence by means of transportation of rigid bodies is based on a previous definition of "rest", that is, on a previous concept of identity of two locuses of space at two different times

"rest" without the concept of congruence of spatial intervals in every S_t.

72. The upshot of these considerations is that, in order to avoid the circularities mentioned above and to minimize the conventional elements in the empirical foundation of our physical theory of space and time, we must avoid using standard measuring rod in the definition of distance and using slow transportation of the standard clock in the definition of time tags. We must also abstain from relying on the concept of rigid body, reference frame, and inertial motion.

Instead, we will use one standard clock and light signals. A light signal should not be understood as a "light ray" or a "light beam", that is, we should not assume—in advance—that the light signal propagates along a "straight line".

Empirical Definition of Space and Time Tags

73. First we chose an *etalon* clock. That is to say, we chose a system (a sequence of phenomena) floating somewhere in the universe. Without loss of generality we may stipulate that this is an equipment having a pointer and the readings are real numbers. (For example, let the clock in the U.S. Naval Observatory, used by the GPS.) There is no assumption that this is a clock measuring "proper time". There is no assumption that it "runs uniformly". And there is no assumption that it is "at rest" relative to anything, or that it is of "inertial motion". The reason is that none of these concepts is defined yet.

Consider the experimental arrangement in Fig. 15. The standard clock emits a radio signal at clock-reading t_1 (event A). The signal is received by another equipment (marker) which immediately emits another signal (event B). This "reflected" signal is detected by the standard clock at t_2 (event C). Without loss of generality we may assume that these signals are modulated radio waves, containing some minimal information to identify

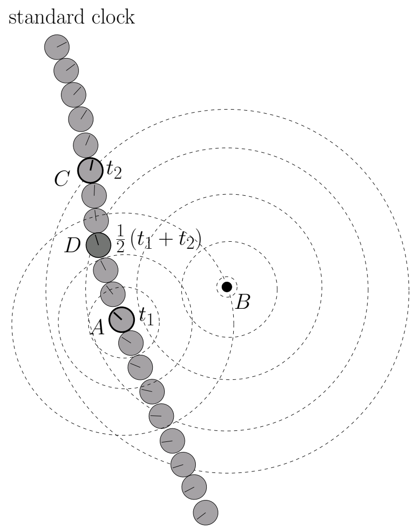

Figure 15: *Operational definition of time tags. (This is just a symbolic sketch, not a real "two dimensional space-time diagram" or the like.)*

them. We also assume, as an empirical fact, that the clock we have chosen is such that a given reflected signal is received by the standard clock only once, at reading t_2, and

$$t_2 \geq t_1 \tag{89}$$

by which we have chosen, conventionally, an "arrow of time" (not the arrow of physical processes in time; see Price 1996, p. 16 and 58). (In fact, we made two choices here. One is the choice of the direction of the parametrization of the clock's pointer positions (89). There is however a more important one: by applying the terms "sending" and "receiving" a signal, we previously determined the causal order of events A and C. To what extent this causal order is purely conventional? How can we—without prior spatiotemporal conceptions—distinguish whether an event is a "sending" or a "receiving" of a signal? How is this choice of causal order related to the change of information content of the signal? To what extent this choice is determined by our free will and free action experience at the modulation of the radio waves? Is this freedom an objective openness of future or merely a subjective experience? These are delicate metaphysical question; into the discussion of which it is not our present purpose to enter.)

Definition (A1) The *absolute time* tag of event B is the following:

$$\tau(B) := t_1 + \frac{1}{2}(t_2 - t_1) \tag{90}$$

The definition is about event B consisting in the "reflection" of the radio signal emitted by the standard clock. That is to say, we assigned an absolute time tag to a definite physical phenomenon which we called "event". It is far from obvious, however, what must be regarded as an event in general—prior to the concepts of time and distance—, and how one can extend the definition for the physical events of other kinds. (See Brown 2005, pp. 11-14.) We do not dwell on this problem here. The reader can easily imagine various operational solutions of how to use the B-type "reflection" events for marking other physical events/phenomena. So we will assume that definition (A1) is extended for all physical events.

74. At this point, one might think that we are ready to define the distance between simultaneous events in the usual way. Surely, we can define the distance between the simultaneous events D and B (Fig. 15) as $\frac{1}{2}(t_2 - t_1)\,c$, where the value of c is taken as a convention. In this way, however, we can define the distance only *from the standard clock*. But there is no way to extend this definition for arbitrary pair of simultaneous events. In order to define the distance between arbitrary simultaneous evens we need further preparations.

Denote S_τ the set of simultaneous events with time tag τ.

Definition (A2) A one-parameter family of events $\gamma(\tau)$ is called *time sequence* if $\gamma(\tau) \in S_\tau$ for all τ.

One has to recognize that a time sequence is a clock-like process. For every event, one can define a time-like tag in the same way as (A1): Event A (Fig. 16) is marked with the emission of a radio signal at time $\tau(A)$. The signal is reflected at event B. Event C is the first detection of the reflected signal at time $\tau(C)$. We define the following time-like tag for event B:

$$\tau^\gamma(B) := \tau(A) + \frac{1}{2}\left(\tau(C) - \tau(A)\right)$$

(If there is no detection of the reflected signal at all, then, say, $\tau^\gamma(B) := \infty$.)

It is an empirical fact that $\tau^\gamma(B) \neq \tau(B)$ in general. It is another empirical observation however that for some particular cases $\tau^\gamma(B) = \tau(B)$.

Definition (A3) A time sequence $\gamma(\tau)$ is a *synchronized copy of the standard clock* if for every event B $\tau^\gamma(B) = \tau(B)$.

Whether or not there exist synchronized copies of the standard clock is an empirical question. We stipulate the following:

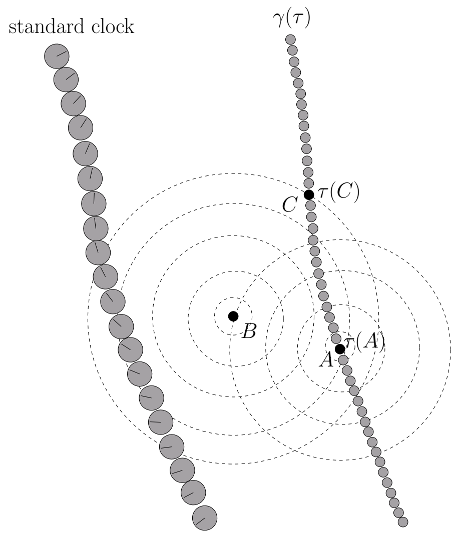

Figure 16: *Clock-like time sequence*

Empirical fact (E1) For any event A there exists a unique synchronized copy of the standard clock $\gamma(\tau)$ such that

$$A = \gamma(\tau(A))$$

75. Now we are ready to define the distance between simultaneous events.

Definition (A4) The *absolute distance* between two simultaneous evens $A, B \in S_\tau$ is operationally defined in the following way. Take a synchronized copy of the standard clock γ such that $A = \gamma(\tau)$. (See Fig. 17) Let $U = \gamma(\tau(U))$ is an event marked with the emission of a radio signal at absolute time $\tau(U)$, such that the signal is received and reflected at event B. The detection of the reflected signal marks the event $V = \gamma(\tau(V))$ of time

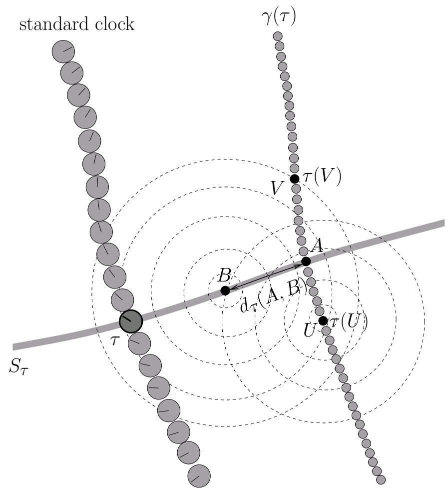

Figure 17: *The distance between two simultaneous events*

tag $\tau(V)$. The absolute distance is

$$d_\tau(A,B) := \frac{1}{2}\left(\tau(V) - \tau(U)\right) c \tag{91}$$

where $c = 299792458\frac{m}{s}$ by convention.

76. We know from (89) that for all $A, B \in S_\tau$

$$d_\tau(A,B) \geq 0 \tag{92}$$
$$d_\tau(A,A) = 0 \tag{93}$$

However, the following fact cannot be known *a priori*:

Empirical fact (E2) For all $A, B, C \in S_\tau$

$$d_\tau(A,B) + d_\tau(B,C) \geq d_\tau(A,C) \tag{94}$$

Some other propositions are however derivable from the definitions.

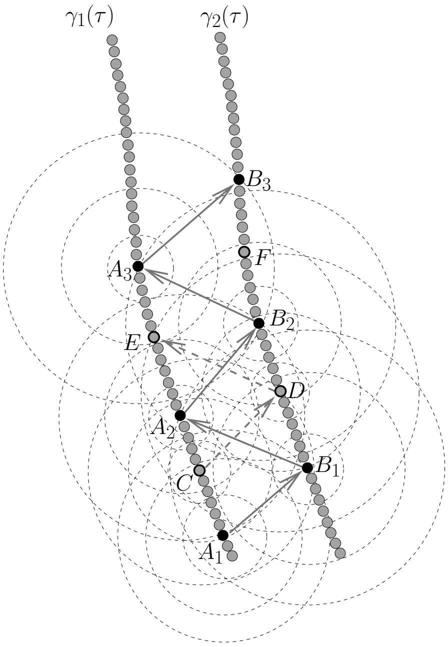

Figure 18: *The distance between the simultaneous points of two synchronized copies of the standard clock is a periodic function of the absolute time*

Lemma 1 Consider two synchronized copies of the standard clock γ_1 and γ_2 (Fig. 18). For any moment of absolute time τ_0

$$d_{\tau_0}\left(\gamma_1(\tau_0), \gamma_2(\tau_0)\right) = d_{\tau_0}\left(\gamma_2(\tau_0), \gamma_1(\tau_0)\right) \tag{95}$$

and

$$d_{\tau_0}\left(\gamma_1(\tau_0), \gamma_2(\tau_0)\right) = d_{\tau_0+T}\left(\gamma_1(\tau_0+T), \gamma_2(\tau_0+T)\right) \tag{96}$$

where

$$T = \frac{d_{\tau_0}\left(\gamma_1(\tau_0), \gamma_2(\tau_0)\right)}{c}$$

Proof Let $\gamma_1(\tau_0)$ be event A_2. Consider the following events: a radio signal is emitted at A_1, then reflected at B_1, then it is reflected again at A_2 and reflected again at B_2, and so on. Let $\tau(E) = \tau(B_2)$ and $\tau(C) = \tau(B_1)$. Taking into account that both γ_1 and γ_2 are synchronized copies of the standard clock, we have the following equations:

$$\begin{aligned} \tau(A_2) &= \frac{\tau(B_2) + \tau(B_1)}{2} \\ \tau(B_2) &= \frac{\tau(A_3) + \tau(A_2)}{2} \\ \tau(B_1) &= \frac{\tau(A_2) + \tau(A_1)}{2} \end{aligned}$$

From the above three equations we have

$$\tau(A_3) - \tau(A_2) = \tau(A_2) - \tau(A_1) \tag{97}$$

and

$$\tau(B_2) - \tau(B_1) = \tau(A_2) - \tau(A_1) \tag{98}$$

Therefore,

$$\tau(E) - \tau(C) = \tau(A_2) - \tau(A_1) = \tau(B_2) - \tau(B_1)$$

Imagine now a radio signal emitted from C, reflected at D and detected at E. Again, taking into account that both γ_1 and γ_2 are synchronized copies of the standard clock, we have

$$\tau(D) = \frac{\tau(E) + \tau(C)}{2} = \frac{\tau(B_2) + \tau(B_1)}{2} = \tau(A_2) = \tau_0$$

Therefore,

$$\begin{aligned} d_{\tau_0}(\gamma_1(\tau_0), \gamma_2(\tau_0)) &= \frac{\tau(E) - \tau(C)}{2} c \\ &= \frac{\tau(B_2) - \tau(B_1)}{2} c \\ &= d_{\tau_0}(\gamma_2(\tau_0), \gamma_1(\tau_0)) \end{aligned}$$

Taking into account this symmetry, (96) immediately follows from (97). ■

In other words, as it follows from (95), for any $A, B \in S_\tau$

$$d_\tau(A, B) = d_\tau(B, A) \tag{99}$$

One has to recognize that a function $S_\tau \times S_\tau \to \mathbb{R}$ with properties (92)–(94) and (99) is what the mathematician calls metric on S_τ. Thus, we can stipulate that (S_τ, d_τ) is a metric space for every moment of absolute time τ.

77. Having metric defined on S_τ, we can define the concept of a straight line in S_τ (Fig. 19).

Definition (A5) *A* subset $\sigma \subset S_\tau$ is called (straight) *line* if satisfies the following conditions:

1. for any $A, B, C \in \sigma$ exactly one of the following three relations hold:

$$\begin{aligned} d_\tau(A, C) + d_\tau(C, B) &= d_\tau(A, B) \\ d_\tau(A, B) + d_\tau(B, C) &= d_\tau(A, C) \\ d_\tau(B, A) + d_\tau(A, C) &= d_\tau(B, C) \end{aligned}$$

2. σ is maximal for property 1.

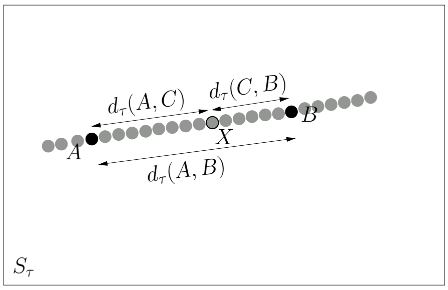

Figure 19: *Straight line*

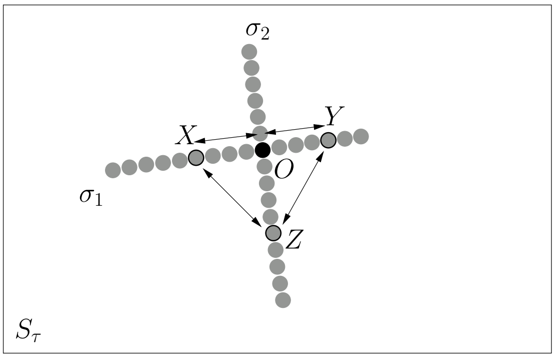

Figure 20: *Orthogonal lines*

Empirical fact (E3) For every $A, B \in S_\tau$ there exists a unique line containing A and B.

Definition (A6) Let σ_1 and σ_2 two lines in S_τ such that $\sigma_1 \cap \sigma_2 = \{O\}$ (see Fig. 20). σ_2 is *orthogonal* to σ_1 if for every $Z \in \sigma_2$ and for every $X, Y \in \sigma_1$

$$d_\tau(X,O) = d_\tau(O,Y) \Leftrightarrow d_\tau(X,Z) = d_\tau(Y,Z)$$

Empirical fact (E4) If σ_1 is orthogonal to σ_2 then σ_2 is orthogonal to σ_1.

Empirical fact (E5) For every $O \in S_\tau$ there exist three lines σ_1,σ_2 and σ_3 such that they are pairwise orthogonal and $\sigma_1 \cap \sigma_2 \cap \sigma_3 = \{O\}$.

Empirical fact (E6) Let $O \in S_\tau$ an arbitrary event and three lines σ_1,σ_2 and σ_3 such that they are pairwise orthogonal and $\sigma_1 \cap \sigma_2 \cap \sigma_3 = \{O\}$. There is no line $\sigma \subset S_\tau$ orthogonal to each of σ_1,σ_2 and σ_3, such that $\sigma_1 \cap \sigma_2 \cap \sigma_3 \cap \sigma = \{O\}$.

We usually express this fact by saying that space is three dimensional.

Empirical fact (E7) Let $A \in S_\tau$ be an arbitrary event and $\sigma_1 \subset S_\tau$ an arbitrary line. There always exists a line σ_2 orthogonal to σ_1, such that $A \in \sigma_2$.

Definition (A7) Using the notations in (E7), let $\sigma_1 \cap \sigma_2 = \{O\}$. Distance of $d_\tau(A, O)$ is called *the distance of* A *from* σ_1.

Definition (A8) Let $\sigma_1 \subset S_\tau$ be a line. A line σ_2 is *parallel* to σ_1 if for all $X \in \sigma_2$ the distance of X from σ_1 is the same.

Empirical fact (E8) Let $\sigma_1 \subset S_\tau$ be a line and let $C \in S_\tau$ an arbitrary event. There exists exactly one line σ_2 such that $C \in \sigma_2$ and σ_2 is parallel to σ_1.

Definition (A9) Let $A, B \in \sigma$ two events on line σ. *Line segment* between events $A, B \in S_\tau$ is the following subset of σ:

$$\sigma(A, B) := \{ X \in \sigma |\, d_\tau(A, X) + d_\tau(X, B) = d_\tau(A, B)\} \qquad (100)$$

78. Now, we have everything at hand to define the usual Cartesian coordinates in S_τ. First we need a 3-frame.

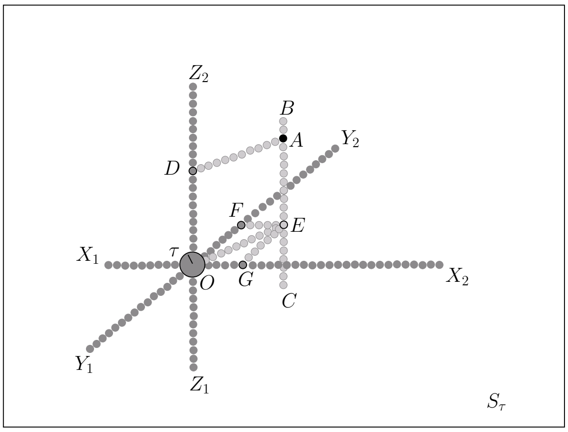

Figure 21: *Cartesian coordinates in S_τ*

Definition (A10) A *3-frame* in S_τ consists of three pairwise orthogonal line segments, $\sigma\,(Y_1, Y_2)$, $\sigma\,(Z_1, Z_2)$, such that

$$\sigma\,(X_1, X_2) \cap \sigma\,(Y_1, Y_2) \cap \sigma\,(Z_1, Z_2) = \{O\}$$

where O is the origin of the frame (Fig. 21).

The end points play marginal role, but we do not assume that these segments have "infinite" length. The segments are supposed to be long enough for the purposes of the empirical coordination of the physical events in question. The origin of the 3-frame is arbitrary, although it could be a natural choice to take the "τ-event" of the standard clock as origin.

In the following definition we give the operational definition of the three absolute space tags of an event $A \in S_\tau$.

Definition (A11) Take a line segment $\sigma(B, C) \ni A$ parallel to $\sigma\,(Z_1, Z_2)$. (See Fig. 21.) Take another line segment $\sigma(A, D)$ orthogonal to $\sigma\,(Z_1, Z_2)$ such that $D \in \sigma\,(Z_1, Z_2)$. Let $\sigma(O, E)$ be a line segment parallel to $\sigma(A, D)$ such that $E \in \sigma(B, C)$. Finally, take the line segments $\sigma(E, F)$ and $\sigma(E, G)$ such that

$\sigma(E,F)$ is parallel to $\sigma(X_1,X_2)$ and $F \in \sigma(Y_1,Y_2)$, and $\sigma(E,G)$ is parallel to $\sigma(Y_1,Y_2)$ and $G \in \sigma(X_1,X_2)$. Now, the space tags are defined as follows:

$$x_\tau(A) := \begin{cases} d_\tau(G,O) & \text{if} \quad G \in \sigma(O,X_2) \\ -d_\tau(G,O) & \text{if} \quad G \in \sigma(O,X_1) \end{cases}$$

$$y_\tau(A) := \begin{cases} d_\tau(F,O) & \text{if} \quad F \in \sigma(O,Y_2) \\ -d_\tau(F,O) & \text{if} \quad F \in \sigma(O,Y_1) \end{cases}$$

$$z_\tau(A) := \begin{cases} d_\tau(D,O) & \text{if} \quad D \in \sigma(O,Z_2) \\ -d_\tau(D,O) & \text{if} \quad D \in \sigma(O,Z_1) \end{cases}$$

79. It must be emphasized that with the above definitions we only defined the space tags in a given set of simultaneous events S_τ. Yet, we have no connection whatsoever between two S_τ and $S_{\tau'}$ if $\tau \neq \tau'$. In principle, there exist infinitely many possible bijections between the different S_τ's, but without any natural physical meaning. This is true, even if we prescribe that the bijection must be an isomorphism preserving distances.

According to some vague intuition, a time sequence $\gamma(\tau)$ satisfying that

$$x_\tau(\gamma(\tau)) = \text{const.} \tag{101}$$

$$y_\tau(\gamma(\tau)) = \text{const.} \tag{102}$$

$$z_\tau(\gamma(\tau)) = \text{const.} \tag{103}$$

corresponds to a localized physical object being at rest. “At rest”—relative to what? The actual behavior described by these equations very much depends on how the different 3-frames are chosen in the different S_τ's. One might think that an object is at rest if equations (101)–(103) hold in one and the same 3-frame in all S_τ. But, what does it mean that “one and the same 3-frame in all S_τ”? When can we say that a line segment $\sigma(X_1',X_2')$ in

$S_{\tau'}$ is *the same* 3-frame axis as $\sigma(X_1, X_2)$ in S_τ? When can we say that an event A' is in the same place in $S_{\tau'}$ as event A in S_τ?

In asking these questions, it is necessary to be careful of a possible misunderstanding. Although they are close to each other, the problem we are addressing here is different from the problem of persistence of physical objects (Butterfield 2005). What we would like to define is the identity of two locuses of space at two different times, and not the genidentity of the physical objects occupying them. One might think that some definition of genidentity of physical objects must be prior to our operational definition of space and time tags, at least in the case of the standard clock. This is, however, not necessarily the case. The standard clock is just an ordered (ordered by the clock readings) sequence of physical events, but without the further metaphysical assumption that these events belong to the same physical object. (We definitely do not make such assumption in the case of a synchronized copy of the standard clock.)

80. In order to establish connection between arbitrary two sets of simultaneous events we need some preparations.

Lemma 2 Let γ_1 and γ_2 be arbitrary two synchronized copies of the standard clock. For any two moments of absolute time τ and τ'

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) = d_{\tau'}\left(\gamma_1(\tau'), \gamma_2(\tau')\right) \tag{104}$$

Proof The proof will be based on (96). Let us assume that $\tau < \tau'$. Denote T the period in (96), that is

$$T = \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{c}$$

First we will prove that

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \geq d_{\tau'}\left(\gamma_1(\tau'), \gamma_2(\tau')\right)$$

Let n be the smallest integer such that $\tau' < \tau + nT =: \tau_1$ (Fig. 22). It follows from (96) that

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) = d_{\tau_1}(\gamma_1(\tau_1), \gamma_2(\tau_1))$$

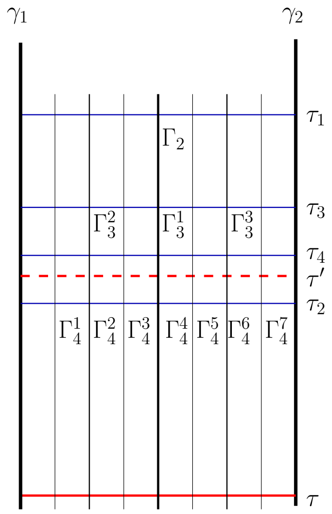

Figure 22: *Proof of Lemma 2*

Let $\tau_2 := \frac{\tau_1+\tau}{2}$. Consider the synchronized copy of the standard clock Γ_2 that goes through the middle point of line segment $\sigma(\gamma_1(\tau), \gamma_2(\tau))$. Taking into account that $\tau_2 = \tau + m_2 \frac{T}{2}$ for some integer m_2 (namely, $m_2 = n$), and also that $\frac{T}{2}c = \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{2}$, one can apply (96) for the synchronized copies of the standard clock γ_1 and Γ_2. Therefore,

$$d_{\tau_2}(\gamma_1(\tau_2), \Gamma_2(\tau_2)) = d_\tau(\gamma_1(\tau), \Gamma_2(\tau)) = \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{2}$$

The same argument can be repeated for γ_2 and Γ_2. Therefore,

$$d_{\tau_2}(\Gamma_2(\tau_2), \gamma_2(\tau_2)) = d_\tau(\Gamma_2(\tau), \gamma_2(\tau)) = \frac{d_\tau(\gamma_1(\tau), \gamma_2(\tau))}{2}$$

It follows from (94) that

$$d_\tau(\gamma_1(\tau), \gamma_2(\tau)) \geq d_{\tau_2}(\gamma_1(\tau_2), \gamma_2(\tau_2))$$

Assume that $\tau' > \tau_2$. Therefore, take $\tau_3 := \frac{\tau_2+\tau_1}{2}$. Again, consider the synchronized copies of the standard clock Γ_3^1, Γ_3^2, Γ_3^3 dividing line segment $\sigma\left(\gamma_1(\tau), \gamma_2(\tau)\right)$ into 4 pieces of equal length. Taking into account that $\tau_3 = \tau + m_3 \frac{T}{4}$ for some integer m_3 and also that $\frac{T}{4}c = \frac{d_\tau(\gamma_1(\tau),\gamma_2(\tau))}{4}$, one can apply (96) for the synchronized copies of the standard clock γ_1 and Γ_3^1. Therefore,

$$d_{\tau_3}\left(\gamma_1(\tau_3), \Gamma_3^1(\tau_3)\right) = d_\tau\left(\gamma_1(\tau), \Gamma_3^1(\tau)\right) = \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{4}$$

Similarly,

$$\begin{aligned}
d_{\tau_3}\left(\Gamma_3^1(\tau_3), \Gamma_3^2(\tau_3)\right) &= \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{4} \\
d_{\tau_3}\left(\Gamma_3^2(\tau_3), \Gamma_3^3(\tau_3)\right) &= \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{4} \\
d_{\tau_3}\left(\Gamma_3^3(\tau_3), \gamma_2(\tau_3)\right) &= \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{4}
\end{aligned}$$

Consequently, from (94),

$$d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right) \geq d_{\tau_3}\left(\gamma_1(\tau_3), \gamma_2(\tau_3)\right)$$

Assume $\tau' < \tau_3$. Therefore, take $\tau_4 := \frac{\tau_3+\tau_2}{2}$. Again, consider the synchronized copies of the standard clock $\Gamma_4^1, \Gamma_4^2, \Gamma_4^3, \ldots \Gamma_4^7$ dividing line segment $\sigma\left(\gamma_1(\tau), \gamma_2(\tau)\right)$ into 8 pieces of equal length. Taking into account that $\tau_4 = \tau + m_4 \frac{T}{8}$ for some integer m_4 and also that $\frac{T}{8}c = \frac{d_\tau(\gamma_1(\tau),\gamma_2(\tau))}{8}$, one can apply (96) for the synchronized copies of the standard clock γ_1 and Γ_4^1. Therefore,

$$d_{\tau_4}\left(\gamma_1(\tau_4), \Gamma_4^1(\tau_4)\right) = d_\tau\left(\gamma_1(\tau), \Gamma_4^1(\tau)\right) = \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{8}$$

Similarly,

$$\begin{aligned}
d_{\tau_4}\left(\Gamma_4^1(\tau_4), \Gamma_4^2(\tau_4)\right) &= \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{8} \\
d_{\tau_4}\left(\Gamma_4^2(\tau_4), \Gamma_4^3(\tau_4)\right) &= \frac{d_\tau\left(\gamma_1(\tau), \gamma_2(\tau)\right)}{8}
\end{aligned}$$

$$\vdots$$
$$d_{\tau_4}\left(\Gamma_4^7\left(\tau_4\right),\gamma_2\left(\tau_4\right)\right) \quad = \quad \frac{d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right)}{8}$$

Consequently, from (94),

$$d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right) \geq d_{\tau_4}\left(\gamma_1\left(\tau_4\right),\gamma_2\left(\tau_4\right)\right)$$

And so on and so forth,

$$d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right) \geq d_{\tau_i}\left(\gamma_1\left(\tau_i\right),\gamma_2\left(\tau_i\right)\right)$$

On the other hand,

$$\lim_{i\to\infty}\tau_i = \tau'$$

therefore

$$d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right) \geq d_{\tau'}\left(\gamma_1\left(\tau'\right),\gamma_2\left(\tau'\right)\right)$$

Exactly in the same way one can prove that

$$d_\tau\left(\gamma_1\left(\tau\right),\gamma_2\left(\tau\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right),\gamma_2\left(\tau'\right)\right)$$

One simply has to change the roles of τ and τ'. Denote T', this time, the period

$$T' = \frac{d_{\tau'}\left(\gamma_1\left(\tau'\right),\gamma_2\left(\tau'\right)\right)}{c}$$

Let n' be the smallest integer such that $\tau > \tau' - n'T' =: \tau_1'$ Then, it follows from (96) that

$$d_{\tau'}\left(\gamma_1\left(\tau'\right),\gamma_2\left(\tau'\right)\right) = d_{\tau_1'}\left(\gamma_1\left(\tau_1'\right),\gamma_2\left(\tau_1'\right)\right)$$

Let $\tau_2' := \frac{\tau_1'+\tau'}{2}$. Consider the synchronized copy of the standard clock Γ_2' that goes through the middle point of line segment $\sigma\left(\gamma_1\left(\tau'\right),\gamma_2\left(\tau'\right)\right)$. Taking into account that $\tau_2' = \tau' - m_2'\frac{T}{2}$ for some integer m_2, and also that $\frac{T}{2}c = \frac{d_{\tau'}\left(\gamma_1\left(\tau'\right),\gamma_2\left(\tau'\right)\right)}{2}$, one can apply (96) for the synchronized copies of the standard clock γ_1 and Γ'_2. Therefore,

$$\begin{aligned} d_{\tau'_2}\left(\gamma_1\left(\tau'_2\right), \Gamma'_2\left(\tau'_2\right)\right) &= d_{\tau'}\left(\gamma_1\left(\tau'\right), \Gamma'_2\left(\tau'\right)\right) \\ &= \frac{d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)}{2} \end{aligned}$$

Similarly,

$$d_{\tau'_2}\left(\Gamma'_2\left(\tau'_2\right), \gamma_2\left(\tau'_2\right)\right) = \frac{d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)}{2}$$

Therefore,

$$d_{\tau'_2}\left(\gamma_1\left(\tau'_2\right), \gamma_2\left(\tau'_2\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)$$

And so on and so forth,

$$d_{\tau'_i}\left(\gamma_1\left(\tau'_i\right), \gamma_2\left(\tau'_i\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)$$

At the same time,

$$\lim_{i\to\infty} \tau'_i = \tau$$

Consequently,

$$d_{\tau}\left(\gamma_1\left(\tau\right), \gamma_2\left(\tau\right)\right) \leq d_{\tau'}\left(\gamma_1\left(\tau'\right), \gamma_2\left(\tau'\right)\right)$$

■

81. The following isomorphism can be regarded as a natural one.

Definition (A12)

$$\begin{aligned} \mathbb{T}_{\tau}^{\tau'} : S_{\tau} &\rightarrow S_{\tau'} \\ A &\mapsto \mathbb{T}_{\tau}^{\tau'}(A) = \gamma(\tau') \end{aligned}$$

where γ is a synchronized copy of the standard clock such that $A = \gamma(\tau)$. Let us call $\mathbb{T}_{\tau}^{\tau'}$ the *time shift* between S_{τ} and $S_{\tau'}$.

It follows from (E1) and Lemma 2 that this definition is sound and $\mathbb{T}_{\tau}^{\tau'}$ is a distance preserving bijection.

82. Now we have everything at hand to define the space tags of events.

Definition (A13) Let A be an arbitrary event. The *absolute space tags* of A are defined as follows:

$$\begin{aligned} \xi_1(A) &:= x_0\left(\mathbb{T}^0_{\tau(A)}(A)\right) \\ \xi_2(A) &:= y_0\left(\mathbb{T}^0_{\tau(A)}(A)\right) \\ \xi_3(A) &:= z_0\left(\mathbb{T}^0_{\tau(A)}(A)\right) \end{aligned}$$

Thus we have defined four absolute space-time tags for every event: $\tau(A), \xi_1(A), \xi_2(A), \xi_3(A)$.

83. For example, the *absolute velocity* of a time sequence $\gamma(\tau)$ is obviously defined as

$$\mathbf{v}(\tau) := \begin{pmatrix} \frac{d\xi_1(\gamma(\tau))}{d\tau} \\ \frac{d\xi_2(\gamma(\tau))}{d\tau} \\ \frac{d\xi_3(\gamma(\tau))}{d\tau} \end{pmatrix}$$

I omit the further (but straightforward) definitions.

84. I call $\tau(A)$ "absolute time" not in the sense of what Newton called "absolute, true and mathematical time", that is independent of any empirical definition (see Scholium II in chapter "Definitions" of the *Principia.*), but in the sense of what the 20th century physics calls absolute time; it is "independent of the position and the condition of motion of the system of co-ordinates" (Einstein 1920, p. 51). The space-time tags $\tau(A), \xi_1(A), \xi_2(A), \xi_3(A)$ are *absolute* in the sense that they are not relative to a reference frame but prior to any reference frame (actually the concept of "reference frame" is still not defined).

Our concepts of absolute time and space tags are, of course, "relative" to the trivial semantical convention by which we define the meaning of the terms. Namely, they are "relative" to the *etalon* clock-like process we have chosen in the universe. *This kind of "relativism" is however common to all physical quantities having empirical meaning.* (Beyond the choice of the *etalon* clock, the space tags $\xi_1(A), \xi_2(A), \xi_3(A)$ have some additional conventional element; they also are relative to the chosen 3-frame in S_0. This additional conventionality is, however, of marginal importance; it is nothing more than what we would call in our usual language "the choice of a 3-coordinate basis in a given reference frame".)

85. As it was already mentioned in Point **38**, there has been a long discussion in the literature about the conventionality of simultaneity. (See, for example, Reichenbach 1956; Bridgeman 1965; Grünbaum 1974; Salmon 1977; Malament 1977; Friedman 1983; Ben-Yami 2006.) Without entering in the details of the various arguments, the following facts must be pointed out here.

As it is obvious from (90), we chose the standard "$\varepsilon = \frac{1}{2}$-synchronization". (Of course, it could be a contingent fact of nature that $t_2 = t_1$ in Fig. 15. In that case the choice of the value of ε would not matter.) This choice was entirely conventional; it was *a part of the trivial semantical convention* defining the term "absolute time tag". This choice is prior to any claims about the one-way or even round-trip speed of electromagnetic signals, because there is no such a concept as "speed" prior to the definition of time and space tags; it is, of course, prior to "the metric of Minkowski space-time", in particular to the "light-cone structure of the Minkowski space-time", because we have no words to tell this structure prior to the space-time tags; and it is prior to the causal order of physical events, because—even if we could know this causal order prior to temporality—we cannot know in advance how causal order is related with temporal order (which we have defined here). It is actually prior to any discourse about two locuses in space, because there is no "space" prior to definition (A1) and there is no concept of a "persistent space locus" prior to definition (A12).

Inertial motion

86. A remark is in order on the empirical facts (E1)–(E8) to which we refer in constructing the space-time tags. In claiming these statements as empirical facts I mean that they ought to be true according to our ordinary physical theories. The ordinary physical theories are however based on the ordinary, problematic, space-time conceptions, relaying on "reference frames realized by rigid bodies" and the likes, without proper, non-circular, empirical definitions. Thus, especially in the context of defining the two most fundamental physical quantities, distance and time, we must not regard our ordinary physical theories as empirically meaningful and empirically confirmed claims about the world. Whether these statements are true or not is, therefore, an empirical question, and it is far from obvious whether they would be completely confirmed if the corresponding experiments were performed with higher precision, similar to the recent GPS measurements, especially for larger distances. Strangely enough, according to my knowledge, these very fundamental facts have never been tested experimentally; no textbook or monograph on space-time physics refers to such experimental results; actually, with a very few exceptions (for example, Milne 1935 Part I; Bridgman 1965), it is not even attempted to provide a clear, non-circular empirical definition of "time" and "distance" in one single (inertial) frame, as if it would be a problem only in the case of an accelerated observer (cf. Märzke and Wheeler 1964; Pauri and Vallisneri 2000).

So, the best we can do is to *believe* that our physical theories based on the usual sloppy formulation of space-time concepts are true (in some sense) and to consider the predictions of these theories as empirical facts. However, as the following analysis reveals, it is far from obvious whether the predictions of the believed theories really imply (E1)–(E8).

87. Throughout the definition of space-time tags, we avoided the term "inertial", and because of a good reason. First of all, if "inertial" is regarded as a kinematical notion based on the concept of straight line and constancy of velocity, then it cannot be antecedent to the concept of space-time tags. If, on the other hand, it is understood as a manner of existence of a physical object in the universe, when the object is undergoing a free floating, in other words, when it is "free from forces", then the concept is even more problematic. The reason is that "force" is a concept defined through the deviation from the trajectory of inertial motion (first circularity), and neither the inertial trajectory nor the measure of deviation from it can be expressed without spatiotemporal concepts, that is, they cannot be antecedent to the definition of space-time tags (second circularity). So there is *no* precise, non-circular definition of inertial motion. (And this is—in my view—the major difficulty with Märzke and Wheeler's (1964) "geodesic clock" approach, too.) It is to be emphasized that this operational/logical circularity is a problem even in a special relativistic/flat/local space-time.)

88. According to our believed special relativistic physical theory, space-time is a 4-dimensional Minkowski space and inertial trajectory is a time-like straight line in the Minkowski space. Since we are prior to the empirical definitions of the basic spatiotemporal quantities, we cannot regard this claim as an empirically confirmed physical theory. Nevertheless, let us assume for a moment that our special relativistic theory is the true description of the world "from God's point of view". It is straightforward to check that all the facts (E1)–(E8) are true if 1) the standard clock moves along an inertial world line in the Minkowski space-time and 2) it reads the proper time, that is, it measures the length of its own word line, according to the Minkowski metric. However, we human beings can know neither whether the standard clock (chosen by us) is of inertial motion in God's Minkowskian space-time nor whether it reads the proper time. What if these conditions fail? What does special relativistic kinematics say about (E1)–(E8) if the standard clock is accelerated and/or it does not read the proper time?

In order to answer this question, we have to follow up the operational definitions (A1), (A2),... and *calculate* whether statements (E1), (E2),... are true or not if the standard clock moves along a given world line γ and the "time" it reads is, say, a given function of the Minkowskian coordinate time, $\chi(t)$. Although the task is straightforward, the calculation is too complex to give a general answer by analytic means. But the problem can be efficiently solved by computer. One finds the following—perhaps surprising—results.

For the sake of the contrast, let me first mention that one obtains a very misguiding result if, for the sake of simplicity, the calculation is made in a *2-dimensional* Minkowski space-time:

No matter if the standard clock moves along a non-inertial world line γ, no matter if it reads a time $\chi(t)$ which is an arbitrary monotonic function of the Minkowskian coordinate time, different from the proper time along its world line, facts (E1)–(E8) are always true.

If this 2-dimensional result were the final truth, one would conclude that no spatiotemporal measurement can ascertain whether the standard clock moves inertially or not; the very concept of "inertial" motion would remain a purely conventional one; facts (E1)–(E8) would always be true, independently of the "objective" fact of how the standard clock moves in God's Minkowski space-time.

In contrast, the real *4-dimensional* calculation leads to the following results:

(A) *Facts (E1)–(E8) are always true if the standard clock moves along an inertial world line, no matter if the clock reads a time $\chi(t)$ which is an arbitrary monotonic function of the Minkowskian coordinate time, different from the proper time along its world line.*

(B) *If the standard clock moves along a non-inertial world line γ, facts (E1)–(E8) are never true, no matter if the clock reads the proper time or not.*

The whole thing hinges on (E1); there are no synchronized copies of the standard clock if the standard clock moves non-inertially.

89. There are remarkable consequences of the above results:

1. Result (A) implies that no objective meaning can be assigned to the concept of "proper" time. "Time" is what the *etalon* clock reads, by definition.

2. Contrary to the misguiding 2-dimensional result, (B) shows that the notion of "inertial motion" is not entirely conventional. In accord with our intuition based on the believed physical theories, we can give an objective meaning to "inertial motion" by means of correct—neither logically nor operationally circular—experiments: *the standard clock is of inertial motion if statements (E1)–(E8) are true.* Assuming that the standard clock is inertial, one can extend the concept for an arbitrary time sequence $\gamma(\tau)$ of events: $\gamma(\tau)$ corresponds to an inertial motion if the absolute space tags $\xi_1(\gamma(\tau)), \xi_2(\gamma(\tau)), \xi_3(\gamma(\tau))$ are linear functions of the absolute time tag τ.

3. On the basis of our believed physical theories, one cannot, however, predict whether (E1)–(E8) are true or false. It is still an open *empirical* question.

4. Imagine that (E1)–(E8) are not satisfied. It not only means that the standard clock we have chosen is non-inertial but it also means that the chosen clock is inappropriate for the definition of space-time tags. More exactly, one has to stop at definition (A1). One can define the time tags but cannot define the spatial notions, in particular the distances between simultaneous evens.

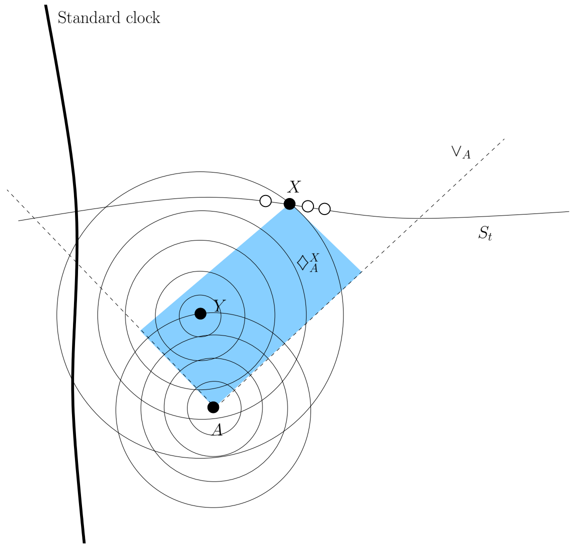

Figure 23: *The test of inertiality*

5. Consequently, it is meaningless to talk about "non-inertial reference frame", "space-time coordinates (tags) defined/measured by an accelerated observer", and the likes.

90. In the light of these consequences, it is an intriguing question whether the standard clock contemporary physical laboratories use for coordination of physical events satisfies conditions (E1)–(E8), in particular (E1). It is quite implausible that it does—taking into account the Earth's rotation, the Earth's motion around the Sun, the Solar System's motion in our Galaxy, etc.

Consider first what in fact has to be tested (Fig. 23). (E1) would require the existence of a unique synchronized copy of the standard clock through every event. Let therefore A be an arbitrary event with absolute time tag $\tau(A)$. Introduce the following notations:

$$
\begin{aligned}
\vee_A &:= \left\{ X \,\middle|\, \begin{array}{c} \text{Radio signal from } A \\ \text{is received at } X. \end{array} \right\} \\
\wedge^A &:= \left\{ X \,\middle|\, \begin{array}{c} \text{Radio signal from } X \\ \text{is received at } A. \end{array} \right\} \\
\diamondsuit_A^B &:= \vee_A \cap \wedge^B
\end{aligned}
$$

Consider the following quantity:

$$
N := \max_{t,A} \begin{cases} \min\limits_{X \in \vee_A \cap S_t} \ \max\limits_{Y \in \diamondsuit_A^X} \left| \tau(Y) - \frac{\tau(A)+\tau(X)}{2} \right| & t > \tau(A) \\ \min\limits_{X \in \wedge^A \cap S_t} \ \max\limits_{Y \in \diamondsuit_X^A} \left| \tau(Y) - \frac{\tau(A)+\tau(X)}{2} \right| & t < \tau(A) \end{cases}
$$

$N = 0$ is a necessary condition of inertiality of the standard clock. In this case, for every event A there exists a unique synchronized copy of the standard clock. That is, for every time $t > \tau(A)$ there is a unique event $X \in \vee_A \cap S_t$ such that $\tau(Y) = \frac{\tau(A)+\tau(X)}{2}$ for all $Y \in \diamondsuit_A^X$ and for every time $t < \tau(A)$ there is a unique event $X \in \wedge^A \cap S_t$ such that $\tau(Y) = \frac{\tau(A)+\tau(X)}{2}$ for all $Y \in \diamondsuit_X^A$.

91. Let us outline how the experimental test could be realized. Our standard clock is transmitting, say in every few nanoseconds, a radio signal encoding the actual clock reading (Fig. 24). We need a huge number of little devices $e_1, e_2, \ldots e_i, \ldots$ with the following functions:

1. They continuously receive the regular time signals from the standard clock.
2. They can transmit radio signals containing the following information: a) an ID code of the device and information about the standard clock reading, so from the signal they send it always can be known which device was the transmitter and what was the standard clock reading received

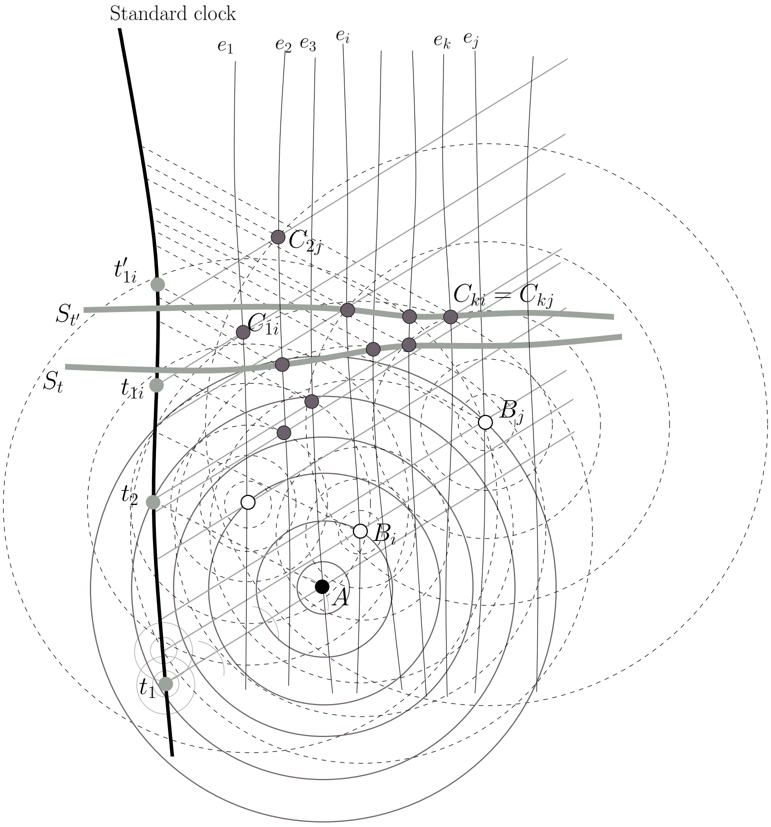

Figure 24: *The sketch of a realistic measurement to decide whether the standard clock is inertial or not*

by the transmitter at the moment of the emission of the signal, b) information about the type of event on the occasion of which the signal was transmitted.

3. They can receive the signals transmitted by the others.

We install these devices everywhere in a certain region of the universe. Now, the following events will happen.

1. Assume that e_3 is programed such that it transmits a radio signal (event A) when receives the time signal of t_1 from the standard clock. Let us call it A-signal. The A-signal will arrive back to the standard clock at time t_2.

2. The A-signal sweeps through the whole region and triggers the other devices to transmit a B-signal. For example, event B_i consists in that e_i receives the A-signal from e_3 and emits its own B_i-signal with the needed information. B_j is a similar event for e_j, etc.

3. The B-signals will be received by some other devices. For example, C_{1i} is the event when e_1 receives the B_i-signal transmitted by e_i and sends out his own signal (C_{1i}-signal) with the corresponding information. This information arrives back to the standard clock at time t_{1i}.

In this way, a huge amount of data is recorded, from which we can ascertain the absolute time tags of all events in question. We can determine $\diamondsuit_A^{C_{lm}}$ for every C_{lm}. For example, say, it turns out that $C_{ki} = C_{kj}$ and, therefore, $B_i, B_j \in \diamondsuit_A^{C_{ki}}$, etc. One also can determine the sets of simultaneous events. Now, the standard clock is inertial only if in every S_t there is a unique $C_{lm} \in S_t$ such that for every event $B_i \in \diamondsuit_A^{C_{lm}}$

$$\tau(B_i) = \frac{\tau(A) + \tau(C_{lm})}{2}$$

92. Assume, for example, that the center of Earth is at rest in the Minkowski space-time, and the standard clock is located at the equator, that is, it is orbiting together with the given point of the surface of the spinning Earth. Computer simulation shows, that this non-inertial motion of the standard clock causes a discrepancy of $N \approx 0.1\,\mathrm{ns}$ (10^{-10} second) from inertiality within a region of size 1 light-minute around the Earth, which is, in principle, an observable effect within the Solar System. Of course, the discrepancy increases with the distances. (The relevant value is about $3 \cdot 10^{-4}$ second within a region of size 1 light-day.)

93. One must recall, however, that the above calculation is merely a kind of "metaphysical" speculation without any empirically confirmed basis. It is based on the assumption that our world is a Minkowski space-time in which the standard clock moves in a certain way; but there is no empirical evidence for this assumption. Not because of the possible gravitational effects (Minkowski space-time is only an approximation—according to our believed theories), but because of the logical/operational circularity: in order to confirm or falsify that Minkowskian geometry (or some general relativistic space-time geometry) is the true theory describing all relationships between the space and time tags of all physical events, we need to know, first, how to ascertain, empirically, the space and time tags.

Again, whether or not the standard clock used in contemporary physics satisfies conditions (E1)–(E8) is still an open empirical question.

The life in absolute space and time

94. Nevertheless, assume that the empirical facts (E1)–(E8) hold. Let us also assume the following:

Empirical fact (E9) The empirically confirmed laws of physics (expressed, of course, in terms of absolute space and time) are exactly the same as the ordinary laws of (special) relativistic physics, expressed in one single space-time coordinates, namely in the ones we called absolute space and time tags (i. e., "in the frame of reference of the standard clock", in our usual relativistic terms).

If so, then, as it can be easily seen, the whole relativistic physics can be reconstructed within the framework of absolute space and time. As an example, consider how a moving observer describes the "space" and "time" coordinates of an event in his/her own "frame of reference". (I use the term "reference frame" only symbolically. The concept of reference frame as a rigid system of material points—rigid body of a spacecraft, three orthogonal rigid rods co-moving with the observer, etc.—is a vague and very problematic notion which ought to be expelled from the conceptual vocabulary of physics.) We will assume that the observer moves along a time sequence the absolute velocity of which is smaller than the speed of light. Now, imagine that the observer has a clock-like device and, naively, performs exactly the same operational procedure as (A1)–(A13). If (s)he can go through all the steps, then—according to assumption (E9) and Point **88**—(s)he is an inertial observer. (Otherwise it would be meaningless to talk about the "space" and "time" coordinates in his/her "frame of reference".) What can we say about the "space" and "time" tags so obtained?

Of course, we can say nothing in the general case when the observer's device has nothing to do with the standard clock. Assume, however, that the observer's clock-like device is an identical copy of the standard clock, which was gently accelerated up to the velocity of the observer; therefore, it is almost like a clock, except that it runs slower by the factor $\sqrt{1-\frac{v^2}{c^2}}$, due to assumption (E9). In this case, the observer obtains the same result as one would obtain from the Lorentz transformation. Let me illustrate this with a simple two-dimensional calculation.

Imagine that a radio signal is emitted (event B) when the observer meets the standard clock (Fig. 25). Let $\tau(B) = 0$.

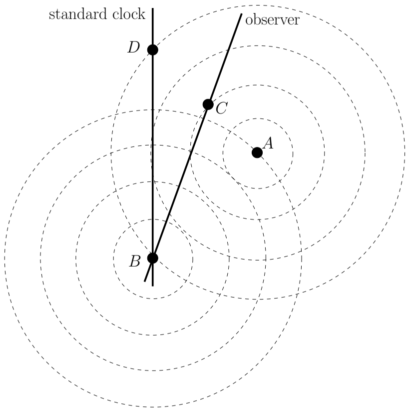

Figure 25: *When the observer meets the standard clock, a radio signal is emitted (event B). Event A is marked with the reflection of the signal. The reflected signal first arrives at the observer (event C) and then at the standard clock (event D)*

Event A is marked with the reflection of the signal at time $\tau(A)$. The reflected signal first arrives at the observer (event C) and then at the standard clock (event D). By definition, $\tau(A) = \frac{\tau(D)}{2}$. We know that

$$v\tau(C) = \xi(A) - c\,(\tau(C) - \tau(A))$$

where, by definition, $\xi(A) = c\tau(A)$. Therefore,

$$\tau(C) = \frac{2c\tau(A)}{c+v}$$

Taking into account assumption (E9), the observer's "clock"-reading at C is

$$t(C) \quad = \quad \tau(C)\sqrt{1-\frac{v^2}{c^2}}$$

Therefore, the "time" and "space" coordinates (s)he obtains is

$$t(A) \quad = \quad \frac{t(C)}{2} = \frac{c\tau(A)}{c+v}\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{\tau(A) - \frac{v}{c^2}\xi(A)}{\sqrt{1-\frac{v^2}{c^2}}}$$

and

$$x(A) = \frac{\xi(A) - v\tau(A)}{\sqrt{1-\frac{v^2}{c^2}}}$$

The "time" and "space" coordinates defined in this way are nothing but the $\widetilde{\text{time}}$ and $\widetilde{\text{space}}$ coordinates $\left(\tilde{t}^{K'}(A), \tilde{x}^{K'}(A)\right)$ in Point **38**. (Note that the above derivation—without reference to the behavior of a rigid measuring-rod—could replace the similar calculation in Point **42**.)

95. How can a moving observer ascertain the absolute time and space tags of an arbitrary event A (in order, for example, to assign to A the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\left(\hat{t}^{K'}(A), \hat{x}^{K'}(A)\right)$ defined in Point **38**)? This is actually very easy. For that we only need to equip the standard clock and the marking devices with functions similar to the ones described in Point **91**. In addition, let the standard clock be continuously writing and broadcasting a "log file", containing all the relevant information: when a signal was transmitted and when it was received back from which marker, etc. By reading off this "log file", the remote moving observer can reconstruct the absolute time and space tags of all events.

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